John Theymer

1609/345.

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THE

John Heymer

THE ACCOMPLISHED TUTOR.

CHAP. VII.

Of ALGEBRA.

SECT. I.

OF NOTATION.

Difinitions.

1. ALGEBRA is the art of reſolving difficult queſtions, incapable of being reſolved by the rules of common arithmetic.

In Algebra the value of quantities are expreſſed by ſome letters of the alphabet, which have ſometimes figures, and certain characters added to them, whereby their value is increaſed or diminiſhed; and each letter may repreſent any quantity at pleaſure. But generally, the firſt letters in the alphabet *a*, *b*, *c*, *d*, &c. are uſed to ſignify quantities, the value whereof is known; and the latter letters, as, *w*, *x*, *y*, *z*, &c. for quantities which are unknown; the letters are then managed according to the rules of the art.

2. The ſign + ſignifies *addition*, and in algebra it is called *plus*, it denotes that the characters or letters placed on each ſide of it, are to be added together, thus, $a + b$ ſignifies that the quantity expreſſed by *a* is to be added to that repreſented

 by

by b. Thus, if a ſtand for 3, and b for 6, the ſum of theſe two added together is 9.

3. The ſign — ſignifies ſubtraction: thus, $a - b$ ſignifies that the quantity repreſented by b, is to be ſubtracted from that repreſented by a; as if a was 8, and b was 3, then $a - b$ would be equal to 5: this ſign is called *minus*.

4. The ſign + repreſenting addition, is called a *poſitive*, or an *affirmative* ſign. The ſign — ſubtraction is called a *negative* ſign.

5. Like ſigns are when ſeveral quantities have all the ſign + or —; and unlike ſigns are quantities where ſome have the ſign +, and others the ſign —

6. The ſign = denotes equality, and is placed between two quantities to ſhew they are equal: thus, $a=b$ ſignifies that a and b are equal to each other.

7. The ſign × ſtands for multiplication, and ſignifies that the quantities placed on each ſide are to be multiplied together: thus, $a \times b$ ſignifies the quantity a is to be multiplied by the quantity b, as if a be 5, and b 6, they will, with the product, ſtand thus $5 \times 6 = 30$, which ſignifies that 5 multiplied by 6 is equal to 30. But the product of two or more ſimple quantities is generally reckoned by merely joining the letters. Thus, the product of the above quantity is expreſſed $a\,b$, and if there be three or more quantities to be multiplied together, as $a \times b \times c$, they will be expreſſed by $a\,b\,c$.

8. The ſign ÷ expreſſes diviſion: thus, $a \div b$ ſignifies that a is to be divided by b, but this ſign is not much uſed; for diviſion is generally expreſſed in the manner of a fraction: thus, $\frac{a}{b}$ and $\frac{a-b}{c+d}$ ſignifies that a is to be divided by b, and $a - b$ divided by $c+d$.

9. The ſign ∽ ſignifies the difference between two quantities: thus, $a \backsim b$ ſtands for the difference between a and b.

10. The ſign ⊏ or ⁊ are ſigns of majority, and ſhews that the quantity placed before the ſign is greater than that which follows it: thus, $a \sqsubset b$ ſhews that a is greater than b.

11. The

11. The sign ⌐ or ∠ signifies minority, and shews that the quantity placed before the sign is less than that which follows it; thus, $a \angle b$ signifies that a is less than b.

12. The sign $\surd$ is the sign of the square root. It also expresses the Cube root, Biquadrate root, &c. by placing 3, or 4 &c. over it; thus, $\surd a$ $\sqrt[3]{a}$ $\sqrt[4]{a}$ and denote the square root, cube root, and biquadrate root of a respectively.

13. Involution is the raising of a quantity to any power, according as it is joined to the figures 2, 3, 4, &c. respectively.

14. The sign *lu* signifies evolution, and denotes that the quantity to which it is joined is the square and cube root &c. as it is joined to the numbers 2, 3, &c. respectively.

The power of a quantity is often expressed in Algebra by placing a figure over the quantity; thus, a^2, a^3, and a^4, denotes the square cube and biquadrate of a respectively, or the second, third, and fourth power; and the figures 2, 3, and 4, placed over a, are called the indices or exponents of a.

15. Like quantities are those that consist of the same letters as a, $4a+2a-3a$ and $b-2b+3bb$, &c.

16. Unlike quantities consist of different letters; as, a, $2b$ $3c$; or $2a$, $cd-d$.

17. Simple quantities consist of one term only; as, $4b$, $3a^2$, $12d$, &c.

18. Compound quantities consist of several terms; as, $a+c$, $2b-d$, &c.

19. A Line drawn over several quantities, shews that they are to be taken as a compound quantity; as, $\overline{a+b-c}$.

20. The Coefficient of a quantity is the number prefixed to it; as, $6d$; here 6 is the coefficient, and signifies that the quantity d is multiplied thereby.

21. A Binomial Quantity consists of two terms; as, $b+c$. A Trionomial Quantity of three terms; as, $a+b+c$. A Quadrinomial Quantity, of four terms; as, $a+b+c+d$.

22. A Refidual Quantity is a binomial, where one of the terms is a negative one; as, $a-b$.

23. A Rational Quantity has no radical fign.

24. A Surd Quantity, is that which has not a proper root, as, the fquare root of b, ($\sqrt{b}$,) the cube root of bb ($\sqrt[4]{bb}$.)

25. The fign : :: : fignifies proportion; as, 5 : 10 :: 40:80 that is, as 5 to 10, fo is 40 to 80.

26. An Equation, is the comparifon of two quantities which are equal to one another, having the fign of equality between them, of which there are feveral forts:

1. A Dependant Equation, which may be deduced from fome others. 2. An Independant Equation, which cannot be deduced from another. 3. A Pure Equation which contains but one power of the unknown quantity. 4. An Affected Equation, which has feveral powers of the unknown quantity.

Axioms.

1. If equal quantities be added to equal quantities, the fums will be equal. And if equal quantities be taken from equal quantities, the remainders will be equal.

2. If equal quantities be multiplied by equal quantities, the products will be equal. And if equal quantities be divided by equal quantities, the quotient will be equal.

3. If equal quantities be raifed to equal powers, the products will be equal.

4. Quantities equal to any other quantity, are equal to one another.

5. The whole is equal to all its parts taken together.

SECT.

SECT. II.

OF THE FOUR SINGLE RULES OF ALGEBRA.

Of ADDITION.

Rule 1. When like quantities having like ſigns, are to be added together, add together the coefficients, (if there be any,) and to the ſum prefix the ſign, and ſubjoin the common quantity.

2. When the quantities are like, but have unlike ſigns, take the difference between the ſum of the affirmative coefficients, and the ſum of the negative ones, to which difference prefix the ſign of the greater ſum, and annex the common quantity.

3. But when the quantities are all unlike, they cannot be brought into one ſum; but muſt be wrote down one after another; prefixing to each, his proper ſign; as in the following examples:

$7ab$		$12a^2c+2ab$		$-10+{}^2y^2-2a^2$	
$12ab$		$9a^2c+5ab$		$-7x^2y^2-7a^2$	
$9ab$		$10a^2c+2ab$		$-4x^2y^2-2a^2$	
$2ab$		$4a^2c+3ab$		$-31x^2y^2-a^2$	
$30ab$	Sum	$35a^2c+12ab$	Sum	$-52x^2y^2-12a^2$	Sum

When there is no coefficient prefixed to a quantity, the coefficient is 1. And when there is no ſign prefixed, the quantity is affirmative, as in the firſt quantities of the firſt of the foregoing examples:

Examples

Examples of like Quantities, and unlike Signs.

$$\begin{array}{l} -3ac-7cd+de \\ +7ac+4cd-de \\ -6ac-2cd+de \\ \hline -2ac-5cd+de \end{array} \qquad \begin{array}{l} +12a^2b^2-de-12x^2 \\ +\ 4a^2b^2+de+6\ x^2 \\ +\ 5a^2b^2-de-10x^2 \\ \hline +21a^2b^2-de-16x^2 \end{array}$$

Unlike Quantities, and unlike Signs.

$$\begin{array}{l} +15a-6a^2-2b^2+5cd \\ -\ 2de+4xy-x^2\ +xy \\ +3b^2-2x\ +y^2 \\ \hline +15a-6a^2+b^2+5cd-2de+5xy-x^2-2x+y^2 \end{array}$$

SUBTRACTION.

RULE. Place the quantities one under the other, and change all the ſigns of the ſubtrahend, that is where there is an affirmative ſign, place a negative one and *vice verſa.* Then add the quantities together as in addition.

EXAMPLES.

$$\begin{array}{ll} \text{From} & +a^2b^2-cd \\ \text{Take} & +a^2b^2-2c \\ \hline & +a^2b^2-2cd \end{array} \qquad \begin{array}{ll} \text{From} & -ab+6a^2-9cd \\ \text{Take} & +4ab-2cd+x^2 \\ \hline & -ab+6a^2-7cd-x^2 \end{array}$$

Subtraction, as well as each of the other four rules in Algebra, is proved in the ſame manner as in common arithmetic*.

MULTIPLI-

* The reaſon of this rule is evident from hence, that if a decrement or a negative quantity be taken away from an affirmative quantity, the remainder will be the ſame as if an increment or an affirmative quantity of equal quantity be added to the original quantity. For every negative quantity always decreaſes the value of any quantity with which it is joined. Thus, if --b be taken from a--b there will remain --a, and if +b, be added to a--b, the ſum is likewiſe a.

MULTIPLICATION.

RULE. Multiply each term of the multiplier into every term of the multiplicand. That is, the coefficients into the coefficients, and the letters into the letters, and to each prefix its sign; viz. + to like signs, — to unlike signs.

EXAMPLES.

Multiply	$7a$	Multiply	$3a^2$	Multiply	$-5c^2a^2$
By	$3b$	By	$5c^2$	By	$+7c^2a^2$
Product	$21ab$	Product	$15a^2c^2$	Product	$35c^4a^4$

Mult. $7a-3c$
By $12a+4c$
$84aa+28ac$
$+28ac-12cc$
Pro. $+84aa-56ac-12cc$

Multiply a^2-ab+c^2
By $-a+2b^2$
$-a^3+a^2b-ac^2$
$+2a^2b^2-2ab^3+2c^2b^2$
P. $a^3+a^2b+2a^2b^2-ac^2-2ab^3+2c^2b^2$

The foregoing examples may be proved by division, as in common arithmetic*.

DIVISION.

* This Rule depends upon the same principle as Multiplication in common arithmetic. And that two quantities having like signs, should give a product with the sign +, and two quantities of unlike signs, the sign —, may be proved from hence; viz. 1. If an affirmative quantity be multiplied by an affirmative quantity, the product must of course be an affirmative quantity. 2 If a negative quantity be multiplied by an affirmative one, the negative quantity must be taken as often as there are units in the affirmative one, and the sum of any number of negative quantities will be negative. And if an affirmative quantity be multiplied by a negative one, the affirmative quantity must be subtracted as often as there are units in the negative one, and the sum of any number of negatives will be negative 3. Again, if a negative quantity be multiplied by a negative quantity, the multiplicand is to be subtracted as often as there are units in the multiplier; but, to subtract a negative quantity, is the same thing as to add an equal affirmative one. Therefore, the product will be affirmative, from hence, the general rule; that like signs produce + and unlike signs —.

DIVISION.

Rule. If the quantities be ſimple, divide the coefficient of the dividend, by the coefficient of the diviſor, and place the anſwer in the quotient, annexing thereto thoſe letters in the dividend, which are not found in the diviſor, obſerving that like ſigns produce $+$, and unlike ſigns $-$.

2. But if the quantities be compound, divide the firſt term of the dividend by the firſt term of the diviſor, and place the reſult in the quotient. Then multiply the whole diviſor thereby, and ſubtract the product from the dividend, and to the remainder bring down the next term in the dividend. And repeat the operation as in common arithmetic. But the terms in the dividend ſhould be ranged in a proper order, that is according to the dimenſions of ſome letter.

EXAMPLES.

$$\begin{array}{l} 9ac)\,36abcd\,(4bd \\ \quad \underline{36abcd} \\ \qquad\quad 0 \end{array} \qquad\qquad \begin{array}{l} 4a^2b^2)-24a^2b^3d\,(-6bd \\ \qquad\quad \underline{24a^2b^3d} \\ \qquad\qquad\quad 0 \end{array}$$

$$\begin{array}{l} 4a-bc)\,16abb-24cbb\,(4bb \\ \qquad\quad \underline{16abb-24cbb} \\ \qquad\qquad\qquad\quad 0 \end{array}$$

$$\begin{array}{l} 3a-b)\,3a^3-12a^2-ba^2+10ab-ab2\,(a^2-4a+2b \\ \qquad \underline{3a^3 \qquad\qquad ba^2} \\ \qquad\quad -12a^2 \qquad +10ab \\ \qquad\quad \underline{-12a^2 \qquad +\ 4ab} \\ \qquad\qquad\qquad\quad +\ 6ab-2b^2 \\ \qquad\qquad\qquad\quad \underline{+\ 6ab-2b^2} \\ \qquad\qquad\qquad\qquad\qquad 0 \end{array}$$

When

When the divifor will not exactly divide the dividend, as is often the cafe; the dividend is to be placed over the divifor, and a line drawn between them like a fraction throwing out fuch letters as are found in both the divifor and dividend. Thus, If $ab+c^3$ was to be divided by $a\,b-c^2$, it would ftand thus, $\frac{ab+c^3}{ab-c^2}=\frac{+c^3}{-c^2}$. When the power of a quantity is to be divided by any other power of the fame quantity, it is done by fubtracting the exponent of the divifor, from that of the dividend:

Thus, $\frac{5b^8}{b^4}=b^4$*

SECT. III.

OF FRACTIONAL QUANTITIES.

BEFORE the ftudent proceed to equations, it is neceffary that he know how to manage fractional quantities, and to raife a quantity to any given power; and, on the contrary, to extract the root of any quantity, to manage furd quantities, &c.

The rules for managing Algebraic fractions are exactly the fame as thofe for Vulgar Fractions in arithmetic, and therefore, need not be repeated; as few perfons would attempt algebra, till they were fufficiently fkilled in common arithmetic. An example or two, may, however, be of fervice.

* To prove the reafon of this rule, that like figns give +, and unlike figns —, it is only neceffary that the divifor be multiplied by the quotient and the product will be equal to the dividend.

EXAMPLE 1. Reduce the mixed quantity $a\frac{-c}{d}$, to a fraction. Here $\frac{da-c}{-d}$, is the fraction required.

EXAMPLE 2. Let the fraction $\frac{bd-b^2}{d}$, be reduced to a mixed quantity. Here dividing $bd-b^2$ by d, the quantity will be b, and the remainder $-b^2$, and therefore the mixed quantity will be $b-\frac{b^2}{d}$.

EXAMPLE 3. Reduce the fractions $\frac{a}{b}$ $\frac{c}{d}$ and $\frac{e}{f}$ to fractions of the ſame value, having a common demonſtrator. Here $\frac{adf}{bdf}$ $\frac{cbf}{bdf}$ $\frac{ebd}{bdf}$ are the fractions required.

Involution; or to find any given Power of any given Quantity.

RULE. Multiply the quantity into itſelf as often as the index contains units, except one, and the laſt product will be the required power; or, which is more convenient, multiply the index of the quantity by the index of the power.

Thus, let it be required to raiſe the quantities $b+c$ and $b-c$ to the third power, or the cube.

$$\begin{array}{ll}
b+c & \text{root} \\
b+c & \\
\hline
b^2+bc & \\
\quad +bc+c^2 & \\
\hline
b^2+2bc+c^2= & \text{Square} \\
b+c & \\
\hline
b^3+2b^2c+bc^2 & \\
\quad b^2c+2bc^2+c^3 & \\
\hline
b^3+3b^2c^2+3bc^2+c^3= & \text{Cube.}
\end{array}$$

$b-c$ Root

$$\begin{array}{ll}
b-c & \text{root} \\
b-c & \\
\hline
b^2-bc & \\
\quad -bc+c^2 & \\
\hline
b^2-2bc+c^2= & \text{Square} \\
b-c & \\
\hline
b^3-2b^2c+bc^2 & \\
\quad -b^2c+2bc^2-c^3 & \\
\hline
b^3-3b^2c^2+3bc^2-c^3 & \text{Cube}
\end{array}$$

These quantities are raised to a third power only, but by the same method of proceeding, quantities may be raised to any higher power.

If a trinomial, or quadrinomial, &c. be required to be raised to any power, it will be best done by taking the first or the last term of the quantity, and for all the other terms substitute any single term. These two terms being raised to the required power, the answer will be obtained by replacing, instead of the substituted term, the proper value.

Thus, if it be required to raise $b-c+d-ef+g$ to the fourth power, for the terms $b-c+d-ef$, substitute the term a, then the quantity will be $a+g$, which being raised to the fourth power, the quantity a is to be taken away, and the proper value placed instead thereof in the product, which the learner may prove at his leisure.

In involing a fractional quantity, both the numerator and denominator must be raised to the required power, by which a new fraction will be obtained, being the answer of the question. Thus, the third power $\frac{a^2}{b}=\frac{a^6}{b^3}$.

The *Binomial Theorem*, invented by *Sir Isaac Newton*, is the most elegant and concise method of raising a quantity to any power, and is as follows; let n denote any number at plea-

ſure, and let $a + b$ be a binomial, then the n*th* power thereof will be as follows:—

$$\text{by } a^n + na^{n-1}b + \frac{n\,.\,n-1}{1\,.\,2}.a^{n-2}b^2 + \frac{n.n-1.n-2}{1.\,2.\,3}a^{n-3}b^3$$

$$+ \frac{n.n-1.n-2.n-3}{1\,.\,2\,.\,3\,.\,4.}.a^{n-4}b^4 + \frac{n.n-1.n-2.n-3.n-4}{1\,.\,2\,.\,3\,.\,4\,.\,5}.a^{n-5}b^5, \text{ \&c.}$$

And the n*th* power of $a-b$, is expreſſed in the ſame manner, except that the ſigns of every other term will be negative.

To illuſtrate this theorem, let $a+b$ be involved to the third power. In this caſe, the index is 3, which muſt be placed in the theorem inſtead of n, then the firſt term will be a^3, the ſecond term $a^{3-1}b=3a^2b$; the third term, $\frac{3\times2}{2}a^{3-2}b^2=3ab^2$, the fourth term $\frac{3\times2\times1}{2\times3}a^{3-3}b^3=b^2$, the fifth and following terms are equal to nothing. Therefore theſe four terms together, or the third power of $a+b$ is $a^3+3a^2b+3ab^2+b^3$.

It may here be obſerved that the coefficients increaſe till the indices of the two letters a and b become equal or change values; then they return or decreaſe again in the ſame order; thus, having the coefficients of half the terms, the reſt are known.

Evolution; or, to extract the Root of the given Power.

RULE I. If the quantities be ſimple, extract the root of the coefficient for the new coefficient, and divide the index of the letters by the index of the power, and the quotient will be the root required.

Thus, the ſquare root of $25b^2$ will be $5b^2=4b$, and the cube root of $64x^3=3x^3=3x$.

RULE

RULE 2. If the quantities are compound after ranging the terms according to the dimenſions of ſome letter, ſo that the higheſt power of that letter may ſtand firſt in order, and the lower powers of the ſame letter follow, according to the dimenſions of their power; take the root of the firſt term, and place it in the quotient, and if it be the ſquare or cube root, ſubtract the ſquare or cube thereof from the firſt term, bring down two or three of the next terms for the dividend, according as the caſe ſhall be the ſquare or cube root, then proceed to find the diviſor as in extracting the ſquare or cube root in common arithmetic. And if the root of a higher power is to be extracted, it is performed in the ſame manner as in common arithmetic.

EXAMPLES.

EXAMPLE 1. What is the ſquare root of

$$
\begin{array}{rl}
 & 36x^4+108x^2+81(6x^2+9 \\
 & 36x^4 \\
\hline
12x^2+9) & +108x^2+81 \\
 & +108x^2+81 \\
\hline\hline
\end{array}
$$

EXAMPEE 2. What is the ſquare root of $9x^4+16a^4-4b^4-24a^2x^2+12b^2x^2-16a^2b^2$. Anſwer $3x^2-4a^2+2b^2$,

EXAMPLE 3. What is the cube root of

$$
\begin{array}{rl}
 & x^3-6x^2y+12xy^2-8y12xy^2(x-2y \quad \text{Root} \\
 & x^3 \\
\hline
3x^26xy+4y^2) & -6x^2y+12xy^2-8y^3 \\
 & -6x^2y+12xy^2-8y^3 \\
\hline\hline
\end{array}
$$

Surd Quantities.

When a quantity has not a perfect root, it is called a ſurd quantity; and the root cannot be expreſſed any other way, then by either inſerting the quantity with its proper radical ſign, or throwing it into an infinite ſeries. Thus, the ſquare root

root of a, can be expreſſed in no other way than by $\sqrt{a}$, or $a^{\frac{1}{2}}$, the cube root of b^2 by $\sqrt[3]{b^2}$ or $b^{\frac{2}{3}}$, the cube root of $\frac{a^2b}{c^2}$ by $\sqrt[3]{\frac{a^2b}{c^2}}$.

To Reduce a Rational Quantity to the form of a Surd.

RURE. Multiply the index of the quantity by the index of the ſurd, and over the product place the radical ſign, and it will be the form required. Thus, let 3 be reduced to the form of a ſurd. Here $3^1 \times {}^2$, or $3^2 = 9$, therefore, $\sqrt{9}$ is the ſurd requird.

Again, let a^2 be reduced to the form of a cube ſurd or $\sqrt[3]{b}$. Here $a^2 \overset{3}{\times} = 2^6$ and $\sqrt[3]{a^6}$ is the ſurd quantity.

To Reduce Quantities of different Indices to othe Quantities equal in value, and having one given Index.

RULE. Divide the indices of the quantities by the given index, and the quotient will be the new indices of thoſe quantities. Then over the ſaid quantities place the given index, and they will be the equivalent values required.

EXAMPLE. Let $12^{\frac{1}{2}}$, and $9^{\frac{1}{4}}$, be reduced to equal quantities, having the commom index $\frac{1}{3}$. Here $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2} =$, the index of the firſt quantity, and $\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} =$, the index of the ſecond quantity.

Reduce a Surd Quantity to its most simple Terms.

RULE. Divide the ſurd by the greateſt power which it contains, and place the root of ſuch power before the quotient with the radical ſign between them.

Thus,

Thus, let $\sqrt{32}$ be reduced to its moſt ſimple terms. Here 16 is the greateſt ſquare contained in 32, which being divided by 16, the quotient is 2; therefore, $4\sqrt{2}$ is the ſurd required.

Again, the moſt ſimple terms of the ſurd $\overline{81^2ba)}^{\frac{1}{2}}$ is $9a\sqrt{b}$.

To Find whether Surds are Commensurable, or not.

RULE. Reduce the ſurds to the leaſt common index; and the quantities, if fractions to a common denominator, except, when like terms are commenſurable; then divide them by the greateſt common diviſor, or by ſuch a diviſor as will give one rational quotient, and if both the quantities are rational, the ſurds are commenſurable; otherwiſe, not.

EXAMPLE. Let $\sqrt{30}$ and $\sqrt{18}$ be given to find whether they are commenſurable, theſe two ſurds have already one common index, and are equal to $\sqrt{3\times10}$, and $\sqrt{3\times6}$, reſpectively. Therefore, divide 30 and 18 by 3, and the quotients are 10 and 6, that is, 5 and 3; therefore, they are commenſurable.

To Add, or Subtract Surd Quantities.

RULE. If the quantities have unlike indices, reduce them to quantities with like indices; and fractional quantities muſt be reduced to a common denominator, or to other fractions that have rational denominators or numerators; then reduce the quantities to their ſimpleſt terms, and if the ſurd part be the ſame, in all, annex it to the ſum or difference with the ſign $\times$, but if the ſurd part is not the ſame in all, the quantities muſt be added or ſubtracted by joining them together with the ſign + or —.

EXAMPLE

EXAMPLE 1. Let $\sqrt{32}\sqrt{72}$ be added together. Here $\sqrt{32} = \sqrt{\overline{16\times 2}} = 4\sqrt{2}$; and $\sqrt{72} = \sqrt{36\times 2} = 6\sqrt{2}$, and the ſum $= \overline{4+6}\times\sqrt{2} = 10\sqrt{2}$.

EXAMPLE 2. Let $\sqrt{4a}$, and $\sqrt[4]{a^6}$ be added together. Here $\sqrt{4a} = \sqrt[4]{\overline{16a^2}} = 2\sqrt[4]{a^2}$ and $\sqrt[4]{a^6} = \sqrt[4]{\overline{a^4\times a^2}} = a\sqrt[4]{a^2}$; therefore, their ſum $= \overline{a+2}\times\sqrt[4]{a^2} = \overline{a+2}\times\sqrt{a}$. If it were required to have ſubtracted $\sqrt{4a}$ from $\sqrt[4]{a^6}$, the remainder would have been $\overline{a-2}\times\sqrt{a}$, alſo if $\sqrt[3]{a^2} - \sqrt{a^3} + \sqrt{7}$ be added to $2\sqrt[3]{a^2} + \sqrt{a} - \sqrt{}$, the ſum will be $3\sqrt[3]{a^2} - \sqrt{a^3} + \sqrt{7} + \sqrt{a} - \sqrt{3}$.

To Multiply, and Divide Surds.

RULE. Reduce the ſurds to the ſame index; and the product or quotient of the rational quantities being annexed to the product or quotient of the ſurds, will give the product or quotient required.

EXAMPLE 1. Multiply $2\sqrt{2}$ by $3\sqrt{3}$, theſe ſurds have the ſame index already, therefore, $2\times 3=6$ and $2\times 3=6$; thus, $6\sqrt{6}$ is the product required.

EXAMPLE 2. Multiply $a^{\frac{1}{3}}$ by $b^{\frac{1}{4}}$. Here $a^{\frac{1}{3}} = \overline{a^4}|^{\frac{1}{12}}$, and $b^{\frac{1}{4}} = \overline{b^3}|^{\frac{1}{12}}$, therefore, their product is $\overline{a^4b^3}|^{\frac{1}{12}}$.

EXAMPLE 3. Let $x^{\frac{1}{2}}$ be divided by $\overline{x^{\frac{1}{3}}+y^{\frac{1}{2}}}$, this is the ſame as if the dividend $x^{\frac{1}{2}}$ was multiplied by $x^{\frac{1}{3}}+y^{\frac{1}{2}}$; therefore, the quotient $\overline{\dfrac{x^3}{x^2+y^3}}\Big|^{\frac{1}{6}}$.

Involution, or Evolution of Surd Quantities.

RULE. If the ſurd be a ſimple quantity, multiply the index of the quantity by the index of the power, to which the ſurd is to be involved; or by the fraction, expreſſing the root to which it is to be involed, and if there is a rational part its proper power or root is to be prefixed thereto.

Compound

Compound surds are involved and extracted as integers or rational quantities, having regard to the operation of simple surds.

EXAMPLE 1. What is the square of $a\ \sqrt{x}$. Here the square of $a=aa=a^2\sqrt{x}=x$; therefore, the square of $a\sqrt{x}=a^2x$.

EXAMPLE 2. What is the cube of $a\sqrt{x^{\frac{5}{2}}}$: Here the cube of $a=a^3$, and the cube of $\sqrt{x^{\frac{5}{2}}}=\sqrt{x^{15}}$, therefore, the cube of $a\sqrt{x^{\frac{5}{2}}}=a^3\sqrt{x^{15}}$.

EXAMPLE 3. What is the cude root of $\overline{a+x}\big|^{\frac{1}{2}}$. Here $\overline{a+x}\big|^{\frac{1}{2}}=\overline{a+x}\big|^{\frac{1}{2}\times\frac{1}{3}}=\overline{a+x}\big|^{\frac{1}{6}}$.

SECT. IV.

OF EQUATIONS.

AN EQUATION is the mutual comparing of two equal quantities, having the sign $=$ between them. Thus, if a be equal to 3, and b to 6, and c to 4, and d to 13, then a added to b will be equal to d made less by c, and is thus expressed in algebra $a+b=d-c$.

To Reduce an Equation.

When a question is brought to an Equation, in order to understand the value thereof, the quantity or quantities sought must be placed on one side of the equation, and the known quantities on the other side. For this purpose, the following rules must be attended to:—

First. When any quantity is expressed in the same manner, in both sides of the equation it may be intirely rejected or thrown out of both. Thus, if $3d+7x=4c-2b+7x$. Here $7x$ should be rejected from both sides of the equation, then it will stand thus, $3d=4c-2b$.

Second. When known and unknown quantities are both on the same side of the equation, the known quantities must be transposed to the contrary side, and the signs changed. That is, those which have the sign $+$, must, after they are transposed to the other side, have the sign $-$. And those which have the $-$, must, after transposition, have the $+$. Thus, if $10+5=x-5$, then if x be the quantity sought -5 must be transposed to the other side of the equation with the sign $+$, it will then stand thus, $+5+10+5=x$, therefore, $x=20$. Again, what is the value of x in this equation $24-4x+10=60-12x$, Here $+24$ and $+10$ in the first side of the equation be transferred to the other side, and $+12x$ in the second side of the equation; being transferred to the first side, it will stand thus, $-4x+12x=60-24-10$, and by subtracting $4x$ from $12x$, and 24 and 10 from 60, the equation will be $8x=26$, therefore, $x=3\frac{1}{4}$.

Third. If there be fractions in the equation, multiply both sides of the equation by the denominators of the fractions, and the product will be the true integral quantities.

Example 1. Reduce the fractional equation $a+\frac{b^2}{x}=c$ to integral quantities. Here by multiplying the whole by x, we shall have $ax+b^2=cx$.

Again, If there be given $\frac{a^2}{a+x}+\frac{b^2}{x}=c$, then will $a^2x+b^2\overline{a+x}=\overline{ca+x}\times x$, that is $a^2x+ab^2+b^2x+acx+cxx$.

Fourth. If in the unknown quantity there be a surd, all the other terms must be transposed to the contrary side, and each side of the equation involved according to the index of the

the furd; and if there be more furds than one, the operation muft be as often repeated as there are furd quantities. Thus, if $\sqrt{x^2+ax}+d=c$, by tranfpofing d, the equation will be fquare root $\sqrt{x^2+ax}=c-d$, and by fquaring both fides, the equation is $x^2+ax=c^2-2cd^2+d^2$, thus, the equation is freed from the furd.

Fifth. When any quantity is multiplied into both fides of the equation, or into the higheft term of the unknown quantity, divide the whole equation thereby. Thus, the equation $5bx^2=3bc$ is divided by b, and it becomes $5x^2=3c$; again if it were divided by 5 it would be $x^2=\frac{3c}{5}$.

Sixth. When the fide of the equation is a pure power of the quantity, or when it has a rational root, extract the root of both fides of the equation. Thus in the equation $a^2=b^2+ax$, the equation will be $a=\sqrt{b^2+ax}$. Again, if $x^2+4x+9=25c$ be given by taking the fquare root we have $x+2x+3\times5c$.

Each or all of the foregoing rules are to be ufed as may be neceffary, till the equation be brought to a proper form.

Examples, wherein the foregoing Rules appear.

Example 1. What is the value of x, in the equation $10+\frac{36}{12-x}=16$.

By fubtracting 10 from each fide of the equation, we have $\frac{36}{12-x}=6$, both fides of which divided by 6, the quotient is $\frac{6}{12-x}=1$, this multiplied by $12-x$, give $6=12-x$, whence by tranfpofing x and 6, we have $x=12-6$, or $x=6$.

EXAMPLE 2. What is the value of x in the equation $\frac{ax^2+ac^2}{a+x}=ax+b^2$. Here multiplying by $a+x$, there comes out $ax^2+ac^2=\overline{ax+b^2}\times\overline{a+x}$, or $ax^2+ac^2=a^2x+ab^2+ax^2+b^2x$, which transposed and ordered according to the foregoing rules is $\overline{a^2+b^2}\times x=-ab^2+ac^2$, wherefore, $x=\frac{-ab^2+ac^2}{a^2+b^2}$, so that if a $=1$, $b=2$, $c=3$, then will $x=\frac{-4+9}{1+4}=1$.

EXAMPLE 3. What is the value of x in this equation, $\sqrt{x}+\sqrt{a+x}=\frac{2a}{\sqrt{a+x}}$. Multiply the whole equation by $\sqrt{a+x}$, then it will be $\sqrt{ax+xx}+a+x=2a$; and subtracting $a+x$ from both sides it is $\sqrt{ax+xx}=a-x$; this squared, gives a $x+xx=a^2-2ax+x^2$, which reduced and transposed gives $3ax=a^2$, and consequently $x=\frac{a^2}{3a}=\frac{a}{3}$

EXAMPLE 4. What is the value of x in the equation $\sqrt{a^2+x^2}+x=\sqrt{ax+a}$; in this equation there are two irreducible surds, but $\sqrt{a^2+x^2}$, being the most compounded, I throw it out first by transposing x in the first side of the equation, and then squaring the equation, and bringing all the rational terms on one side, and contracting, we have $ax=2\sqrt{ax}\times\overline{a-x}$, and by squaring again $a^2x^2=4ax\times\overline{a^2-2ax+x^2}$, which divided by ax, gives $ax=4a^2-8ax+4x^2$, which transposed according to the former rules, and divided by 4 gives $x^2-\frac{9ax}{4}=-a^2$.

EXAMPLE 5. What is the value of z in this equation $z=\sqrt{a^2+z\sqrt{b^2+z^2}}-a$, by transposing a the equation is $a+z=$

$z=\sqrt{a^2+z\sqrt{b^2z^2}}$ and by squaring both sides $a^2+2az+z^2=a^2+z\sqrt{{}^2b+z^2}$, or $2az+z^2=z\sqrt{b^2+z^2}$, this divided by z is $2a+z=\sqrt{b^2+z^2}$, and by squaring again $4a^2+4az+z^2=b^2+z^2$, which ordered by the foregoing rules gives $4az=b^2-4a^2$, therefore, $x=\dfrac{b^2}{4a}-a$.

To Exterminate an unknown Quantity out of Several Equations; or, to Reduce two or more Equations to a single one.

Rule. If the quantity to be exterminated, has but one dimension in the equation, find the value of it in two equations, and put those values equal to each other; or having found the value in one equation, substitute it in the room of the quantity in the other equations. Proceed in the same manner with every unknown quantity. But if the unknown quantity be of several dimensions, find the value of its highest power in two equations. Then if the coefficients are not the same, multiply the less quantity, so that it may become equal to the greater. Put these values equal to each other, and there will arise a new equation, with a less power of the unknown quantities, and the operation must be repeated till the quantity be exterminated.

EXAMPLES.

Example I. What is the value of x and y in these two equations, $7x-5y=28$ and $3x+4y=55$, by transposing 28 in the first equation, and $5y$, we have $7x-28=5y$, therefore, the value of y is $\dfrac{7x-28}{5}$.

In the second equation by proceeding in the same manner, viz:—By transposing $3x$, the value of y is found to be

be $\frac{55-3x}{4}$; therefore, these two quantities being put equal to each other, we have the equation $\frac{7x-28}{5}=\frac{55-3x}{4}$. In this equation only x is concerned. Multiply this equation by 20, which is the product of 4 and 5; or, which is the same thing, multiply the numerators and the denominators cross ways; and there will arise the equation $28x-112=275-15x$, which by transposition, becomes $43x=387$, or $x=\frac{387}{43}$, therefore, $x=9$, therefore, 9 being substituted in either of the given equations instead of x, the value of y will be found. Thus, in the first equation if 9 be substituted for x, it will be $63-5y=28$, which transposed is $\frac{63-28}{5}=y$, or $7=y$.

Example 2. Require the value of x, y, and z in the three following questions:—

$x+100=y+z.y+100=2x+2z.z+100=3x+3y$, by transposing 100 in the first equation, $x=y+z-100$ arises, which value substituted in the other two equations, instead of x, we have the two following;—

$y+100\ (=2y+2z-200+2z)=2y+4z-200$

$z+100\ (=3y+3z-300+3y)=6y+3z-300$, then by transposing y and $4z-200$, in the first of these two equations we have $300-4z=y$, which substituted for the y in the last equation, is $z+100=1800-24z+3z-300$, that is $z+100=1500-21z$, wherefore $22z=1400$, or $z=\frac{1400}{22}=63\frac{7}{11}$ therefore, $y=300-4z=45\frac{5}{11}$, and $x=y+z-100=9\frac{1}{11}$.

Of

Of the Nature and Composition of Equations, containing different Dimensions of the same Unknown Quantity.

It often happens that the unknown quantity will be of ſeveral different dimenſions, then ſuch equation is called a quadratic, a cubic, a biquadratic equation, &c. according as the dimenſions of the higheſt power is a ſquare, cube, or biquadrate, in ſuch equations we muſt diſcover the root or value of the unknown quantities.

All equations are derived, (or may be conſidered ſo,) from thoſe of a more ſimple form. Thus, if $x-b=0$, which is a ſimple equation, be raiſed to the ſecond power, there ariſes $x^2-2bx+2b=0$, which is called a quadratic equation, if the former equation be raiſed to the third power, we have $x^3-3bx^2+3b^2x-b^2=0$, which is called a cubic equation, and ſo on. It is but ſeldom that equations occur in this regular form, for the coefficients of the terms will generally be more or leſs than thoſe produced by the involution of one quantity, as $x-b$, and therefore, a quadratic equation is a compound one, generally derived from $\overline{x-b}\times\overline{x-c}$. A cubic equation is derived from $\overline{x-b}\times\overline{x-c}\times\overline{x-d}$. A biquadratic equation from $\overline{x-b}\times\overline{x-c}\times\overline{x-d}\times\overline{x-e}$, or from a quadratic ſquared, &c. But, the letters b, c, d, &c. may have either affirmative or negative ſigns.

In equations of this nature, as the whole is equal to nothing, it is obvious that ſome or other of the factors muſt be equal to nothing. It is alſo evident that any ſuch equation may be divided by its factors, till there remain only one factor; and as each of the inferior equations obtained by ſuch diviſion, muſt ſtill be equal to nothing, it muſt follow that each of theſe factors themſelves are equal to nothing; therefore, b, c, d, e, &c. exhibit ſo many different values of x with

with contrary ſigns; therefore, every equation has as many roots as there are dimenſions of the unknown quantity in its higheſt power. And where b, c, d, e, &c. are found negative, x is affirmative, and where any of theſe are affirmative, x is negative. By multiplying the factors or roots together it is obſervable than when b, c, d, &c. are all negative, or, which is the ſame thing when all the values of x are affirmative, the ſigns in the equation are $+$, and $-$ alternately. But when there is a negative root, one affirmative quantity will follow another; therefore, there will be as many affirmative roots in the equation as there are changes of the ſigns from $+$to$-$, and from $-$to$+$, and all the reſt will be negative.

What is here delivered, concern only poſſible roots. An impoſſible root is when b, c, d, &c. denote the ſquare or any other, even root of a negative quantity; an equation, derived from ſuch roots is an impoſſible, or imaginary one.

In the multiplication of the roots of ſuch equations, the coefficient of the ſecond term is the ſum of all the roots with contrary ſides; the coefficient in the third term is equal to the ſum of the rectangles of thoſe roots; or, of all the products that can poſſibly ariſe by combining them two and two; the coefficient of the fourth term is equal to the ſum of all the products that can poſſibly ariſe by the combination of them three and three, &c. and the laſt term is always equal to the product of all the roots with contrary ſigns.

The Resolution of Quadratic Equations.

If it be a pure quadratic as $x^2 = b^2$, or $x^2 - b^2 = o$ it is produced from the rectangle of $x-b$ and $x+b$, and therefore has one affirmative, and one negative root, and the affirmative root is equal in number to the negative. The root in this caſe is found by extracting the ſquare root of b^2. Thus, if $x^2 + = 576$, then $x = \sqrt{576} = 24$.

All

All other quadratics are comprehended under ſome of the following forms :—viz. $x^2+2bx=+d=o$; or, $x^2-2bx\pm d=o$; or, $2bx-x^2\pm d=o$; and this laſt form, by tranſpoſition, becomes the ſame as the ſecond form, only the negative roots are changed into affirmative roots, and the affirmative into negative; therefore we may conſider the two other forms, as applicable to all caſes, and in the ſolution of them, it will be more commodious to tranſpoſe d, and then they will ſtand thus: $x^2+2bx=\pm d$ and $x^2-2bx\pm d$. Now if to each of theſe equations be added b^2 (the ſquare of half the coefficient of the ſecond term) we ſhall have in the former caſe $x^2+2bx+b^2=\pm d+b^2$, and in the latter caſe $x^2-2bx+b^2=\pm d+b^2$, and by extracting the ſquare roots, the equations become $x+b=\sqrt{\pm d+b^2}$ and $x-b=\sqrt{\pm d+b^2}$ reſpectively; and x in the former caſe $=\sqrt{\pm d+b^2}-b$ and in the latter $=\sqrt{\pm d+b^2}+b$ which expreſſions give the affirmative values of x; but the ſquare root of the above equations may alſo be $-x-b=-\sqrt{\pm d+b^2}$ and $-x+b=-\sqrt{\pm d+b^2}$ reſpectively; and therefore in the former caſe $x=-\sqrt{\pm d+b^2}+b$, and in the latter $=-\sqrt{\pm d+b^2}-b$. That is, if $B=\sqrt{\pm d+b^2}$, where d is + or —, according as it is + or — in the ſecond ſide of the given equation; then in the firſt caſe, where $x^2+2bx=\pm d$ the values of $\pm x$ are $\pm B\mp b$, where b is — or + according as x is affirmative or negative; and in the ſecond caſe, where $x^2-2bx=\pm d$, the values of $\pm x$ are $\pm B\ \pm b$, where b is + or —, as x is + or —.

EXAMPLES.

Example 1. What is the value of x in this equation, $x^2+6x=2295$? Here $b=3$, and $2295=+d$, and $\sqrt{+d+b^2}=\sqrt{2295+9}=48=B$, and $\pm B+b=\pm 48+3=+45$, and -51 for the two values of x.

Example 2. What are the values of x in $x^2-11x=-28$? Here $b=\frac{11}{2}$, and $-28=-d$, alſo $b^2=30\frac{1}{4}$, and $\sqrt{-d+b^2}=$

$\sqrt{-28+30\,.\,25}=\sqrt{2\,.\,25}=1\,.\,5=B$, and $\pm B\pm b=\pm 1\,.\,5$ $\pm 5\,.\,5=+7$, and $+4$ the two values of x, which here are both affirmative.

To Increase or Diminish the Roots of Equation.

RULE. Subſtitute a new letter for the unknown quantity, or ſubſtitute — the given increment, or + the given decrement; and ſubſtitute the powers thereof in the equation inſtead of the unknown letter.

EXAMPLE. Increaſe the roots of the following equation by 2. $x^3+x^2-10x+8=0$. Let $x+2=z$ or $z-2=x$, then $x^3=\overline{z-2}|^3$ $x^2=\overline{z-2}|^3$ and $-10x=-10\times z-2=-10z+20$ then the powers of x being actually involved, and the ſeveral terms collected, we have $x^3+x^2-10x+8=z^3-5z^2-2z+24=0$ where the roots of z are greater than thoſe of x by 2.

Thus all the negative roots of an equation, may be made affirmative, by increaſing them with a proper quantity.

To Complete a Deficient Equation.

RULE. Increaſe or diminiſh the roots of the equation, by ſome given quantity, as ſhewn in the laſt example.

To Multiply or Divide the Roots of any Equation, by the given Quantity.

RULE. Multiply or divide any new letter by the given number; and ſubſtitute its powers in the equation for the unknown quantity.

EXAMPLE. Divide the roots of the equation $x^3-2x+\sqrt{3}+0=0$ by $\sqrt{3}$. Here by putting $x=y\sqrt{3}$ and ſubſtituting it for x, we have $3y^3\sqrt{3}-2y\sqrt{3}+\sqrt{3}=0$, which by dividing by $\sqrt{3}$, is $3y^3-2y+1=0$ for the equation required.

By this rule fractions and ſurds may be taken out of an equation, viz. by dividing the new letter by the common denominator;

nominator; or by multiplying the new letter by the ſurd quantity.

To take any Term out of an Equation.

Rule. Add an unknown quantity to a new letter, and ſubſtitute this ſum and the powers thereof, for the root in the given equation; then any term, or any of thoſe quantities, wherein the new letter is of the ſame power, being put into an equation, and made equal to nothing, will give the value of the unknown quantity, which being put into the equation, with the new letter, the power of the new letter which was equated, will vaniſh.

EXAMPLE.

Suppoſe $x^4-3x^3+3x^2-5x-2=0$

Put $y+e=x$ then $x^4=y^4+4y^3e+6y^2e^2+4ye^3+e4$

$-3x^3=-3y^3-9y^2e-9ye^2-3e^3$

$+3x^2=+3y^2+6ye+3e^2$

$-5x=-5y-5e$

$+2=+2$

If the ſecond term is to be taken away, we have $4y^3e-3y^3=0$, and dividing by y^3, $4e-3=0$, or, $e=\frac{3}{4}$, which ſubſtituted for e, the ſecond term will vaniſh. If the third term is to be taken away, we have $6y^2e^2-9y^2e+3y^2=0$, and dividing by the y^2, we have $6e^2-9+3=0$, from which quadratic equation e may be determined; in like manner the fourth term may be taken away by ſolving the cubic equation; and the fifth term by ſolving a biquadratic equation, &c.

To Resolve, or Extract the Root of a Cubic Equation.

Rule. Take the ſecond term of the equation away as taught in the laſt example; then the equation will be in this form, $x^3+ax=b$, and the following general expreſſion will give the value of x.

$$\left(\frac{b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}\right)^{\frac{1}{3}} \frac{\frac{1}{3}a}{\left(\frac{-b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}\right)^{\frac{1}{3}}}$$

There are ſeveral other particular rules for the ſolution of biquadratic and other higher equations, but the method of approximating roots of equations, has ſuperceded all the other methods, on account of its diſpatch.

To Approximate the Roots of Equation, in general.

Rule. By ſeveral trials chooſe ſome number to repreſent the unknown quantity, and ſuch a number that approaches pretty near to the true value. Then aſſume ſome letter, as v, to denote the defect of exceſs of the number ſo found, and put that number + or $-v$, inſtead of the unknown quantity in the equation; by which, a new equation will ariſe, affected with v only, and known quantities, wherein all the terms that contain two or more dimenſions of v may be rejected as inconſiderable in reſpect to the reſt. This being done, the value of v will be found by a ſimple equation, which, added to, or ſubtracted from the ſaid number, according as it was taken, too little, or too big, will give a number ſtill nearer the truth. Then with this number and the letter v, proceed as before to find another value of v, which muſt be applied as above: repeat the operation till the unknown quantity be diſcovered to a ſufficient degree of exactneſs.

Example. Let it be required to find the value of x, in this equation, $x^3+24x=587814$. Here by a few trials it will be found that x is ſomething above 80; wherefore, let $80+v=x$; then, $x^3=512000+19200v+1680v+v^3$, and $24x=1920+24v$, therefore, (rejecting thoſe terms, affected with v^2 and v^3,) we have $513920+19224v=587914$, and

$v=$

$$v = \frac{587914 - 513920}{19224} = \frac{73994}{19224} = 3.8,$$ this added to 80, gives 83.8 for the approximate value of x, this being substituted in the equation, it will be found too great, therefore, for another operation take $83.8 - v = x$, and $x^3 = 588480.472 - 21067.32v$, and $24x = 2011.2 - 24v$, and consequently $590491.672 - 21091.32v = 587914$, and $v = \frac{2577.672}{21091.32} =$ 0.1222 nearly. Hence $x = 83.8 - 0.1222. = 83.6778.$ true to five figures, and the sixth being too much by only two.

If the operation be repeated, it will give the anſwer true to eleven figures. But when five or ſix figures of the roots, have been obtained, and more exactneſs is ſtill required; it will ſhorten the work to ſeek a correction for v, inſtead of one for the whole root, which may be had by ſubſtituting the laſt value of $v+$ or $-$, (new,) v inſtead of the laſt v in the equation, including all the powers of the laſt v, but rejecting thoſe of the new v in the equation, thence ariſing, as before.

Thus, in the laſt example, the whole equation is $513920 + 19224v + 1680v^2 + v^3 = 587914$, or $19224v + 1680v^2 + v^3 = 73994$, and in this equation putting $3.8 - v$, inſtead of v, and rejecting the terms of v^2 and v^3 as before; a correction will be obtained for the laſt found v, which will give the anſwer as above. In like manner a ſecond or third correction may be found, and the operation carried on to any degree of exactneſs. This rule doubles the number of figures true in the root at each operation. If the term wherein v^2 is found to be retained, v will be had by ſolving a quadratic; and then treble the number of figures will be had each time, therefore, if the firſt figure only be taken true, nine or ten figures will be had at two operations. This rule affords various theorems for ſolving particular equations as well as general

neral formulas adapted to all of which I ſhall give two examples.

Example 1. $x^2+6x=61495=c$. Here by trial, it will be found that x is above 200; therefore, let $200=r$, then $-b=r-1200$ and $c-r^2-br=20295$, The diviſion is as follows:—

THE OPERATION.

$$\begin{array}{r l}
2r+b=406 & \qquad (-45=245=x) \\
\underline{v=40} & \Big)\,20295(45=v,\ \text{and}\ r+v-=200 \\
2r+b+v=446 & \Big) \\
40 & \quad \underline{1784} \\
2r\ (\text{new})+b=486 & \Big)\ \ 2455 \\
\underline{v^2=\ \ 5} & \Big)\ \ \underline{2155} \\
2r+b+\underline{\underline{v^2=491}} &
\end{array}$$

Example 2. Again, for a cubic equation, let $x^3+bx^2+dx=c$. Put $r+v=x^2$, ſo will $x^3=r^3+3r^2v+\pm 3rv^2\pm v^3$; $bx^2=br+2brv+bv^2$, and $dx=dr\pm dv$. Hence $r^3+br^2+\overline{3r^2+2br+a}\times v+\overline{3r+b}\times v^2+dr=c$, ($v^3$ being rejected, as ſmall in compariſon of the reſt,) and by tranſpoſing $+\overline{3r^2+2br+d}\times v+\overline{3r+b}\times v^2=c-r^3-br^2-dr$ and $v=+\dfrac{c-r^3-br^2-dr}{3r+2br\pm d+3r+bv}$, which is wrought after the ſame manner as the laſt example.

To Find the Limits of Equations.

When an Equation contain ſeveral unknown quantities, it will admit of an infinite number of ſolutions, when both fractional and negative numbers are admitted, for all of them but one, may be taken at pleaſure, and their value ſubſtituted in the equation, which quantity will be determined. But ſometimes both fractional and negative quantities will be excluded from an equation, and ſuch equation will be confined to a determinate number of ſolutions. I ſhall, therefore,

fore, assign the limits of such equations in the following cases;—

CASE I.

When Several Unknown Quantities are in one Equation; to find their Limits.

RULE. Transpose all the negative quantities to the contrary side, that all the terms may be affirmative; then to find the limits of any one quantity; suppose all the rest to vanish in the equation, then the value of that one will become determinate, and will be one limit thereof. And to know which limit it is, suppose the other quantities to increase, and become of some certain value; then if the value of the unknown quantity under consideration, increase, it is the least limit, but if it decreases it is the greatest.

When fractional quantities are to be excluded, instead of supposing the other quantities to vanish, put each of them $=1$, and an equation will arise, from which the limits of the remaining quantity will be found as before. Proceed in the same manner to find the limits of the other unknown quantities.

EXAMPLE. What is the limits of x and y, in the equation $4x+5y=67$. Let $y=0$ or be supposed to vanish, and then $4x=67$, and $x=16\frac{3}{4}$. Now let y be supposed to be equal to some quantity; then it is evident, that as y increases, x decreases, therefore $16\frac{3}{4}$ is the greater limit; wherefore x is less than $16\frac{3}{4}$.

If $x=0$ then $5y=67$ and $y+13\frac{2}{5}$. Now if x be supposed to increase, y will decrease; and therefore $13\frac{2}{5}$ is the greater limit of y; whence y is less than $13\frac{2}{5}$, and the less limit of both x and y is 0.

CASE

CASE II.

To Determine the Limits of three, or more Unknown Quantities, when they are in two Equations.

Rule. Fix upon a quantity to be limited and expunge one of the other quantities; then there will be had one limiting equation. Do the fame with another unknown quantity, and there will be had another limiting equation; from each of which equations find the limit for the quantity fixed on.

Example. What is the limit of x, in the two following equations. $v+y+z=56$, and $32x+20y+16z=1232$. To exterminate y, multiply the firft equation by 20, and there arifes $20x+20y+20z=1220$, fubtract this from the fecond equation, and we have $12x-4z=112$, then excluding the fractions, the lefs limit of x in this equation is $9\frac{2}{3}$.

Again, to exterminate z, multiply the firft equation by 16, and fubtract the product from the fecond equation, and there remains $16x+4y=336$. And the greater limit of x in this equation is $20\frac{3}{4}$. Hence x is greater than $9\frac{1}{4}$, and lefs than $20\frac{3}{4}$. In the fame manner may y and z be limited; and fo alfo, in any other equation.

Of Indeterminate Problem.

Example. What is the leaft integer for the value of x, that will alfo caufe the value of the following fraction, to be an integer: $\frac{ax+b}{c}$

Rule. Divide the denominator c, by the coefficient a of the indeterminate quantity; then divide the divifor by the remainder, and the laft divifor again, by the laft remainder, and continue this operation till an unit only remains. Write down all the quotients in a line, as they rife under the firft quotient,

quotient write an unit, and under the ſecond quotient write the firſt quotient; then multiply the firſt and ſecond quotients together, to the product, add the firſt term of the lower line, place the ſum under the third quotient: multiply in like manner, the next two correſponding terms of the two lines together, and add the ſecond term of the lower line to the product; put down the ſum under the fourth term of the upper line; proceed in the ſame manner till you have multiplied by every number in the upper line; then multiply the laſt number, thus found by the abſolute number b, in the numerator of the fraction, and divide the product by the denominator, then the remainder will be the true value of x required, provided the number of terms in the upper line be even, and the ſign of b be negative, or that the number be odd, and the ſign of b affirmative: but if the number of terms be even, and the ſign of b affirmative, or *vice verſa;* then the difference between the ſaid remainder and the denominator of the fraction, will be the true anſwer.

OPERATION.

The given fraction $\dfrac{ax+b}{c}=\dfrac{71x+10}{89}$.

$$
\begin{array}{lcl}
71)89(1 & \qquad & 1.\ 3.\ 1\ \text{ Total quotients,} \\
\quad 18)71(3 & & 1.\ 1.\ 4.\ 5 \\
\qquad 17)18(1 & & \qquad\quad 10=b \\
\qquad\quad \underline{1} & & \qquad\quad \underline{50}\ \text{ Product.}
\end{array}
$$

Here if the product 50 be divided by 89, the remainder is 50= the leaſt value of x.

In this rule it is always ſuppoſed, that a is leſs than c; and that they are prime to each other; for if they were to admit of a common meaſure, whereby b is not diviſible; no integer could be aſſigned for x, ſo as to give the value of the fraction.

EXAMPLE 2. What is the leaſt value of x and y in whole numbers, in the equation $24x-13y=16$. Here by tranſpoſing $13y$, and dividing by $24x$, we have $x=\frac{13y+16}{24}$; therefore, the leaſt value of $y=8$, and the leaſt value of $x=5$.

To Find the Value of a Fraction in an Infinite Series.

RULE. Divide the numerator by the denominator, and continue the operation as far as is neceſſary. For in many caſes, after the quotient is continued to a few terms, it may be ſeen how the terms converge, and thus, any number of terms may be aſſigned at pleaſure.

EXAMPLE 1. What is the value of $\frac{1}{1+x^2}$

$$
\begin{array}{l}
1+x^2)\,1\,(1-x^2+x^4-x^6+x^8-\&c.\\
\underline{1+x^2}\\
\quad -x^2\\
\quad \underline{-x^2-x^4}\\
\qquad\quad x^4\\
\qquad\quad \underline{x^4+x^6}\\
\qquad\qquad -x^6\\
\qquad\qquad \underline{-x^6-x^8}\\
\qquad\qquad\qquad x^8\\
\qquad\qquad\qquad \underline{x^8+x^{10}}\\
\qquad\qquad\qquad\quad x^{10}
\end{array}
$$

If a quantity which is not a fraction, is to be thrown into an infinite ſeries, it muſt be brought into a fraction by placing one underneath it, as the denominator.

After a few terms are found in the ſeries, the law by which it converges, will ſoon be diſcovered, and the terms may be continued to any number.

Sometimes the ſeries cannot eaſily be diſcovered by reaſon of the coefficients; then it will be neceſſary to aſſume a ſeries with

with unknown coefficients to represent it, which being multiplied or involved as the question requires, and the quantities of the same dimension being put equal to each other, new equations will be had, wherein the coefficients may be discovered.

EXAMPLE 2. Suppose $\frac{1}{a-x}$ be the given quantity, and the assumed series be $A+Bx+Cx^2+Dx^3+Ex^4$, &c. $=\frac{1}{a-x}$. Multiply both by $a-x$, and there arises $1=aA+aBx+aCx^2+aDx^3+aEx^4$, &c. And $-Ax-Bx^2-Cx^3-Dx^4$, &c. and by equating the coefficients of the same powers of x, $aA=1$, $aB-A=0$, $aC-B=0$, $aD-C=0$, $aE-D=0$, &c. Thus, in the first equation $A=\frac{1}{a}$; in the second equation $B=\frac{A}{a}=\frac{1}{a}$; in the third $C=\frac{B}{a}=\frac{1}{a^3}$; in the fourth $D=\frac{C}{a}=\frac{1}{a^4}$, and in the like manner $E=\frac{1}{a^5}$; therefore, $\frac{1}{a-x}$ brought to a series, is $\frac{1}{a}+\frac{x}{a^2}+\frac{x^2}{a^3}+\frac{x^3}{a^4}+\frac{x^4}{a^5}$, &c.

Some of the Properties of Square Numbers.

1. All even square numbers are divisible by 4, therefore, if a number consists of two even square numbers, it will be divisible by 4.

2 Any odd square number divided by 4, leaves a remainder of 1; therefore, if a number consisting of two odd square numbers be divided by 4, there will be a remainder of 2.

3 Therefore, if a number consisting of an odd and an even square number, be divided by 4, there will be a remainder of 1.

 4. From

4. From hence it follows that if any number compoſed of two ſquare numbers be divided by 4, it cannot leave a remainder of 3; therefore, a number compoſed of two ſquare numbers, cannot fall within this progreſſion 3, 7, 11, 15, 19, 23, &c.

5. Any number ending in 2, 3, 7 or 8 is not a ſquare number.

6. The ſum of any number of terms of the ſeries 1, 3, 5 7, 9, 11, &c. beginning with the firſt, is a ſquare number whoſe root is equal to the number of terms.

7. The difference between any two ſquare numbers is equal to the product of the ſum, and difference of their roots. Thus, if a and b be the roots, then $a+b\times a-b=a^2-b^2$, and the ſame is alſo equal to the ſum of the two roots, together with twice the ſum of the roots of all the intermediate ſquare numbers. Thus, the difference berween 36 and $9=\overline{6+3}+2\times\sqrt{\overline{16+25}}=9+18-27$.

To reſolve queſtions of this nature, the chief point is to make ſuch aſſumptions for the root of the required ſquare or cube as ſhall, when involved, cauſe either the given number or the higheſt power of the unknown quantity to vaniſh from the equation, whereby at length there will be only one dimenſion of the unknown quantities, and ſo the queſtion will be ſolved by reducing the equation.

SECT. V.

THE RESOLUTION OF SEVERAL ALGEBRAIC PROBLEMS.

Qu. 1. What are thoſe two numbers, the ſum whereof is 108, and the proportion of the leſs to the greater, is as 5 to 7?

Let

Let x represent the greater number, then $108-x$ is equal to the less number, and the proportions of the numbers will be as follows: $108-x : x :: 5 : 7$, these four quantities being in a direct proportion the product of the two means $5x$ is equal to the product of the two extremes $756-7x$, therefore, we have this equation $5x=756-7x$, and by transposing $7x$ we have $12x=756$. Hence, by dividing 756 by 12, x is found equal to 63, which is the greater number, therefore, $108-63=45$, the less number.

Qu. 2. A servant bought apples at 6 for a penny, and pears at 5 for two-pence. The number of apples and pears together was one hundred; the money given for the whole, was 2*s.* 2*d.* how many were there of each sort?

Let a represent the number of apples, then $100-a$ will be the number of pears; and as $6:1d.::a:\frac{a}{6}=$, price of the apples. Also, as $5:2d.::100-a:\frac{200-2a}{5}=$, price of the pears; then $\frac{a}{6}+\frac{200-2a}{5}=2s.\ 6d.$ This equation multiplied by thirty, gives $5a+1200-12a=780$. By transposing and dividing $7a=1200-780=420$, and by division $a=60$, the number of apples; and $100-60=40$, the number of pears.

Qu. 3. It is required to divide the number 128 into four such parts, that if the first part be added to 7, the second subtracted from 7, the third multiplied by 7, and the fourth divided by 7, the results may be equal among themselves.

Let the four parts into which the number is divided be represented by the letters v, x, y, z. Then $v+7=7-x=7y=\frac{z}{7}$; and from the equality of the two first equations we have $x=y-14$; from the equality of the first and third equations $y=\frac{v+7}{7}$, and from the equality of the first and fourth

$z=$

$z=\overline{v+7}\times 7=7v+49$; therefore, by collecting these together, there arises $v+\overline{v+14}+\frac{v+7}{7}+\overline{7v+49}=128$. By collecting the terms and transposition $9v+\frac{v+7}{7}=65$; and multiplying this by 7, collecting the terms, transposing and dividing we have $v=7$. And hence $x=7+14=21$; $y=\frac{7+7}{7}=2$, and $z=\overline{7+7}\times 7=98$, the several parts required.

Qu. 4. There are two cubical pieces of marble, the side of one exceeding the side of the other by 3 inches, the solid inches of both are 2457 inches; what is the length of the side of each piece.

Let the side of the less piece be represented by x, then the side of the greater will be $x+3$, and $x^3+\overline{x+3}^3=2547$ inches; therefore, $2x^3+9x^2+27x=2430$; this equation solved, gives $x=9$, and consequently $y=12$.

Qu. 5. A Gentleman left a sum of money to be divided among three servants, in such proportion, that $\frac{1}{2}$ of the share of the first, $\frac{1}{3}$ of the second share and $\frac{1}{4}$ of the share of the third should be equal to £62; and one third of the first, one fourth of the second, and one fifth of the third equal to 47, and one fourth of the first, one fifth of the second, and one sixth of the third, equal to £38, what is each servants share?

Put $a=62$, $b=47$, and $c=38$, and let the three shares required, be denoted by x, y, and z; then the conditions of the question will stand thus:—$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=a$. $\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=b$ $\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=c$. These equations brought

out

out of fractions, give $6x+4y+3z=12a$. $20x+15y+12z=60b$. $15x+12y+10z=60c$. Here by subtracting the second equation from 4 times the first, in order to exterminate z, there arises $4x+y=48a-60b$; then by taking three times the third equation from 10 times the first, we have $15x+4y=120a-180c$, which subtracted from 4 times the last equation, leaves $x=72a-240b+180c=24$; wherefore, $y=48a-60b-4x=60$ and $z=\frac{12a-6x-4y}{3}=120$. demonstrated thus:—

$$\frac{24}{2}+\frac{60}{3}+\frac{120}{4}=12+20+30=62$$
$$\frac{24}{3}+\frac{60}{4}+\frac{120}{5}=8+15+24=47$$
$$\frac{24}{4}+\frac{60}{5}+\frac{120}{6}=6+12+20=38$$

Qu. 6. A grocer bought 120 pounds of tea, and as many pounds of coffee, he had one pound of coffee more for 20 shillings than of tea, and the whole price of the tea exceeded that of the coffee by six pounds; I demand how many pounds of tea he had for 20 shillings, and how many pounds of coffee?

Let the number of pounds of tea bought for 20 shillings, be represented by x, then the number of pounds of coffee, for 20 shillings, will be $x+1$, and the whole price of the tea will be $\frac{120}{x}$ pounds, and that of the coffee $\frac{120}{x+1}$ pounds; therefore, $\frac{120}{x}-\frac{120}{x+1}=6$; wherefore, $120x+120-120x=6x^2+6x$, therefore, $x^2+x=20$; which resolved gives $x=4$ the pounds of tea for 20 shillings, and $x+1=5$ the pounds of coffee for 20 shillings.

Qu. 7.

QU. 7. Two travellers ſet out on a journey at the ſame time, the one ſets out from C to go to B, the other from B to go to C, they both travel uniformly, and in ſuch proportion that he that ſet out from B, four hours after meeting of the other, arives at C, and the other arrives at B, nine hours after meeting. How many hours did each perſon take to perform his journey?

In this figure D is the place of their meeting; put $a=4$, $b=9$, and x for the number of hours which they travel before they meet. Then the diſtances they have travelled, with the ſame uniform pace, will be to each other as the times in which they are deſcribed; therefore, B D : D C : : x, (the time in which the traveller, who ſet out from B goes the diſtance B D,) : $a=$ the time in which he travels from D to C; and by the ſame manner as B D : D C : : b, (the time the other traveller goes from B to D,) : $x=$ the time he goes from D to C; now as x is to a in the ratio of B D to D C; and b to x in the ſame ratio it will follow that as $x:a::b:x$, whence $x^2=ab$ and $x=\sqrt{ab}=6$, therefore, $a+\sqrt{ab}=10$, and $b+\sqrt{ab}=15$, are the two numbers required.

QU. 8. The ſum of three numbers in geometrical proportion and the ſum of the ſquares of three numbers being given; to find the numbers themſelves.

Put a for the ſum of the three numbers, and b for their ſquares, and x, y and z for the numbers themſelves, then we ſhall have $x+y+z=a$, and $x^2+y^2+z^2=b$ and $xz=y^2$, whence by tranſpoſing y in the firſt equation, and involving both ſides to the ſecond power, there ariſes $x^2+2xz+z^2=a^2-2ay+y^2$, from which ſubtracting the ſecond equation we have $2xz-y^2=a^2-2ay+y^2-b$; but $2xz$ by the third equation is $=2y^2$, therefore, $2y^2-y^2=a^2-2ay+y^2-b$ or $a^2-2ay-b=0$, whence, $y=$

$\frac{a}{2}=$

$\frac{a}{2}-\frac{b}{2a}$. Now to find x and z, we muſt look upon y as a known quantity, and then by the ſecond equation we ſhall have $x^2+z^2=b-y^2$ from which ſubtracting $2xz=2y^2$, we have $x^2-2xz+z^2=b-3y^2$, and by taking the root we have $x-z=\sqrt{b-3y^2}$, but by the firſt equation $x+z=a-y$, therefore, $x=\frac{a-y+\sqrt{b-3y^2}}{2}$ and $z=\frac{a-y-\sqrt{b-3y^2}}{2}$

Qu. 9. A Farmer ſold as many ſheep and oxen as brought him £100; for the ſheep he received 17 ſhillings each, and for the oxen £7 each. It is required to know how many he ſold of each?

Let the number of ſheep be x, and that of the oxen y; then we have this equation $17x+140y=2000$, and conſequently $x=\frac{2000-140y}{17}=117-8y+\frac{71-4y}{17}$ which being a whole number $\frac{11-4y}{17}$ or $\frac{4y-11}{17}$ muſt therefore be a whole number likewiſe; whence by proceeding as above, we have $y=7$, and $x=60$, and this is the only anſwer the queſtion will admit of.

Qu. 10. What is the dimenſions of a cubical block of marble, whoſe ſide in inches is expreſſed by two digits; the ſuperficies of the block is equal to 864 times the ſum of the ſaid digits; and its ſolidity is equal to 576 times the ſquare of the ſum of the ſaid digits?

Put x for the digit in the place of tens, and y for the digit in the place of units. then $10x+y$ is equal to the ſide of the cube and $\overline{10x+y}|^2\times6=864\times\overline{x+y}$ or $\overline{10x+y}|^2=144\times\overline{x+y}$. Alſo, $\overline{10x+y}|^3=576\times\overline{x+y}|^2$ per queſtion; and multiplying theſe equations croſs ways, we have $\overline{10x+y}|^2\times576\times\overline{x+y}|^2=144\times\overline{x+y}\times\overline{10x+y}|^3$. Then dividing both ſides by $144\times\overline{x+y}$

$\overline{x+y}\times\overline{10x\times y}\big|^2$, we have $\overline{x+y}\times 4=10x+y$, or $4x+4y=10$ $x+y$, and by tranſpoſition $3y=6x$, therefore, $y=2x$, this being ſubſtituted for y in the former equation, we have $\overline{10x+2x}\big|^2=144\times\overline{x+2x}$ or $144x^2=144\times 3x$; and dividing by $144x$ we have $x=3$ and $y=2x=6$, thererefore, the ſide of the cube is 36.

CHAP. X.

OF THE VALUE OF LIVES;

OR,

DOCTRINE OF *ANNUITIES.*

SECT. I.

THE VALUE OF AN ANNUITY FOR A SINGLE LIFE.

AN ANNUITY is a ſum of money payable yearly, half yearly, or quarterly; to continue either for life, for a certain number of years, or for ever.

When an Annuity remains unpaid after it it due, it is ſaid to be in *arrear*. When the purchaſer of an Annuity does not immediately enter upon poſſeſſion, the annuity is ſaid to be in *reverſion*.

The

The intereſt upon Annuities in arrear may be computed either in the way of ſimple or compound intereſt. But compound intereſt being found moſt equitable, both for buyer and ſeller, is in moſt general uſe.

Annuities may be divided into certain and uncertain.

A certain annuity is that which continues for a certain time, or for ever. An uncertain annuity depends upon one or more lives.

Before I proceed to give the Doctrine of Contingent Annuities, it will be neceſſary to deliver the rules for calculating of Annuities certain.

PROBLEM I.

To find the Amount of an Annuity for a given Term of Years, at a given Rate of Intereſt.

Example. What will an Annuity of £50 amount to, at the end of 8 years, at the rate of 5 per cent. per annum, Simple Intereſt.

In this example, the intereſt being at 5 per cent. multiply the rate of intereſt of £1 for 1 year, or .05 by 50 the annuity, and the product by 8 the number of years, and the product hence ariſing is 20; the half whereof (10) multiplied by the number of years, made leſs by one, (7,) produces 70, the ſimple intereſt; which added to the product of 50, and 8 (400,) give 470, the amount requried.

PROBLEM II.

To find the Amount of an Annuity, at Compound Intereſt.

Rule. Multiply the amount of £1, for 1 year, as often into itſelf as there are years, except one; or which is the ſame, raiſe it to the power whoſe index is equal to the number of years and from the reſult, ſubtract 1; then divide

 the

the remainder by the intereſt of £1, for 1 year, and multiply the quotient by the annuity, and the product will be the amount required.

Example. What is the amount of an Annuity of £50 for 3 years, at 5 per cent. Compound Intereſt? Here the amount of £1 for 1 year is 1.05, which multiplied twice into itſelf, produces 1.157625 and 1, ſubtracted from this, the remainder is .157625, which divided by .05, the quotient is 3.1525, this multiplied by 50, produces 157.625, or £157 12*s.* 6*d.* the anſwer required.

Note. If the payments are half yearly, or quarterly, the amount and intereſt of £1 muſt be taken for a half or a quarter of a year. And then the double or quadruple of the time muſt be taken. And the amount of £1 for half a year at Compound Intereſt is equal to the ſquare root of the amount for a year; and the amount for a quarter of a year, is equal to the ſquare root of that for half a year

PROBLEM III.

To find the Present Value of an Annuity, having the Time and Rate.

Rule. Multiply the amount of one year as often into itſelf as there are years, leſs 1; or involve it to the power, denoted by the time; by this reſult, divide 1, and ſubtract the quotient from 1, divide the remainder by the intereſt of £1 for the year; then multiply this laſt quotient by the annuity, and the product will be the preſent value.

Example. What is the preſent value of an Annuity of £40 for 5 years, diſcounting 5 per Cent. per Annum, Compound Intereſt. Here 1.05 involved to the fifth power is 1.27628. By which dividing 1, the quotient is .78353, which ſubtracted from 1, leaves .21647, this divided by .05 gives 4.3294, which multiplied by 40 is 173.176 or £173 3*s.* 6½*d.* the preſent worth.

PROBLEM

PROBLEM IV.

Having the present Worth, Rate and Time, to Find the Annuity.

RULE. Find the present value of £1, annuity at the given rate and time; and then by the rule of three, say, as the present worth, thus found, is to £1 annuity, so is the present worth given to its annuity; that is, divide the given present worth by that of £1 annuity.

EXAMPLE. What Annuity will £173 3*s.* 7*d.* purchase to continue 5 years, allowing Compound Interest at 5 per cent?

.05:1::1:£20

$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$

1.2762815625)20.0000000(15.6705

20

15.6705

4.3295 present worth of £1 annuity.

4.329)173.179(£40 annuity, Answer.

Annuities for ever, or Freehold Estates.

In calculating the value of an Annuity for ever, commonly called an *Annuity in fee Simple,* three things are to be considered. 1. The annuity or yearly rent. 2. The price or present worth. 3. The rate of Interest.

PROBLEM I.

Having the Rent and Rate of Interest, to find the Price or value.

RULE. As the Interest of £1 is to £1, so is the rent to the price or value.

EXAMPLE.

EXAMPLE. What is the present worth of an Annuity of £40 per annum in fee Simple, Compound Interest 3½ per cent? As .035 the interest of £1 for the year is to £1; so is £40 the rent of the Annuity to 1142.857142 or £1142 17s. 1½.

PROBLEM II.

Having the Price and Rate of Interest, to find the Annuity.

RULE. As £1 is to its Interest, so is the price to the annuity.

EXAMPLE. What Annuity will £4000 purchase, at 4½ per cent?

As £1 is to .045, so is £4000 to £180, the annuity.

PROBLEM III.

Having the Price and Rent of the Annuity, to find the Rate of Interest.

RULE. As the price is to the rent, so is £1 to the rate of Interest.

EXAMPLE. If an Annuity of £180 cost £4000, what is the rate of Interest Compound at,

As 4000:180::1: .045 or 4½ per cent. rate of Interest,

PROBLEM IV.

Having the Rate of Interest, to find how many Years purchase an Estate is worth.

RULE. Divide 1 by the rate of Interest, and the quotient is the answer.

EXAMPLE. How many years purchase is an annuity, when the purchaser has 2½ per cent. for his money.

.025)1.000(40 years purchase.

PROBLEM

PROBLEM V.

Having the number of Years purchase, to find the Rate of Interest.

RULE. Divide 1 by the number of years purchaſe, and the quotient is the rate of Intereſt.

EXAMPLE. What Intereſt has a purchaſer, who gives 40 years purchaſe for an annuity.

40)1.000(.025 intereſt required.

Though the foregoing examples are moſtly performed by a ſingle diviſion or multiplication, yet they give the anſwers at Compound Intereſt; but in caſes, where there is a reverſion, recourſe muſt be had to the tables of Annuities, on Compound Intereſt, as in the following Problems:—

PROBLEM VI.

Having the Rate of Interest, and the Annuity, to find the present Value of the Reversion.

RULE. Find the preſent Value of the Annuity by Problem I. then by the Tables, find the preſent Value of the Annuity for the years before the reverſion takes place. Subtract this value from the former value, and the remainder is the preſent value of the reverſion.

EXAMPLE. What is the value of an Eſtate or an Annuity of £110 per annum, to continue 20 years? What is the value of the ſame, after the expiration of 20 years, to continue for ever? and what is the value of the whole, at 6 per cent. Compound Intereſt?

.06)130.00(2166.6666	Value of the whole.
1491.0896	Value of the poſſeſſion.
.675.5770	Value of the reverſion.

PROBLEM

PROBLEM VII.

Having the Value of an Annuity in Reversion the time prior to the commencement of the Reversion and Rate of Interest, to find the Annuity.

Rule. Find the amount of the price of the reverſion, for the years prior to the commencement, by the Tables: then find the annuity, which that amount will purchaſe.

Example. If the reverſion of an Annuity to commence 20 years hence, be bought for £675.577, which is the annuity, Compound Intereſt, at 6 per cent?

Here by the tables, the amount of £675.577 for 20 years, at 6 per cent. is £2166.6, which will purchaſe an annuity of £130.

Life Annuities.

Annuities for Lives, are calculated from obſervations made on the bills of mortality, which, however imperfect they may ſeem to be in themſelves, when individually applied, they are nevertheleſs become in very general uſe, and are found to anſwer pretty accurately, for ſuch perſons, or public offices, who buy and ſell annuities.

It muſt however be obſerved, that calculations of this nature are at beſt but bare probabilities, or mere chance work; yet every perſon muſt be ſenſible, that a perſon of 60 years of age has not ſo great a chance of living as a perſon of 30, provided both be in good health; therefore, an Annuity to continue for the life of the former, cannot be worth near ſo much as an equal annuity for the life of the latter.

The principal writers in calculations of this kind are *Dr. Halley, Mr. Simpſon, Monſieur de Moivre,* and *Dr. Price,* each of whom have formed tables on the probability of human life, deduced from the bills of mortality.

Dr. Halley,

Dr. Halley from the bills of mortality at Breſlaw, conſtructed a Table in which he ſhews how many perſons died in every year out of 1000 till the death of the laſt. He choſe the city of Breſlaw, the capital of Sileſia, for this purpoſe, which he thought might ſerve as a ſtandard to the reſt of Europe, being a central town, at a diſtance from the ſea, and not much crowded with foreigners. His Table is, however, found not to be ſo well adapted to the probability of human life in England, as thoſe of ſome others ſince his time. I ſhall, however, give his table, as well as that of *Mr. Simpſon*, and *Dr. Price*, that the reader may ſee the comparative merit of each.

Dr. HALLEY's TABLE,

CALCULATED

From the Bills of Mortality, at Breslaw.

TABLE I.

Age.	Persons living.	Decrement of Life.	Age.	Persons living.	Decrement of Life.	Age.	Persons living.	Decrement of Life.
1	1000	145	32	515	8	63	212	10
2	855	57	33	507	8	64	202	10
3	798	38	34	499	9	65	192	10
4	760	28	35	490	9	66	182	10
5	732	22	36	481	9	67	172	10
6	710	18	37	472	9	68	162	10
7	692	12	38	463	9	69	152	10
8	680	10	39	454	9	70	142	11
9	670	9	40	445	9	71	131	11
10	661	8	41	436	9	72	120	11
11	653	7	42	427	10	73	109	11
12	646	6	43	417	10	74	98	10
13	640	6	44	407	10	75	88	10
14	634	6	45	397	10	76	78	10
15	628	6	46	387	10	77	68	10
16	622	6	47	377	10	78	58	9
17	616	6	48	367	10	79	49	8
18	610	6	49	357	11	80	41	7
19	604	6	50	346	11	81	34	6
20	598	6	51	335	11	82	28	5
21	592	6	52	324	11	83	23	3
22	586	7	53	313	11	84	20	5
23	579	6	54	302	10	85	15	4
24	573	7	55	292	10	86	11	3
25	567	7	56	282	10	87	8	3
26	560	7	57	272	10	88	5	2
27	553	7	58	262	10	89	3	2
28	546	7	59	252	10	90	1	1
29	539	8	60	242	10	91	0	0
30	531	8	61	232	10			
31	523	8	62	222	10			

TABLE II.

SHEWING

The Probabilities from 10 *Years observation on the Bills of Mortality in London, by Mr. Simpson.*

Age.	Perfons living.	Decrement of Life.	Age.	Perfons living.	Decrement of Life.	Age.	Perfons living.	Decrement of Life.
0	1000	320	27	321	6	54	135	6
1	680	133	28	315	7	55	129	6
2	547	51	29	308	7	56	123	6
3	496	27	30	301	7	57	117	5
4	469	17	31	294	7	58	112	5
5	452	12	32	287	7	59	107	5
6	440	10	33	280	7	60	102	5
7	430	8	34	273	7	61	97	5
8	422	7	35	266	7	62	92	5
9	415	5	36	259	7	63	87	5
10	410	5	37	252	7	64	82	5
11	405	5	38	245	8	65	77	5
12	400	5	39	237	8	66	72	5
13	395	5	40	229	7	67	67	5
14	390	5	41	222	8	68	62	4
15	385	5	42	214	8	69	58	4
16	380	5	43	206	7	70	54	4
17	375	5	44	199	7	71	50	4
18	370	5	45	192	7	72	46	4
19	365	5	46	185	7	73	42	3
20	360	5	47	178	7	74	39	3
21	355	5	48	171	6	75	36	3
22	350	5	49	165	6	76	33	3
23	345	6	50	159	6	77	30	3
24	339	6	51	153	6	78	27	2
25	333	6	52	147	6	79	25	2
26	327	6	53	141	6	80	23	2

TABLE III.

The Probabilities of Life, calculated from 46 *Years observation, on the Bills of Mortality, at Northampton: viz. from* 1735 *to* 1780.

Age.	Perfons living	Decrement of Life.	Age.	Perfons living	Decrement of Life.	Age.	Per fons living.	Decrement of Life.
0	11650	1894	34	4085	75	69	1312	80
½	9756	1106	35	4010	75	70	1232	80
1	8650	1367	36	3935	75	71	1152	80
2	7283	502	37	3860	75	72	1072	80
3	6781	335	38	3785	75	73	992	80
4	6446	197	39	3710	75	74	912	80
5	6249	184	40	3635	76	75	832	80
6	6065	140	41	3559	77	76	752	77
7	5925	110	42	3482	78	77	675	73
8	5815	80	43	3404	78	78	602	68
9	5735	60	44	3326	78	79	534	65
10	5675	52	45	3248	78	80	469	63
11	5623	50	46	3170	78	81	406	60
12	5573	50	47	3092	78	82	346	57
13	5523	50	48	3014	78	83	289	55
14	5473	50	49	2936	79	84	234	48
15	5423	50	50	2857	81	85	186	41
16	5373	53	51	2776	82	86	145	34
17	5320	58	52	2694	82	87	111	28
18	5262	63	53	2612	82	88	83	21
19	5199	67	54	2530	82	89	62	16
20	5132	72	55	2448	82	90	46	12
21	5060	75	56	2366	82	91	34	10
22	4985	75	57	2284	82	92	24	8
23	4910	75	58	2202	82	93	16	7
24	4835	75	59	2120	82	94	9	5
25	4760	75	60	2038	82	95	4	3
26	4685	75	61	1956	82	96	1	1
27	4610	75	62	1874	81		299198	11650
28	4535	75	63	1393	81			
29	4460	75	64	1712	80			
30	4385	75	65	1632	80			
31	4310	75	66	1552	80			
32	4235	75	67	1472	80			
33	4160	75	68	1392	80			

TABLE IV.

The Expectation of Life at every Age, according to the Bills of Mortality, for both London and Northampton.

Age.	Expectation. London	Expectation. North.	Age.	Expectation. London	Expectation. North.	Age.	Expectation. Lon.	Expectation. North.
0	19.5	25.18	34	22.4	26.20	68	9.9	9.50
1	27.5	32.74	35	22.0	25.68	69	9.6	9.05
2	32.5	37.79	36	21.6	25.16	70	9.3	8.60
3	34.5	39.55	37	21.2	24.64	71	8.9	8.17
4	36.1	40.58	38	20.8	24.12	72	8.6	7.74
5	36.5	40.84	39	20.4	23.60	73	8.3	7.33
6	36.5	41.07	40	20.1	23.08	74	8.0	6.92
7	36.3	41.03	41	19.7	22.56	75	7.7	6.54
8	36.1	40.79	42	19.3	22.04	76	7.3	6.18
9	35.7	40.36	43	19.0	21.54	77	6.9	5.83
10	35.3	39.78	44	18.6	21.03	78	6.5	5.48
11	34.8	39.14	45	18.3	20.52	79	6.0	5.11
12	34.2	38.49	46	17.9	20.02	80	5.5	4.75
13	33.6	37.83	47	17.5	19.51	81		4.41
14	33.0	37.17	48	17.2	19.00	82		4.09
15	32.4	36.51	49	16.8	18.49	83		3.80
16	31.8	35.85	50	16.5	17.99	84		3.58
17	31.2	35.20	51	16.1	17.50	85		3.37
18	30.6	34.58	52	15.7	17.02	86		3.19
19	30.0	33.99	53	15.4	16.54	87		3.01
20	29.4	33.45	54	15.0	16.06	88		2.86
21	28.8	32.90	55	14.7	15.58	89		2.66
22	28.2	32.39	56	14.3	15.10	90		2.41
23	27.7	31.88	57	13.9	14.63	91		2.09
24	27.1	31.36	58	13.6	14.15	92		1.75
25	26.6	30.85	59	13.2	13.68	93		1.37
26	26.1	30.33	60	12.9	13.21	94		1.05
27	25.6	29.82	61	12.5	12.75	95		0.75
28	25.1	29.30	62	12.1	12.28	96		0.50
29	24.6	28.79	63	11.7	11.81			
30	24.1	28.27	64	11.3	11.35			
31	23.6	27.76	65	11.0	10.88			
32	23.1	27.24	66	10.6	10.42			
33	22.8	26.72	67	10.2	9.06			

TABLE V.

The Value of £1 Annuity on a single Life, according to the probabilities of Life, at London and Northampton, from the Age of 6 *to* 75 *Years; at* 3, 4, *and* 5 *per Cent.*

	3 per Cent.		4 per Cent.		5 per Cent.			3 per Cent.		4 per Cent.		5 per Cent.	
Age.	Lon.	North	Lon.	North	Lon.	North	Age.	Lon.	North	Lon.	North	Lon.	North
6	18.8	20.73	16.2	17.48	14.1	15.04	41	13.0	14.62	11.4	13.02	10.2	11.70
7	18.9	20.85	16.3	17.61	14.2	15.17	42	12.8	14.39	11.2	12.84	10.1	11.55
8	19.0	20.89	16.4	17.66	14.3	15.23	43	12.6	14.16	11.1	12.66	10.0	11.41
9	19.0	20.81	16.4	17.63	14.3	15.21	44	12.5	13.93	11.0	12.47	9.9	11.26
10	19.0	20.66	16.4	17.52	14.3	15.14	45	12.3	13.69	10.8	12.28	9.8	11.11
11	19.0	20.44	16.4	17.39	14.3	13.04	46	12.1	13.45	10.7	12.09	9.7	10.95
12	18.9	20.28	16.3	17.25	14.2	15.94	47	11.9	13.20	10.5	11.89	9.5	10.78
13	18.7	20.08	16.2	17.10	14.1	14.83	48	11.8	12.95	10.4	11.68	9.4	10.62
14	18.5	19.87	16.0	16.95	14.0	14.71	49	11.6	12.69	10.2	11.47	9.3	10.44
15	18.3	19.69	15.8	16.79	13.9	14.59	50	11.4	12.44	10.1	11.26	9.2	10.27
16	18.1	19.44	15.6	19.62	13.7	14.46	51	11.2	12.18	9.9	11.06	9.0	10.10
17	17.9	19.22	15.4	16.46	13.5	14.33	52	11.0	11.93	9.8	10.85	8.9	9.93
18	17.6	19.01	15.2	16.31	13.4	14.22	53	10.7	11.67	9.6	10.64	8.8	9.75
19	17.4	18.82	15.0	16.17	13.2	14.11	54	10.5	11.41	9.4	10.42	8.6	6.57
20	17.2	18.64	14.8	16.03	13.0	14.01	55	10.3	11.15	9.3	10.20	8.5	9.38
21	17.0	18.45	14.7	17.91	12.9	13.92	56	10.1	10.83	9.1	9.98	8.4	9.19
22	16.8	18.31	14.5	15.80	12.7	13.83	57	9.9	10.61	8.9	9.75	8.2	9.00
23	16.5	18.15	14.3	15.68	12.6	13.75	58	9.6	10.34	8.7	9.52	8.1	8.80
24	16.3	17.98	14.1	15.56	12.4	13.66	59	9.4	10.06	8.6	9.28	8.0	8.60
25	16.1	17.81	14.0	15.44	12.3	13.57	60	9.2	9.78	8.4	9.09	7.9	8.39
26	15.9	17.64	13.8	15.31	12.1	13.47	61	8.9	9.49	8.2	8.80	7.7	8.18
27	15.6	17.47	13.6	15.18	12.0	13.38	62	8.7	9.21	8.1	8.55	7.6	7.97
28	15.4	17.29	13.4	15.05	11.8	13.28	63	8.5	8.91	7.9	8.29	7.4	7.74
29	15.2	17.11	13.2	14.92	11.7	13.18	63	8.3	8.61	7.7	8.03	7.3	7.51
30	15.0	16.92	13.1	14.78	11.6	13.07	65	8.0	8.30	7.5	7.76	7.1	7.28
31	14.8	16.73	12.9	14.64	11.4	12.97	66	7.8	7.99	7.3	7.49	6.9	7.03
32	14.6	16.54	12.7	14.50	11.3	12.85	67	7.6	7.68	7.1	7.29	6.7	6.79
33	14.4	16.34	12.6	14.35	11.2	12.74	68	7.4	7.37	6.9	6.93	6.6	6.54
34	14.2	16.14	12.4	14.20	11.0	12.62	69	7.1	7.05	6.7	6.65	6.4	6.28
35	14.1	15.94	12.3	14.04	10.9	12.50	70	6.9	6.73	6.5	6.36	6.2	6.02
36	13.9	15.73	12.1	13.88	10.8	12.38	71	6.7	6.42	6.3	6.08	6.0	5.76
37	13.2	15.52	12.0	13.72	10.6	12.25	72	6.5	6.10	6.1	5.79	5.8	5.50
38	13.5	15.30	11.8	13.55	10.5	12.12	73	6.2	5.79	5.9	5.51	5.6	5.24
39	13.3	15.08	11.6	13.38	10.4	11.98	74	5.9	5.49	5.6	5.23	5.4	4.99
40	13.2	14.85	11.5	13.20	10.3	11.84	75	5.6	5.20	5.4	4.96	5.2	4.74

THE USE OF THE TABLES.

Table I.

Shews the probability of life, according to *Dr. Halley's* computation; the firſt column ſhews the ages; the ſecond column, the number of perſons living at thoſe ages, and the third column, the decrement of life, or the number of perſons that died each year: thus, oppoſite age 1, is 1000 in the ſecond column, and 145 in the third column, which ſhews that of 1000 perſons born, in the ſame year, 145 died before the expiration of the year; and of 855 the remainder of the perſons living, 57 died the ſecond year and ſo on.

But the calculations according to this table, differ from the probabilities of life in London, partly owing to the different ſituations of theſe two places. Breſlaw being an inland town not much frequented by ſtrangers or foreigners, and London being a mercantile port, and crowded with traffickers and travellers from all parts of the world; and partly owing to the difference of climate, difference of food, and different manners of life, between the inhabitants of theſe two places, which is always found to occaſion a different proportion in the deaths at the ſame ages. Theſe conſiderations induced *Mr. Simpſon* to compoſe a table of the probability of life, calculated from the bills of mortality of London, and which table will conſequently much better anſwer the purpoſe of calculating the value of an annuity for a life at London, than the other table by *Dr. Halley*. This Table may be ſeen page 51.

It muſt here be obſerved that *Dr. Halley's* Table is better adapted for the uſe of all Europe in general, than any other particular table.

Table III.

Shews the probability of life, at all ages, from 46 years obſervation on the bills of mortality, at Northampton: viz. from

from the year 1735 to 1780 inclusive. This table is justly reckoned to be the most correct of any extant, as it is taken from a large manufacturing town, and which consists generally of the same persons, and also that the table is constructed from a larger number of persons born, than any other table being 11650, which affords an opportunity of observing the decrement of life, to a greater exactness.

Table IV.

Shews the expectation of life, or the number of years which any person may be supposed to have a fair chance of living, according to an equality of chance at every age, according to the bills of mortality, for both London and Northampton. Thus, against age 2, stands 32.5 in the second column under London, and 37.79 in the third column under Northampton, which shews that a child of the age of two years, has an equal chance of living 32.5 years, according to the London tables, or 32 years 6 months, and according to the Northampton tables 37,79 years, or 37 years and upwards of 9 months.

Table V.

Shews the value of an annuity for a single life, according to the probabilities of life at London and Northampton, from the age of 6 to 75 years inclusive, at 3, 4 and 5 per cent. Thus, to find the number of years purchase, which an annuity of £1 is worth to a person of the age of 30. Here in the second column and opposite the age 30, stands 15.0 which shews that an annuity of £1, for a person of 30 years of age in London, and at 3 per cent. is worth 15 years purchase, and the same annuity for a Northampton life, is worth 16.92 years purchase; and for a London life, at 4 per cent. the same annuity is worth 13.1 years purchase, and for a Northampton life, at the same rate and interest,

is worth 14.78 years purchaſe, and the ſame annuity at 5 per cent. for a London life, is worth 11.6 years purchaſe; and for a Northampton life, at the ſame rate of intereſt, 13.07 years purchaſe. This table alſo ſhews the value of an annuity of £1 for a ſingle life, at all the above mentioned rates of intereſt; thus, the aforeſaid annuity for a ſingle life, at 30 years of age, according to the London tables, at 3 per cent. is worth £15, and the ſame, according to the Northampton tables, is worth £16, and upwards of 18 ſhillings, &c.

This table is eſteemed the beſt of any extant, and preferable to any other of a different form. But thoſe who ſell annuities have generally a table of 2 years more value than the lives in this table, for purchaſers who are upwards of twenty years of age.

Difinitions.

1. The probability of life is the chance that any perſon or perſons have of living to any certain time, and is denoted by a fraction, whoſe numerator is the chance of living, and denominator that of dying. Thus, if it were required to find the probability of a perſon of the age of 20, attaining to the age of 37, according to *Mr. Simpſon's* Table. Here it muſt be obſerved, that of 360 perſons living at the age of 20, only 252 ſurvives to the age 37; therefore, 108 perſons have died between the two ages. Thus, 252 is the chance of the ſaid perſons living to the age of 37 and 108, the chance of the ſaid perſons dying before he attains the age of 37, and the probability of life of that perſon, is expreſſed by the fraction $\frac{252}{360}$ or $\frac{7}{10}$; therefore, the odds in that perſon's favour or chance that he ſhall live to that age is as 7 to 3.

2. The probability of dying is expreſſed by a fraction, which is the difference between the former fraction and unity. Thus, the probability that the aforeſaid perſon ſhall die before the age of 37 is expreſſed by $\frac{108}{360}$ or $\frac{3}{10}$, which

ſhews that the chance of that perſon's dying before the ſaid age is as 3 to 7.

3. The extremity of life, is the period beyond which there is no probability of ſurviving. In the Northampton tables this is 96 years.

4. The compliment of life is the number of years which any perſons age wants of the full extremity of life.

5. The expectation of life is the number of years due to the life of a perſon of a certain age, upon an equality of chance. And it is the number of years purchaſe, which an annuity for life is worth, in ready money, without allowing any intereſt. And in ſingle lives it is always equal to the ſum of all the probabilities of ſurviving to the extremity of life.

6. The number of years purchaſe of annuities, at any rate of intereſt, is that number which, if multiplied by the annuity, is equal to the preſent value thereof, according to ſuch rate of intereſt; therefore, it is the preſent value of an annuity of £1, according to a given rate of intereſt, as ſeen in Table V.

7. Reverſion of a life annuity, is where two or more lives are in joint poſſeſſion, and the expectation depends upon the probability of one particular life ſurviving the reſt.

PROBLEM I.

To find the Value of an Annuity for the Life of any Person, at a given Rate of Interest.

RULE. Seek the age of the perſon in the firſt column of Table V. and againſt it, under the proper intereſt, is the number of years purchaſe for either the London or Northampton life, or which is the ſame thing, the preſent value of an annuity of £1, during ſuch life. Multiply this by the annuity, and the product is the anſwer.

EXAMPLE.

Example. Suppofe the given age be 49, the rate of intereft 3 per cent. and the annuity £20. Here again 49 and under, 3 per cent. ftands 11.6 according to the London bills which multiplied by £20, gives £232 for the value of a London life. And in the next column ftands 12.69, the value according to the Northampton bills, which multiplied by £20, as before, produces £253 16*s*. for the value of a Northampton life.

SECT. II.

OF THE VALUE OF AN ANNUITY DURING JOINT LIVES.

PROBLEM I.

To find the Value of an Annuity, for the joint continuance of two Lives, that is, one Life failing, the Annuity to cease.

CASE I.

When both persons are of the same Age.

Rule. Find the value of any one of the lives from Table V. Multiply this value by the intereft of £1, for a year; fubtract the product from 2, divide the aforefaid value by this remainder, and the quotient will be the value of £1 annuity, for the number of years purchafe.

Example. What is the value of £100 annuity, for the joint lives of two perfons, aged 40 years each, according to the London tables, reckoning intereft at 5 per cent? Here, by the table, one life for 40 years is

	10.3
Multiply by	.05
Subtract this product	.515
From	2.000
Remains	1.485

And 1.485)10.3(6.9 value of £1 annuity, which multiplied by 100 is £690, the value of an annuity sought.

CASE II.

When the two persons are of different Ages.

Rule. Find the values of the two lives in Table V. Multiply them one into the other, and call the result the first product; then multiply the said first product by the interest of £1 for a year, calling the result the second product; add the values of the two lives together, and from the sum, subtract the second product; divide the first product by the remainder, and the quotient will be the value of £1 annuity, or the number of years purchase.

Example. What is the value of £50 annuity, for the joint lives of two persons; whereof one is 20, and the other 30 years of age, according to the Northampton Tables, interest at 4 per cent?

The value of 20 years is	16.03
And the value of 30 years is	14.78
First product	236.9234
	.04
Second product	9.476936
Sum of the two lives	30.810000
Remainder	21.333064

And 21.333064)236.9234(11.1 value of £1 annuity.

50	
555.0	Value required.

PROBLEM

PROBLEM III.

To find the Value of an Annuity, during the longest Survivor of two Lives.

Rule. From the ſum of the values of the ſingle lives, ſubtract the value of the joint lives, and the remainder will be the value ſought.

Example. What is the value of an annuity of £1, to continue during the longeſt of two lives; the one perſon being 30, and the other 40 years of age; intereſt at 4 per cent. the life of 30 years of age valued according to the London bills, and that of 40 years of age according to the Northampton bills.

By the table, the value of 30 years is - -	13.1
The value of 40 years is - - - -	13.20
	26.30
The value of their joint lives, by Problem 2, Caſe 2, is - - - - - - -	8.9
The value ſought - - - - -	17.4

If the annuity be any other than £1, multiply the above found value, by the given annuity; and if the two perſons be of equal ages, the value of their joint lives muſt be found by Caſe 1, of Problem II.

PROBLEM IV.

To find the value of an Annuity, for the joint continuance of three Lives; that is, one Life failing, the Annuity to cease.

Rule. Multiply the value of the three ſingle lives, continually into each other, calling the reſult the product of the three lives; multiply that product by the intereſt of £1, and that product again by 2, calling the reſult the double product; then from the ſum of the ſeveral products of the ſaid lives, taken

taken two and two, ſubtract the double product; divide the product of the three lives by the remainder, and the quotient will be the value of the three joint lives.

EXAMPLE. What is the value of an annuity of £1, during the joint lives of three perſons, whereof A is 10 years of age, B 20, and C 30, at 4 per cent. according to the London tables?

Here by the Table, the value of A's life is 16.4, that of B's 14.8, and C's 13.1, which three multiplied together, is 3179.63; this multiplied by .04, the intereſt of £1, gives 127.18528, which multiplied again by 2, gives 254.370 for the double product. Then

The product of A and B is	- -	242.72
And the product of A and C is	-	214.84
The product of B and C is	-	203.88
The ſum of all taken two and two		661.44
Double product to ſubtract	-	254.37
	Remainder	407.07

And 407.07)3179.632(7.8 val. ſought

PROBLEM V.

To find the value of an Annuity, for the longest Life of three or more Persons.

RULE. Find an age anſwerable to the value of the longeſt life of any two lives, which ſubſtitute in lieu of the two, and then find the value of the longeſt life of that life and the other, and that will be the value of the longeſt life, among three lives. If there are four or more lives, ſubſtitute the age correſponding to this value, in lieu of the three lives, and find the value of the longeſt life of this age, and the other remaining age, and it will be the value of the longeſt of four lives; proceed in the ſame manner for five or more lives, having regard to the rate of intereſt.

The examples in the foregoing Problems, will be found ſufficient to inſtruct the learner, how to perform all the following Problems;

Problems; I ſhall, therefore, give a few Problems with their rules, leaving their operation for the exerciſe of the learner.

SECT. III.

THE VALUE OF CONTINGENT REMAINDERS AND REVERSIONS.

PROBLEM I.

To find the value of a reverſion of an aſſigned Life, after a given Term.

Rule. Subtract the value of the annuity for the given term of years, from the value of the propoſed life, on the contingency of its ceaſing, upon the extinction of the aforeſaid life, and the remainder will be the anſwer.

PROBLEM II.

To find the value of the reverſion of an Annuity, for the remainder of the given terms of Years, after an aſſigned Life.

Rule. From the value of the annuity, certain for the given term of years, ſubtract the value of the annuity for the ſaid term, on the contingency of its ceaſing, upon the failing of the propoſed life, and the remainder will be the value of the reverſion.

PROBLEM

PROBLEM III.

To find the value of the reversion of one Life after another.

Rule. Subtract the value of the two joint lives, from the value of the life in expectation, and the remainder will be the value of the reversion.

PROBLEM IV.

To find the value of the reversions of two Lives after one.

Rule. Subtract the value of the life in possession, from the value of the longest of the three lives, and the remainder will be the value of the reversion.

PROBLEM V.

To find the value of a reversion of one Life, after two Lives.

Rule. If the two lives are joint lives, subtract the value of the three joint lives, from the value of the life in expectation, and the remainder will be the answer. But if the reversion takes place, after the extinction of two separate lives, subtract the value of the two lives in possession, from the value of the three lives, and the remainder will be the value of the reversion.

PROBLEM VI.

To find the present value of any number of Lives in succession.

Rule. Multiply the value of each life, by the interest of £1, for one year, and subtract each product from unity or 1; multiply all the remainders continually together, and subtract this product from unity; then the remainder multiplied

plied by the perpetuity*, will be the value of all the ſucceſſive lives.

PROBLEM VII.

A given ſum of money is to be received as a legacy on the deceaſe of D, who is at a given age; what is the value thereof in preſent money?

RULE. Subtract the value of the life of D from the perpetuity; then ſay, as the perpetuity is to the remainder, ſo is the propoſed ſum to the preſent value.

PROBLEM VIII.

To find the value of a Sum of Money, to be received at the deceaſe of B, in caſe A is then deceaſed alſo.

RULE. Subtract the value of the oldeſt life from the value of an annuity for as many years as are expreſſed by the complement of B's age; then ſay, as the complement of the elder life is to the remainder, ſo is the propoſed ſum to its preſent value.

ANNUITIES UPON TONTINES.

PROBLEM IX.

To find the value of an Annuity for either perſon of two, who have a joint Annuity, which at the deceaſe of either one, is to become the ſole property of the ſurvivor.

* The perpetuity or value of an annuity, to continue for ever, is found by dividing 100*l.* by the rate of intereſt per cent. or by dividing 1*l.* by the intereſt of 1*l.* for a year; and the quotient is the perpetuity, or value of an annuity of 1*l.* to continue for ever. The perpetuity is alſo equal to the number of years purchaſe, which a perpetual annuity is worth, without allowing any intereſt.

RULE. From the value of the life of either of the two perſons, ſubtract half the value of the two joint lives, and the remainder will be the value of the other perſons.

PROBLEM X.

What is the preſent value of an Annuity to be poſſeſſed by D, and his heirs as ſoon as any two of the three lives, A, B and C become extinct, and to hold the ſame, during the life of the ſurvivor of the three lives A, B, C?

RULE. Add thrice the value of the three joint lives, A, B and C, to the ſum of the three ſingle lives, deducting therefrom, twice the ſum of each two joint lives: viz.—Of A and B, A and C, and B, C, and the remainder will be the anſwer.

PROBLEM XI.

What is the value of the right of any one of the three following perſons: viz.—A, B and C, who enjoy an annuity equally among them, which, upon the deceaſe of any one, is to become the property of the two ſurvivors, during their joint lives, and on the deceaſe of the next perſon to become the property of the laſt ſurvivor, during his life?

RULE. Subtract half the ſum of the values of the joint lives, A and B, and the joint lives of A and C, from the value of the life of A; then to the remainder add one third of the value of the three joint lives, and the ſum will be the anſwer.

PROBLEM XII.

What is the value of the two ſucceſſive lives, A and B, A having an Annuity for life, and to have the nomination of a ſucceſſor, who is to hold the Annuity for his own life, at the deceaſe of A?

RULE.

Rule. Multiply the value of the life of A, by the value of the life put in at his deceaſe; divide the product by the perpetuity, and ſubtract the quotient from the ſum of the ſaid values, and the remainder will be the anſwer.

SECT. IV.

OF ASSURING LIVES.

By Assuring a life, is meant, obtaining ſecurity for receiving a certain ſum of money, ſhould the aſſured life fail in a certain given time: in conſideration of which, a premium is given to the aſſurer, which is a ſufficient compenſation for the loſs he is likely to ſuſtain in caſe the life ſhould drop. The ſum at which this compenſation ſhould be valued, is varied according to the two following cauſes:—Firſt. The rate of intereſt at which the money is ſuppoſed to be improved; and, ſecondly, the probability of the duration of the life to be aſſured. If the intereſt be high, and the probability of life high alſo, the value of the aſſurance will be low in proportion: on the contrary, if the intereſt be low, and the probability of life alſo low, the value of the aſſurance will be proportionably high. For example:—Let £100 be ſuppoſed to be aſſured on a life, for 1 year; that is, let £100 be payable a year hence, provided a perſon of a given age dies in that time. Now, if the intereſt of money be 5 per cent. and the life *ſure* of failing, the value of the aſ-

ſurance would be the ſame as the preſent value of £100, payable at one years end, reckoning intereſt at 5 per cent. and would be that ſum, which being put out to intereſt now, at 5 per cent. would produce the £100 at the end of the year, which ſum is £95 4 8.

If it be an even chance, or the odds are equal, whether the life does or does not fail in the year, the value of the aſſurance will be half as much as the former value, or £47 12 4.

If the odds againſt the perſon's life failing are two to one, which is the caſe when one third of a given number of lives fail in the time, the value of the aſſurance will alſo be one third of the firſt value, (if the intereſt be the ſame) or £31 15 0.

If the odds are nineteen to one againſt the life failing, which is the caſe when a twentieth part of the lives fail in the given time, the value of the aſſurance will be a twentieth part of the firſt value, or £4 16 0.

If the odds are forty-nine to one againſt the life failing, or when only one out of fifty of ſuch lives fail in the given time, the aſſurance will be only a fiftieth part of the firſt value, or £1 18 0.

Now the odds of two to one, according to *Dr. Halley's* table are, that a life aged 87 years will not drop in a year. The odds of nineteen to one are that a life aged 64 will not drop in a year. And the odds of forty-nine to one are that a life aged 39 will not drop in a year, Therefore, the value of the aſſurance of £100 for one year, on a life aged 87 is £31 15 0;—on a life aged 64, £4 16 0;—on a life aged 39, £1 18 0, at 5 per cent intereſt. But if intereſt be reckoned at 3 per cent, theſe three values will be £32 7—£4 17 0—£1 18 10 reſpectively.

This calculation ſuppoſes the value of the aſſurance to be paid in one ſingle preſent payment. But the value may be paid in *annual payments*, and to be continued till the failure of

of the life, ſhould that happen within the given term; or, if not, till the determination of the time.

The value of an aſſurance upon a life, cannot be diſcovered by any one ignorant of the method of calculating the value of life annuities, delivered in the former part of this chapter. But thoſe who underſtand what has been delivered, may form any calculations upon this ſubject, from the following examples:

Example 1. What is the value of an aſſurance for £100, for 26 years, on a life aged 39, intereſt at 5 per cent?

Here the perſon whoſe life is to be aſſured for £100, is 39 years of age, and the aſſurance is to be continued for 26 years, or till he is 65 years of age, provided he lives ſo long.

CASE I.

When the value of the Assurance is to be paid in fixed annual payments.

In this caſe, the firſt year's payment for the aſſurance is to be made immediately, (according to the rule in theſe caſes) and to be continued every year for 26 years, if the life continues ſo long. Now the value for the aſſurance for the firſt year is found to be £1 18 0;—the value of the aſſurance for the twenty-ſixth year, or laſt year of the term, ſuppoſing the aſſured to live ſo long, is found to be £4 16 0. Therefore, if the value of the aſſurance for the whole 26 years, is to be one conſtant ſum, payable at the beginning of each year, that ſum ought to be greater than the firſt payment, and leſs than the laſt, or ſome mean between £1 18 0 and £4 16 0. To find this mean in all caſes, we have recourſe to the following

Rule. From the value of an annuity, certain for the given term, ſubtract the value of the life for the given term, reſerving

reserving the remainder. Multiply the value of £1 due at the end of the given term, by the perpetuity, and that product by the probability of life. The product added to the reserved remainder, and the sum multiplied by the annual interest of the sum to be assured, and then divided by £1 increased by its interest for one year, and also by the value already mentioned, with unity added, will be the required value of the assurance of the life, for the given term, in a fixed annual payment till the expiration of the term, or the failure of the life within the term.

Now, the value of the life of a person aged 39, for 26 years, at 5 per cent, according to *Mr. de Moivere's* table, is 11.113, this value subtracted from 14.375, the value of an annuity certain for 26 years, (by table 2,) leaves 3.262, the remainder, to be reserved.

Again, the value of £1 to be received at the end of 26 years is .2812. The probability that the life of a person aged 39, shall fail in 26 years, is according to *Dr. Halley's* table $\frac{262}{454}$ and the perpetuity is 20; these numbers multiplied continually together, and 3,262 added to the product, make 6 508, which multiplied by £5, the annual interest of £100, gives £32.54, which divided by 1,05, or £1 increased by its interest for one year, gives £31. And 31 divided by 12,113, the value of the life for 26 years, with unity added, gives £2.56, or £2 11 $2\frac{1}{2}$, which is the required value in fixed annual payments.

CASE II.

When the value of the Assurance is to be paid in one present payment.

This sum is evidently equal to an annuity on the life of the person for the given term, and equal to the fixed annual payment,

payment, and is therefore the ſum ariſing in the foregoing operation, before it is divided by the value of the life for the given term, and is £31.

CASE III.

When the Assurance is to be made for the whole duration of life.

RULE. Subtract the value of the given life from the perpetuity, and multiply the remainder by the product of the given ſum into the intereſt of the £100 for a year; and this laſt product divided by £100, increaſed by its intereſt for a year, will give the value in a ſingle preſent payment. And this payment divided by the value of the life, will conſequently give the value of the aſſurance in fixed annual payments during the continuance of life.

EXAMPLE. What is the value of the aſſurance of £100, for a life aged 39, the ſum to be aſſured for the whole duration of life, at 5 per cent intereſt?

In this example, the value of the life, according to *Monſieur de Moivre*, for the whole continuance of life is 11.966. Which ſubtracted from the perpetuity, viz. 20, leaves £8.034, which multiplied by the product of £100, multiplied by 5, or by £500, gives 4017; and this divided by 105, or by £100 increaſed by its intereſt for a year, gives £38.25, which is the value of the aſſurance in a ſingle payment. And which divided by 11.966 is 3.196, the value of the ſame aſſurance in fixed annual payments for life.

Note. When the value of the aſſurance is required in fixed annual payments, it is generally underſtood that the firſt payment is not made till the end of a year. But when the firſt payment is to be made immediately, the value of the whole ſingle payment muſt be divided by the value of the life, increaſed by unity; that is in the preſent inſtance, by £12,966, which will make the required annual value of the aſſurance

aſſurance £2.95, inſtead of £3.196, or £2 19, inſtead of £3 3 11.

When an eſtate, or a perpetual annuity, is to be aſſured for the duration of another life, after the failure of the aſſured life, inſtead of aſſuring a groſs ſum, the value of a ſingle payment will be the value of the life ſubtracted from the perpetuity, and the remainder multiplied by the annuity, or by the rent of the eſtate. And the value in annual payments to begin immediately, will be the ſingle payment divided by the value of the life, increaſed by unity. Therefore, an aſſurance of an eſtate or annuity, after any given life or lives, is worth as much more than the aſſurance of a correſponding ſum, as £100 increaſed by its intereſt for a year, is greater than £100. Thus the preſent values, in ſingle and annual payments, of the aſſurance of an eſtate of £5 per annum for ever, and of £100 in money, are to one another as £105 is to £100. The reaſon of the difference is, that the Algebraical calculations, by which theſe values are determined, ſuppoſe the groſs ſum and the firſt yearly payment of the annuity are to be received at the ſame time, after the expiration of the life or lives.

The examples here given will be found ſufficient to inſtruct any perſon in the method of finding the value of annuities, in all caſes and reverſions, as alſo the principals of aſſurances upon lives.

CHAP.

CHAP. XI.

OF LOGARITHMS.

SECT. I.

OF THE ORIGIN AND NATURE OF LOGARITHMS.

LOGARITHMS are ratios of numbers, or numbers of ratio. And are the indices of the ratio of numbers to one another; or, the ſeries of numbers in an arithmetical proportion, anſwering to as many others in a geometrical proportion, and in ſuch a manner, that, 0 is the index of 1 in the geometricals. Logarithms were invented for the eaſe of arithmetical calculations, where the numbers, or operations, are large.

The nature of Logarithms depend upon theſe axioms; if a ſeries of quantities increaſe, or decreaſe, according to the ſame ratio, it is called a *geometrical progreſſion*, as the numbers 1, 2, 4, 8, 16, 32, which increaſed by 2, which is called the ratio; if the ſeries of quantities increaſe, or decreaſe, according to the ſame *difference*, it is called an *arithmetical progreſſion*, as the numbers 3, 6, 9, 12, 15, 18, &c. which increaſe by 3, which is therefore called their common difference. Now, if underneath the numbers proceeding in a geometrical progreſſion, be placed as many other numbers, pro-

ceeding in an arithmetical progreſſion, theſe laſt are called the Logarithms of the firſt; as in the following

Terms -	1.	2.	4.	8.	16.	32.	64.	128.	256.	512.
Logarithms	0.	1.	2,	3.	4.	5.	6.	7.	8.	9.

In this progreſſion, o is the Logarithm of 1, the firſt term: 1 the Logarithm of the ſecond, which is 2; and 2 the Logarithm of the third term, 4, &c.

Theſe indices, or logarithms, may be adapted to any ſeries in a geometrical progreſſion; and, therefore, there may be as many different kinds of indices, or logarithms, as there can be different kinds of geometrical progreſſions; as may be ſeen in the following ſeries:—

Log. -	.0	1.	2.	3.	4.	5.	&c.
	1.	2.	4.	8.	16.	32.	&c.
	or 2^0.	2^1.	2^2.	2^3.	2^4.	2^5.	&c.
	1.	3.	9.	27.	81.	243.	&c.
	or 3^0.	3^1.	3^2.	3^3.	3^4.	3^5.	&c.
	1.	10.	100.	1000.	10000.	100000.	&c.
	or 10^0.	10^1.	10^2.	10^3.	10^4.	10^5.	&c.

The Geometrical Progreſſion.

Here the ſame indices or logarithms ſerve for any of the ſix under written geometrical ſeries, from which it appears that there may be an endleſs variety of ſets of logarithms adapted to the ſame common numbers, by varying the ſecond term of the geometrical ſeries, as this will change the original ſeries of terms, whoſe indices are the numbers 1. 2. 3. &c. And by interpolotion the whole ſyſtem of numbers may be made to enter the geometric ſeries, and receive their proportionable logarithms, whether they be integers or decimals.

The logarithm of any number is the index of ſuch a power of ſome other number, as is equal to the given one. Thus, if N be equal to r^n, then the logarithm of N is n, which may be either poſitive or negative, and r any number whatever, according to the different ſyſtems of logarithms. When N is one, then n is o, whatever the value of r may be,

be, and consequently the logarithm of 1 is always 0 in every system of logarithms. But in the common logarithms, r is equal to 10, so that the common logarithm of any number is the index of that power of 10, which is equal to the said number; so the common logarithm of $N=10^n$, is n the index of the power of 10.—For example:—1000 being the third power of 10, has 3 for its logarithm: and if 50 be $= 10^{1.69897}$, then 1.69897 is the common logarithm of 50, from which it will follow, that the following decimal series of terms will have the following logarithms respectively.

The Geometric Series.	1000,	100,	10,	1,	.1,	.01,	.001
or	10^3,	10^2,	10^1,	10^0,	10^{-1}	10^{-2}	10^{-3}
Logarithms	3,	2,	1,	0,	−1,	−2,	−3.

The logarithm of a number, which is contained between any two terms of the first series, is included between the two corresponding terms of the latter series; therefore, that logarithm will have the same index, whether positive or negative, as the smaller of these two terms, together with a decimal fraction, which will always be positive. Thus, the number 50 falling between the numbers 10 and 100, its logarithm will fall between 1 and 2, being equal to 1.69897 nearly; and the number .05 falling between .1 and .01, its logarithm will fall between −1 and −2, and is equal to −2 +.69897, the index of the less term, together with the decimal .69896. The index is sometimes called the characteristic of the logarithms, and is always an integer, either positive or negative, or else 0, and shews what place is occupied by the first significant figure of the given number, either above or below the place of units, being in a former case, positive, and in the latter, negative.

When the characteristic of a logarithm is negative, the sign — is commonly set over it to distinguish it from the decimal part, which being the logarithm found in the tables, is always positive; thus, the logarithm of .05 or $-2+.69897$ is written thus, $\bar{2}.69897$. But when it is required to reduce

the whole expreſſion to a negative form, it is done by making the characteriſtic leſs by 1, and taking the arithmetical complement of the decimal, that is, beginning at the left hand, ſubtract each figure from 9, except the laſt ſignificant figure, which is ſubtracted from 10; then will the remainder form a logarithm, wholly negative; thus, the aforementioned logarithm $\bar{2}.69897$ or $-2+69897$ is expreſſed by -1.30103, which is all negative, ſometimes it is convenient to expreſs the logarithm as poſitive, which is done by joining to the tabular decimal, the complement of the index to 10; thus, the above logarithm is expreſſed by 8.69897, which is only increaſing the indices in the ſcale, by 10.

From the foregoing difinitions of logarithms, conſidered either as the indices of a geometric ſeries, or as the indices of the powers of the ſame root: it appears that numbers may be multiplied together by the addition of their logarithms: and they may be divided by the ſubtraction of their logarithms. Alſo a number may be raiſed to any power by multiplying the logarithm of the root by the index of the power: and the extraction of roots may be performed by dividing the logarithm of the given number by the index of the root required to be extracted.

Logarithms conſidered in their theory, are of very ancient origin, and were known to moſt of the ancients; but the celebrated *John Napier*, baron of Merchiſton, in Scotland, was the firſt who applied the uſe of them to Trigonometry; but the form of their conſtruction was not made known till the opinion of the Mathematicians was had concerning them. His ſon *Robert Napier*, in the year 1919, publiſhed a new Edition of his father's work, with the conſtruction of logarithms. And in the ſame year, *Mr. John Speldell*, publiſhed an Improvement of *Napier's* Logarithms.

Other Tables were ſoon after publiſhed by *John Kepler*, and ſome others; all which tables were of that kind, called Hyperbolical, becauſe the numbers expreſs areas between the aſymptote and curve of the hyperbola.

Henry

Henry Briggs, profeſſor af Geometry in Greſham College, ſoon after publiſhed the logarithms of the firſt one thouſand numbers on a new ſcale: viz.—In which the logarithm of the ratio of 10 to 1 is 1, whereas, the logarithm of the ſame ratio in *Napier's* ſyſtem is 2.30258, &c. And in 1624, he alſo publiſhed his Arithmetica Logarithmica, containing the Logarithms of 30,000 natural numbers, to 14 places of figures, beſides the index, which form was recommended to him by *Napier*, and which is a form now in preſent uſe. In 1633, he publiſhed to fourteen places of figures, his Trigonometrial Brittanica, which contained the natural and Logarithmic ſigns, tangents, ſecants, &c.

From the times of *Robert Napier*, to the year 1792, ſeveral Mathematicians publiſhed logarithmic tables, with various improvements, the principal of which were *Gunter*, *Wingate*, *Henrion*, *Miller*, and *Norwood*, *Cavalerius*, *Vlacq*, and *Rowe*, *Frobenius*, *Newton*, *Caramuel*, *Sherwin*, *Gardiner*, and *Dodſon*.

In *Napier's* logarithms, the natural numbers, and their logarithms, are ſuppoſed to be generated by the motions of points, deſcribing two lines, of which one is the natural number, and the other its logarithm. According to this ſuppoſition, the line, or length of the radius to be deſcribed is run over by a point, in ſuch a manner moving along it, that in equal portions of time, it generated or cut off parts in a decreaſing geometrical progreſſion; leaving the ſeveral remainders, or ſines, in geometrical progreſſion alſo; whilſt another point deſcribed equal parts of an indefinite line, in the ſame equal portions of time; ſo that the reſpective ſums of theſe, or the whole line generated, were always the arithmeticals, or logarithms, of the aforeſaid natural ſines. In this conſtruction, o is the logarithm of the greateſt ſine or radius; and, in limiting his ſyſtem, he aſſumed not any particuluar value, or aſſigned number, or part of the radius, but ſuppoſed that the two generating points, by the motions along

along the two lines, describe the natural number and logarithms; and had their velocities equal at the beginning of those lines. This is the reason that the natural sines and their logarithms in his table have equal differences, or increments, at the complete quadrant : and this is also the reason that his scale of logarithms happens accidentally to agree with what is now called the hyperbolical logarithms. which have likewise numeral differences, equal to those of their natural numbers at the beginning; except that these latter increase with the natural numbers, while his, on the contrary, decrease ; the logarithms 2.30258509, &c. being that of the ratio of 10 to 1 in both.

In addition to what has been said, it may be observed, that the indices or characteristic of logarithms correspond to the denominative part of the natural numbers, as the other member of the logarithm does the numerative part of the number ; that is the index shews the denomination of, or place of the left hand figures of the number, and consequently of all the rest. Thus 0 affixed to a logarithm, denotes the first figure of a number to which the logarithm answers to be nothing distant from the place of units. The index 1 shews the first figure of the number to be distant 1 from the place of units ; that is, in the place of tens, and consequently the number itself to be either 10, or some number between 10 and 100 ; and the same may be observed of all the other indices. Therefore, all numbers that have the same denominative, but not the same numerative parts, as all numbers from 1 to 10, from 10 to 100, &c. will have logarithms whose indices are the same, but the other members different. And again, all numbers, which have the same numerative, but not the same denominative parts, will have the same indices ; but the rest of the logarithms will be the same. If a number be purely decimal, to its logarithm is affixed a negative index, shewing the distance of its first significative figure from the place of units. Thus the logarithm of

of the decimal .256 is 1.408240; that of the decimal .0256 is $\bar{2}$.408240. Inſtead of theſe negative indices, ſome uſe their complements, to 10 ſet down with a point on each ſide, thus, .9. and .8. that is, ſuch a figure is made the index as when ſubtracted from 9, leave a remainder expreſſing the number of cyphers prefixed to the decimal, as before obſerved.

SECT. II.

OF THE ARITHMETIC OF LOGARITHMS.

To multiply or divide numbers by Logarithms.

Rule. To multiply two numbers together, add their two Logarithms together, and their ſum will be the logarithm of the product, when the logarithm of unity is 0. The reaſon of this rule is evident, for as unity is to one of the factors, ſo is the other factor to the product. Thus, the the logarithm of the product is a fourth equi-different term to the logarithm of unity and thoſe of the factors; but the logarithm of unity being 0, the ſum of the logarithms of the factors, muſt be the logarithm of the factum, or product.

Therefore, as the factors of a ſquare, are equal to each other, that is, a ſquare is the factum or product of its root multiplied into itſelf, the logarithm of the ſquare will be double the logarithm of the root.

Therefore, unity is to the exponent of the power, as the logarithm of a root to the logarithm of the power.

From hence it appears, that the logarithm of the cube is triple

triple the logarithm of the root; the logarithm of the biquadrate, quadruple that of the root; the logarithm of the fifth power, quintuple that of the root; that of the ſixth power, ſextuple, &c.

From hence we may perceive the great uſe of logarithms, which is to ſhorten the operation of multiplication, and conſequently involution of powers, and extraction of roots; the former of which is performed by mere addition, and the two latter by multiplication and diviſion. Thus 3 the ſum of the logarithms 1 and 2, is the logarithm of the natural number 8, the product of the two natural numbers 2 and 4. In like manner 7, the ſum of the logarithms 2 and 5, is the logarithm of 128, the product of 4 and 32. And 6, the the logarithm of 64, which is the third power of 4, is equal to 3×2. Again 8, the logarithm of 256, which is the fourth power of 4, is equal to 4 multiplied by 2. Moreover, 3 the logarithm of the ſquare root 8, is half the logarithm 6, which is the logarithm of the ſquare 64; and 2, the logarithm of the cube root 4, is ſubtriple the logarithm 6, that of the cube 64.

To perform the viſion by logarithms, the rule is, to ſubtract the logarithm of the diviſor, from the logarithm of the dividend, and the remainder will be the logarithm of the quotient, when the logarithm of unity is 0. For as the diviſor is to the dividend, ſo is unity to the quotient; therefore, the logarithm of the quotient is a fourth equi-different number to the logarithms of the diviſor, the dividend, and the logarithm of unity. The logarithm of unity, therefore, being 0, the difference of the logarithm of the diviſor, and that of the dividend is the logarithm of the quotient. For the difference between 7 and 5, or the remainder, is 2, which is the logarithm of the quotient 4, which quotient is obtained by dividing 128 by 32. Alſo, ſubtracting 3 from 8, the remainder 5 is the logarithm of the quotient 32, which is obtained by dividing 256 by 8.

Examples

Examples in Multiplication.

	Num.	Log.		Num.	Log.
Multiply	68	1.832509	Multiply	9	0.954242
By -	12	1.079181	By -	9	0.954242
Products	816	2.911690		81	1.908484

Examples in Division.

	Numbers	Logarithms
Divide —	816	2.911690
By — —	12	1.079181
Quotients —	68	1.832509

To Extract Roots by Logarithms.

Numbers	Logarithms	
81(9	2)1.908484(0.954242	Sq. Root.
Sq. Root Num.	Logarithms	
Cube Root. 729(9.	3(2.862726(0.954242	Cube Root.

Here to extract the square root, the logarithm of the square number is divided by 2, and the quotient is the square root. And to extract the cube root, the logarithm of the cube number, 729 is divided by 3, and the quotient is the logarithm of the cube root, as before hinted.

To find the Logarithm of any number, and to construct a Canon of Logarithms for natural numbers.

To perform this, it must be observed, that the numbers, 1, 10, 100, 1000, 10000, &c. constitute the geometrical progression of numbers, and, therefore, their logarithms may be taken at pleasure; but to be able to express the logarithms of the intermediate numbers, we must use decimal fractions. Taking the numbers, 0.0000000, 1.0000000, 2.0000000, 3.0000000, 4.0000000, &c. Now it is manifest that the

juſt logarithm for any of thoſe numbers, which are not contained in the ſcale of geometrical progreſſion, cannot be had; yet they may be diſcovered ſo near the truth as to anſwer any purpoſe. Thus if the logarithm of the number 9 were required; as this number falls between 1.0000000 and 10.0000000, we muſt find the mean proportional between their logarithms 0.0000000 and 1.0000000, which will be the logarithm thereof, that is, it will be the logarithm of a number exceeding 3 by $\frac{1622777}{10000000}$, and therefore it will be remote from 9. Therefore, between 3 and 10 find another mean proportional, which may come ſomewhat nearer 9: and between 10 and this mean, another ſtill; and proceed in this manner between the numbers next above and next below 9, till you arrive at 9 $\frac{1}{10000000}$, which not being one millionth part from 9, its logarithm may be taken for 9 itſelf, without any ſenſible error. Thus ſeeking in ſuch caſe for the logarithms of the mean proportionals, we have at laſt 0.954251, which is ſufficiently near the true logarithm of 9.

If we find the mean proportionals in the ſame manner between 1.0000000 and 3.1622777, and aſſign convenient logarithms to each, we have at length the logarithm of the number 2, and ſo of all the reſt.

It is not neceſſary to be at ſo much trouble in inveſtigating the logarithms of all the other numbers; for thoſe numbers that are the ſquare or cube roots of other numbers, and alſo thoſe that are the cubes or ſquares of numbers, have their logarithms eaſily found. Thus if the logarithm of the number 9 be biſſected, we ſhall have the logarithm of the number 3, or 0.47712125, 3 being the ſquare root of 9, and the logarithm of 3 being doubled, gives the logarithm of 6, its ſquare, and triple, gives the logarithm of 27, its cube, &c. &c.

From what has been delivered the chief properties of logarithms are deduced. Logarithms are the meaſures of ratios. The exceſs of the logarithm of a number, above the logarithm of the following number, meaſures the ratio of thoſe

thoſe terms. The meaſure of the ratio of a greater quantity to a leſs is poſitive, as this ratio compounded with any other ratio, increaſes it. The ratio of equality compounded with any other ratio neither increaſes nor diminiſhes it; and its meaſure is nothing. The meaſure of the ratio of a leſs quantity to a greater is negative, as this ratio compounded with any other ratio diminiſhes it. The ratio of any quantity A to unity, compounded with the ratio of unity to A, produces the ratio of A to A, or the ratio of equality; and the meaſures of thoſe two ratios deſtroy each other when added together; ſo that when the one is conſidered as poſitive, the other is to be conſidered as negative.

To find the Logarithm of a Fraction.

RULE. Subtract the logarithm of the numerator from that of the denominator, and to the remainder prefix the negative ſign. Thus ſuppoſe it was required to find the logarithm of the fraction $\frac{3}{7}$.

0.845098	Logarithm of 7.
0.477121	Logarithm of 3.
0.367977	Logarithm of $\frac{3}{7}$.

The reaſon of this rule is evident, for a fraction being the quotient of the numerator divided by the denominator, its logarithm muſt be the difference of the logarithms of theſe two; ſo that the numerator being ſubtracted from the denominator, the difference becomes negative. And the logarithms of proper fractions muſt always be negative, if the logarithm of unity be 0; becauſe a proper fraction is leſs than an unit.

Or the logarithm of the denominator may be ſubtracted from that of the numerator, regard being had to the ſign of the index, which alone in that caſe is negative. Thus,

0.477121	Logarithm of 3.
0.845098	Logarithm of 7.
$\bar{1}$.732023	Logarithm of $\frac{3}{7}$.

 This

This produces the ſame effect in any operation as the logarithm before found, viz. —0,367977, this being to be ſubtracted and the other added.

Or, the fraction may be reduced to a decimal, and its logarithm found; which logarithm differs from that of a whole number only in the index, which is to be negative. For an improper fraction, ſubtract the logarithm of the denominator from that of the numerator, and the remainder is the logarithm of the fraction, as in the fraction $\frac{9}{5}$.

0.9542425	Logarithm of 9.
0.6989700	Logarithm of 5,
0.2552725	Logarithm of $\frac{9}{5}$.

In the ſame manner the logarithm of any mixed number may be found; by reducing the mixed number into an improper fraction.

Or laſtly, an improper fraction may be reduced to a mixed number, and its logarithm muſt be found as if it were a whole number; and its index taken according to the integral part.

In addition, ſubtraction, &c. of Logarithms, with negative and affirmative indices, the ſame rules are to be obſerved as thoſe given in Algebra, for like and unlike ſigns.

In addition of logarithms of this nature, all the figures, except the index, are reckoned poſitive; and, therefore, the figure to be carried to the index from the other part of the logarithms, takes away ſo much from the negative index. Thus if 1.863326 be added to $\overline{3}.698972$, the ſum is $\overline{1}.562298$. And in ſubtraction, if either one or both of the logarithms have negative indices, you muſt change the ſign of the index of the ſubtrahend, after you have carried it to what may ariſe from the decimal part, and then add the indices together; thus, 1.863326 be ſubtracted from $\overline{1},562298$, the remainder will be $\overline{3}.698972$. In multiplication, what is carried from the product of the other parts of the logarithms, muſt

must be subtracted from the product of the indices; thus, if $\bar{2}.477121$ be multiplied by 5, the product will be 8.385605. In division, if the divisor will exactly measure the index, proceed as in common arithmetic. Thus $\bar{4}.924782$ divided by 2, quotes $\bar{2}.462391$; but if the divisor will not exactly measure the index, add units to the index, till you can exactly divide it, and carry these units to the next first number. Thus if $\bar{8}.385605$ be divided by 5, it quotes $\bar{2}.477121$.

SECT. III.

OF THE RULE OF PROPORTION BY LOGARITHMS.

To perform the Rule of Proportion by Logarithms.

Rule. Add the logarithm of the second number to that of the third, and from the sum subtract the logarithm of the first; and the remainder is the logarithm of the fourth number.

Example. Find a fourth proportional to the numbers 4, 68, and 3.

1.832509	Logarithm of 68.
0.477121	Logarithm of 3.
2.309630	Sum.
0.602060	Logarithm of 4,
1.707570	Logarithm of 51, the answer.

This rule is founded on the same reason as the rule of proportion in common arithmetic; for adding the logarithms of

of the ſecond and third numbers together, and ſubtracting the logarithm of the firſt from the ſum, is the ſame thing as multiplying the ſecond and third numbers together, and dividing the product by the firſt.

Or, the operation may be performed by the following rule, viz. Againſt the firſt term write the arithmetical complement of its logarithm, and againſt the ſecond and third terms write their logarithms themſelves; and the ſum of theſe three logarithms, abating 10 in the index, will be the logarithm of the fourth term; thus, in reſolving the aforeſaid queſtion, the operation will ſtand thus:

9.397940	Arithmetical complement of log. of 4.
1.832509	Logarithm of 68.
0.477121	Logarithm of 3.
1.707570	Logarithm of 51, Anſwer.

The reſolution of problems of this nature, is of eminent ſervice in Trigonometry, as will be ſeen hereafter.

CHAP.

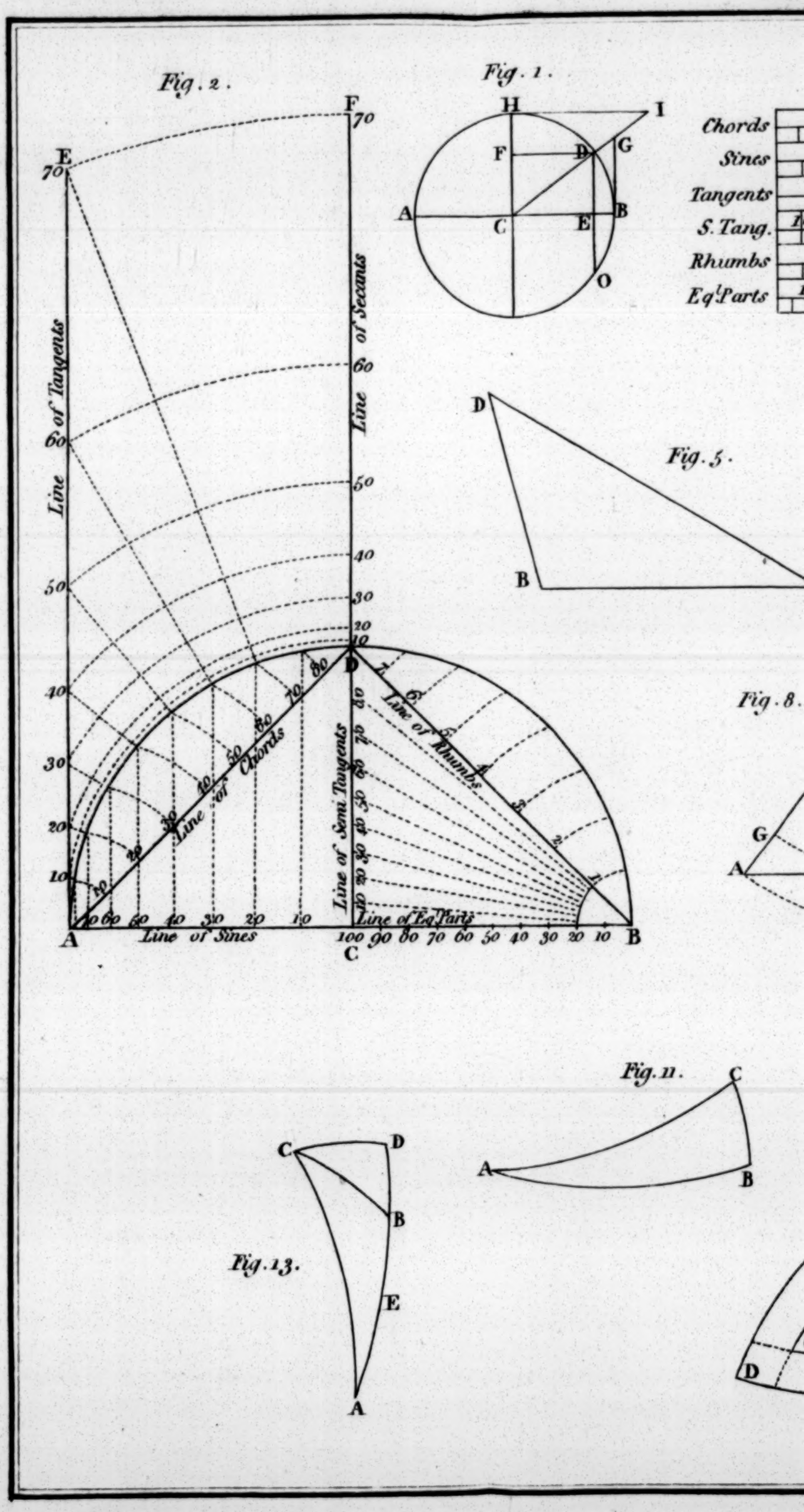


Fig. 2.
Fig. 1.
Chords
Sines
Tangents
S. Tang.
Rhumbs
Eql Parts
Line of Tangents
Line of Secants
Line of Chords
Line of Semi Tangents
Line of Rhumbs
Line of Sines
Line of Eq. Parts
100 90 80 70 60 50 40 30 20 10
Fig. 5.
Fig. 8.
Fig. 11.
Fig. 13.

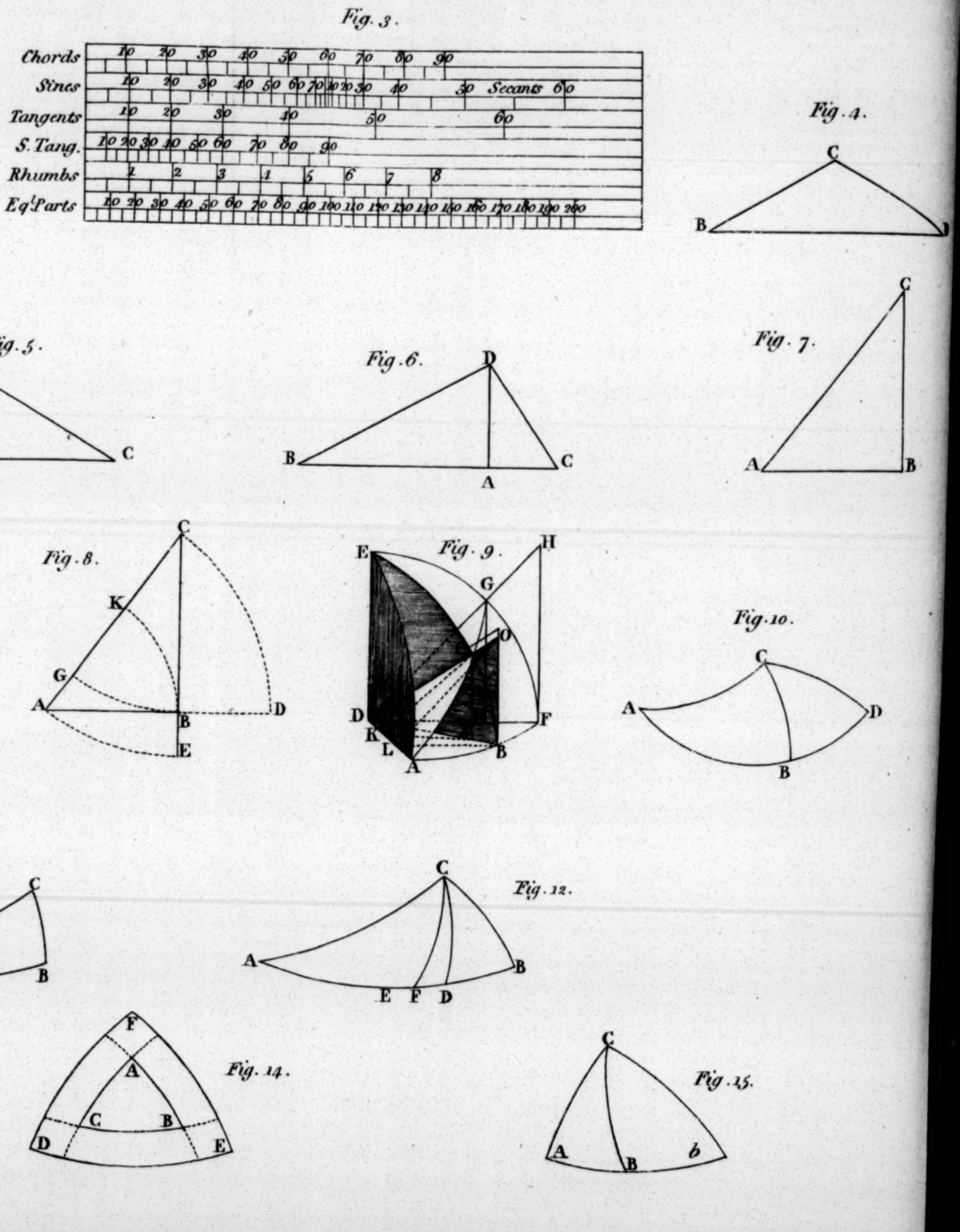


Fig. 3.
Chords
Sines
Tangents
S. Tang.
Rhumbs
Eq. Parts
Secants
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 12.
Fig. 14.
Fig. 15.

CHAP. XII.

OF TRIGONOMETRY.

SECT. I

OF PLANE TRIGONOMETRY.

Difinitions.

1. TRIGONOMETRY, is the art of finding all the fides and angles of a triangle, from having any three of thefe, one of which, (at leaft,) muft be a fide in plane trigonometry. Or, to find the ratios of the fides, when the angles are given. And it is founded on the mutual proportions, which fubfifts between the fides and angles of triangles; which proportions are known by finding the relations between the radius of the circle and certain lines drawn in, and about the the circle, called, *chords*, *fines*, *tangents*, and *fecants*, &c.

2. For this purpofe, the circumference of a circle is divided into 360 parts, called degrees; and every degree fubdivided into 60 other parts, called minutes, and every minute into 60 feconds, and every fecond into 60 thirds, &c. and any angle is faid to confift of fo many degrees, minutes and feconds, as there are contained in the arc, that meafures the angle, or that is intercepted between the legs or fides of the angle.

3. The

3. The complement of an arc, is the difference between the arc and a quadrant.

4. The ſupplement of an arc, is the difference between the arc and a ſemi-circle.

5. The right ſine of an arc, commonly called the ſine, is a perpendicular falling from one end of the arc, to the radius drawn through the other end of the ſame arc, as D, E, (fig. 1, plate 9,) is the ſine of the arc D B, and it is always equal to half the ſubtenſe of double the arc. Thus, D E is equal to half of D O, and the arc D O is double the arc D B. Hence the ſign of an arc of 30 degrees is equal to one half the radius.

6. The ſine complement of an arc is that part of the radius intercepted between the centre, and the right ſine as C E, and is alſo the ſine complement of the arc to a quadrant, for C E is equal to F D, which is the ſine of the arc D H.

7. The coſine of an arc is the ſame as the ſine complement.

8. The verſed ſine is that part of the radius intercepted between the right ſine and the circumference of the circle, as E B.

9. The tangent of an arc is a perpendicular drawn from the extremity of the radius to the ſecant, as B G, which is the tangent of the arc D B.

10. The ſecant of an arc is a line drawn from the centre of the circle, through one end of the arc, till it meets the tangents, as C G.

11. The coſecant and cotangent of an arc, is the ſecant and tangent of that arc, which is the complement of the former arc, to a quadrant. And the chord of an arc, and the chord of its complement to a circle is the ſame, ſo likewiſe the ſine, tangent, and ſecant of an arc, the ſame as the ſine tangent, and ſecant of its complement to a ſemi-circle. Thus, the ſine E D, the tangent B G, and ſecant C G, is the

the ſine, tangent and ſecant of the arc, D A, or the ſupplement of the former arc D B.

12. The Sinus totus, is the greateſt ſine, being the ſine of an arc of 90 degrees, or one quarter of a circle, and is equal to the radius of the circle.

Thus, the ſines always increaſe from B, at which place they are nothing, till they come to the radius C H, which is the greateſt, being the ſine totus. From hence they decreaſe, all the way along the ſecond quadrant from H to A, and at length vaniſh at the point A, thereby, we ſee that the ſine of the ſemi-circle B H A, is nothing. After this, the ſines are negative, as they proceed along the next ſemi-circle A O B, being drawn on the oppoſite ſide, or downwards, from the diameter A B.

As D E is the coſine of D H; the ſine, coſine and radius of any arc form a right angled triangle; as, C D E, or C D F; of which, the radius C D is the hypothenuſe; and therefore, the ſquare of the radius is equal to the ſum of the ſquares of the ſine and coſine of any arc.

The ſines, coſines, tangents, &c. of every degree and minute, in a quadrant, are calculated to a radius of 1, and ranged in tables for uſe. But to ſhorten the operation in calculations in trigonometry, we uſe the logarithms of them, inſtead of the natural numbers, which are called the artificial ſines, tangents, &c. and theſe numbers ſo ranged in tables, form the triangular canon; and contain every ſpecies of right angled triangles; ſo that no ſuch triangle can be propoſed, but one ſimilar to it can be found there, by compariſon, with which, the propoſed one may be computed by analogy of proportion. Laſtly. ſometimes the proportion is not expreſſed in numbers; but the ſeveral ſines, tangents, &c. are actually laid down upon lines of ſcales, from whence the line of ſines of tangents, &c. on the plane ſcale, as follows:—

The Plane Scale is a mathematical inſtrument of moſt extenſive uſe, commonly two feet in length, the lines uſually drawn upon it, are the following:—1. Lines of equal parts.

2. Of Chords.—3. Rhumbs.—4. Sines.—5. Tangents.—6. Secants.—7 Semi-tangents.—8. Longitude.—9. Latitude. All of which, with the method of conſtruction, is ſhewn in fig. 2.

1. The lines of equal parts, are of two kinds: viz.—ſimply divided, and diagonally divided. The firſt of theſe are formed by drawing three parallel lines, and dividing them into any number of equal parts, by ſhort lines drawn acroſs them; and in like manner ſub-dividing the firſt diviſion into ten other equal ſmaller parts, by which numbers or dimenſions of two figures may be taken off. Upon ſome rules ſeveral of theſe ſcales of equal parts are ranged parallel to each other, with figures ſet to them, to ſhew into how many equal parts they divide the inch; as, 20, 25, 30, &c. 2. The diagonal diviſions are formed by drawing eleven long parallel lines, equi-diſtant from each other, which are divided into parts, and croſſed by other ſhort lines, as the former; then the firſt of theſe equal parts have the two outermoſt of the eleven parallel lines divided into 10 equal parts, and the points of diviſion being connected by diagonal lines as ſhewn in Menſuration. The whole ſcale is thus divided into dimenſions of three places of figures.

The other lines upon the ſcale, are commonly uſed in Trigonometry, Navigation, Aſtronomy, Dialling, &c. &c. and are all conſtructed from the diviſions of a circle, as follows:—

2. Deſcribe a circle with any convenient radius, and divide it in four equal parts, by two diameters, drawn at right angles to each other, (fig. 2.) Continue one diameter C D towards F, and draw the tangent line E A, parallel thereto, then draw the chords D A and D B.

3. To conſtruct the line of chords, divide the quadrant A D into 90 equal parts; then on A, as a centre, with the compaſſes, transfer theſe diviſions to the chord line A D, which mark with the correſponding numbers, and it will become

become the line of chords, which may be transferred to the ruler.

4. For the line of rhumbs, divide the quadrant B D into eight equal parts: then with the compaſſes from the centre B, transfer the diviſions to the line B D, which will be the line of rhumbs.

5. For the line of ſines through each of the diviſions of the arc A D draw right lines parallel to the radius D C, which will divide the radius A C into the ſines, or verſed ſines; numbering it from C to A for the ſines, and from A to C for the verſed ſines.

6. For the line of tangents lay a ruler on C, and the ſeveral diviſions of the arc A D; and it will interſect the line E A, which will become a line of tangents, transferring the numbers from the arc A D to the tangent line.

7. For the line of ſecants transfer the diviſions from the tangent line to the line F D with the compaſſes, and from C as the centre, marking the diviſions with the correſponding numbers on the tangent line.

8. For the line of ſemi-tangents, lay a ruler on B and the ſeveral diviſions of the arc A D, which will interſect the radius C D in the ſeveral diviſions of the ſemi-tangents, which are to be numbered according to the arc A D.

9. For the line of longitude, divide the radius A C into 60 equal parts, through each of theſe draw lines parallel to the radius C D; the points where theſe lines interſect the arc A D are to be transferred with the compaſſes from A as a centre to the chord A D, and numbered thereon, which will give the line of longitude.

10. For the line of latitude, the ſemi-circle A D B muſt be completed to a circle, then a ruler laid on the point D, and on the ſeveral diviſions on the line of ſines, A C, will interſect the next quadrant of the circle, in as many points; when from the oppoſite part of the circle to D, as the centre,

the intersections of the arc are to be transferred to its chord, and numbered according to the line of sines.

The chief use of the line of sines, tangents, secants, and semi tangents, are to find the poles and centres of the several circles, represented in a projection of the sphere.

I have been more particular in describing the construction of this scale, as it is an instrument in most general use in mathematics; and by the foregoing directions the learner may construct any lines on the scale himself, where there happens not to be a mathematical instrument maker, nigh at hand.

The three methods of resolving triangles, or cases in trigonometry, are:—1. By geometrical construction. 2. Arithmetical computition. And instrumental operation. In the first method, the triangle, is constructed, by drawing and laying down the several parts of their magnitude: viz.—The side from a scale of equal parts, and the angles from a scale of chords, or other instrument; then the unknown parts are measured by the same scales, and thus they become known.

In the second method, the terms of the proportion are stated according to rule, which terms consist partly of the numbers of the given sides, and partly of the sines, &c, of angles taken from the tables; the proportion is then resolved like all other proportions, in which a fourth term is to be found from three given terms by the Rule of Three.

In the third method of resolving the triangle, by instrumental operation, recourse must be had to the Logarithmic lines, on one side of the two foot scales; extending the compasses from the first term to the second or third, which happens to be of the same kind with it, then that extent will reach from the other term to the fourth term. In this operation for the sides of triangles, is used the line of numbers, and for the angles the line of sines or tangents, according as the proportion respects, sines or tangents.

In

In every case, in plane Trigonometry, there must be given three parts, one of which, at least, must be a side. And every triangle that can be proposed, will fall under one of the three following cases:—

CASE I.

When two of the three given parts are a side, and its opposite angle.

CASE II.

When there are given two sides, and their contained angle.

CASE III.

When the three sides are given.

Rule. For the first Case, viz.—That the sides are proportional to the sines of their opposite angles; that is as the one side given, is to the sine of its opposite angle, so is another side given to the sine of its opposite angle. Or, as the sine of a given angle is to its opposite side, so is the sine of another given angle to its opposite side. Thus, to find an angle, we must begin the proportion with a given side, that is opposite to a given angle; and to find a side, we must begin with an angle opposite to a given side.

Example. In the triangle, B D C, (fig. 4.) having the side B D equal to 106, the side B C equal to 65 and the angle B D C, 31 degrees, 49 minutes, to find the angle B C D, and the side C D.

1. *By Geometrical construction.*

Draw a line B D equal to 106; at D, make an angle of 31° 49′ by drawing D C; take 65 in the compasses, and with one foot at B, extend the other foot to C, in the line D C, then draw the line B C and it is done: for the angle C, will be 120 degrees, 43 minutes; the angle D 31° 49′; and the angle B, 27° 28′; and the side D C, 56.

2. *By*

2. *By Arithmetical computation.*

	Log.
As the fide B C, 65	1.81291
Is to fine angle D, 31° 49′	9.72198
So is the fide B D, 106	2 02531
	11,74729
	1.81291
To fine angle C, 120° 43′	9.93438

To find the fide D C.

	Log.
As the the fine angle D, 31° 49′	9.72198
Is to fide B C, 65	1 81291
So is fine angle B, 27°28′	9.66392
	11 47683
	9 72198
To the fide D C, 56.88	1 75485

Or, it may be wrought as follows :—

180° 0′	The fum of three angles
59° 17′	Supplement of angle C
120° 43′	Angle C
31° 49′	Angle D
152° 32′	Their fum
180° 0′	
152° 32′	
27° 28′	Angle B.

Here it muft be noted, that when the given angle is obtufe, the angle fought will be acute; but when the given angle is acute, and oppofite to a lefs given fide, then the required angle is doubtful whether acute or obtufe; it ought therefore, to be determined before the operation be performed.

For

For the above proportion gives 59° 17′ for the required angle; but as it is obtuſe, its ſupplement to 180° muſt be taken viz.—120° 43′.

3. *By Gunter's line, or Instrumental operation.*

Rule. Extend the compaſſes from 65 to 106 on the lines of numbers, and that extent will reach from 31° 49′ to 59° 17′ on the line of ſines.

Secondly. The extent from 31° 49′ to 27° 28′ on the line of ſines, will reach from 65 to 56.88 on the line of numbers.

CASE II.

When the three given parts are two sides and their contained angles.

Rule. As the ſum of the two given ſides is to the difference of the ſides, ſo is the tangent of half the ſum of the two oppoſite angles or cotangent of half the given angle to the tangent of half the difference of thoſe angles.

Then the half difference added to the half ſum gives the greater of the two unknown angles, and ſubtracted, leaves the leſs of the two angles.

Thus, having all the angles, the remaining third ſide will be found by the former caſe.

Example. Having the ſide B C, equal to 109, B D equal to 76, (fig. 5,) and the angle C B D, 101° 30′ to find the angle B D C or B C D, and the ſide C D.

1. *By Geometrical construction.*

Draw the line B C equal to 109 and B D, ſo as to make an angle with B C, of 101° 30′, and make B D equal to 76, join B C and B D with a right line, and it is done; for the angle D being meaſured, is found to be equal to 47° 32′, the angle C, 30° 58′, and the ſide D C 144.8.

2. *Arith-*

2. *Arithmetically by Logarithms.*

Side B C	109		109	$180^\circ\ 0'$	
Side B D	76		76	$101^\circ\ 30'$	
Their ſum	185	their	33 difference	$78^\circ\ 30$	Sum of the angle D & C.
				$39^\circ\ 15'$	half the ſum.

Then to find the angles D and C.

	Log.
As the ſum of the ſides B C and B D = 185	2.26717
Is to their difference 33	1.51851
So is tan. of ½ ſum of the angles C & D $39^\circ\ 15'$	9.91224
	11.43075
	2.26717
To tangent of ½ the diff. of the angles C & D $8^\circ 17'$	9.16358
To ½ the ſum of the angles D and C	$39^\circ\ 15'$
Add half the difference of the angles C and D	$8^\circ\ 17'$
Gives the greater angle D	$47^\circ\ 32'$
Subtracted, gives the leſſer angle C	$30^\circ\ 58'$

To find D C.

	Log.
As ſine angle D $47^\circ\ 32'$	9.86786
Is to the ſide B C 109	2.03743
So is ſine angle B $101^\circ\ 30'$	9.99119
	12.02862
	9.86786
To the ſide D C, required 144.8	2.16076

3. *By Gunter.*

The extent from 185 to 33 on the line of numbers will reach from $39^\circ\ 15'$ to $8^\circ\ 17'$ on the line of tangents. Secondly, the extent from the angle D $47^\circ\ 32'$ to $78^\circ\ 30'$, (the ſupple-ment

ment of angle B,) on the line of ſines will reach from the ſide B C 109 to 144.8 the ſide D C on the line of numbers.

CASE III.

When the three sides are given: to find the three angles.

Rule. Let fall a perpendicular from the greateſt angle upon the oppoſite ſide or baſe, dividing it into two ſegments, and the whole triangle into ſmaller right angled triangles; then the proportion will be, as the baſe, or ſum of the two ſegments, is to the ſum of the other two ſides; ſo is the difference of thoſe ſides to the difference of the ſegments of the baſe.

Then half the difference of the two ſegments added to half the baſe, or half the ſum of the two ſegments, gives their greater ſegment; and ſubtracted, gives the leſs. Thus, in each of the two right angled triangles, there are given the hypothenuſe and the baſe; therefore, the other angles may be found by the firſt Caſe.

Example. Having the ſides B C equal to 105, (fig. 6,) B D equal to 85 and C D equal to 50; to find the three angles D, C, and B.

1. *Geometrically by construction.*

1. Draw the line B C, equal to 105; with the compaſſes open to 50, and having one foot on the point C, deſcribe an arc; then with the compaſſes open to 85 and one foot in B, cut the former arc in D, join B D and C D, and it is done; for the angles meaſured, B will be found equal to 28° 4′, and C 53° 7′, which being added together, and ſubtracted from 180° leaves 98° 49′, for the angle D.

2. *Arithmetically, by logarithms.*

The two ſhorteſt ſides of the triangle B D and C D added together, is 135, and their difference 35. The ſegments of the baſe B C are found in the following manner:—

	Log.
As the ſide B C equal to 105	2.02119
Is to the ſum of the ſides B D and D C 135	2.13033
So is their difference 35	1.54407
To the difference of the ſegments of B C 45	1.65321

Thus, having the ſum and difference of the ſegments of the baſe, it is only neceſſary to add half that ſum $52\frac{1}{2}$ to half the difference $22\frac{1}{2}$, and it will give the greater ſegment which is 75; and which ſubtracted from 105 leaves 30, the leſſer ſegment. Then to find the angle B D A.

	Log.
As the hypothenuſe B D 85	1.92942
Is to the radius	10.00000
So is the greater ſegment 75	1.87506
To the ſum of the angle B D A	9.94564

Therefore, the angle B D A is equal to 61° 56′.

To find the Angle A D C.

	Log.
As the hypothenuſe D C 50	1.69897
Is to the radius	10.00000
So is the leſs ſegment 30	1.47712
To the ſine of A D C	9.77815

Therefore, the angle A D C is equal to 36° 53′, and the whole angle B D C equal to 98° 49′.

To

To find the angle B, it is only neceſſary to ſubtract the angle B D A or 61° 56′ from 90°, and the remainder 28° 4′, is the angle B, and the angle C is equal to 53° 7′.

3. *By Gunter.*

1. The extent from 105 to 135 will reach from 35 to 45 on the line of numbers. Secondly. The extent from 85 to 75 on the line of numbers, will reach from the radius to 61° 56′ or the angle B D A on the line of ſines. Thirdly. The extent from 50 to 30 on the line of numbers, will reach from the radius to 36° 53′, the angle of A D C on the line of ſines.

The three foregoing caſes of plane triangles, contain all the variety of both right and oblique triangles. But there are ſome other theorems, ſuited to ſome particular forms of triangles, which are often more expeditious in uſe, than the foregoing general ones; particularly the following theorem, for right angled triangles, being a caſe which frequently occurs.

CASE IV.

When there are given the angles and one leg of a right angled triangle.

Rule. As the radius is to the given leg A B, (fig. 7,) ſo is the tangent of the adjacent angle A, to the oppoſite leg B C; and ſo is the ſecant of the ſame angle A, to the hypotheneuſe A C.

Example. In the triangle A B C, having the leg A B equal to 162, and the angle A equal to 53° 7′48″, and conſequently the angle C 36° 52′12″. To find the ſides B C and A C.

1. *Geometrically.*

Draw A B equal to 162 and erect the indefinite perpendicular B C; make the angle A 53° 7′48″; then the side A C will cut B C in the point C, and form the triangle A B C, which by measuring, A C is found equal to 270, and B C to 216.

2. *Arithmetically.*

	Log.
As radius 10	10.0000000
Is to A B 162	2.2095150
So is the tangent A 53° 7′48″	10.1249372
	12.3344522
	10.0000000
To B C 216	2.3344522
So is the secant A 53° 7′48″	10.2218477
To A C 270	2.4313627

3. *By Gunter.*

Extend the compasses from 45° at the end of the tangents, (the radius,) to the tangent of 53° $\frac{1}{8}$; and that extent will reach from 162 to 216 on the line of numbers for B C; then extend the compasses from 36°52′ to 90 on the sines; and that extent will reach from 165 to 270 on the line of numbers for A C.

There is also another method of frequent use in trigonometry; called, *making every side radius*, which is as follows:

Let A B C, (fig. 8,) be a given triangle; make the hypothenuse A C radius first, that is, with the extent of A C as a radius, and on A C, as two centres describe the two arcs C D and A E; then each leg A B, B C will represent the sine of its opposite angle: viz.—The leg B C will be the sine of the arc C D or of the angle A, and the leg A B, the sine of the arc A E, or of the angle C.

Again,

Again, making either leg radius, the other leg will represent the tangent of its opposite angle, and the hypothenuse the secant of the same angle; thus, with the radius A B and center A, describe the arc B K; and B C will represent the tangent of that arc, or of the angle A, and the hypothenuse A C the secant of the same; or, with the radius B C and centre C, describe the arc B G; then the other leg A B is the tangent of that arc B G or of the angle C; and the hypothenuse C A is the secant of the same.

Then the proportions are obvious for the sides in this figure, bear the same proportions to each other, as the parts they represent.

SECT. II.

OF SPHERICAL TRIGONOMETRY.

SPHERICAL Trigonometry teaches the resolution of a spherical triangle, having three given parts; and like plane trigonometry may be either right angled or oblique angled; but before the learner can proceed to the analogies of a spherical triangle, it is necessary to be acquainted with the six following theorems :—

Theorem I.

In all right angled spherical triangles the sine of the hypothenuse is to the radius, as the sine of a leg is to the sine of its opposite angle. And as the sine of a leg is to the radius, so is the tangent of the other leg to the tangent of its opposite angle.

Let E D A F G, (fig. 9,) represent the eighth part of a sphere, where the quadrantal planes E D F G, and E D B C are both perpendicular to the quadrantal plane A D F B, and the

the quadrantal plane A D G C is perpendicular to the plane E D F G, and the ſpherical triangle A B C, is right angled at B, therefore, C A is the hypothenuſe, and B A, D C are the legs.

Draw the tangents H F and O B to the arches G F and C B, and the ſines G M, C I on the radii D F and D B; alſo draw B L, the ſine of the arc A B, and C K the ſine of A C; then join I K and O L. Now H F, O B, G M, and C I are all perpendicular to the plane, A D F B. And H D, G K and O L lie all in the ſame plane A D G C. Therefore, the right angled triangles H F D, C I K, and O D L having the equa-triangles H D F, C K I, O L B are ſimilar. And as C K is to D G, ſo is C I to G M, that is, as the ſine of the hypothenuſe is to the radius, ſo is the ſine of a leg to the ſine of its oppoſite angle. For G M, is the ſine of the arc G F, which meaſures the angle C A B. Alſo as L B is to D F, ſo is B O, to F H; that is, as the ſine of a leg is to the radius, ſo is the tangent of the other leg to the tangent of its oppoſite angle Q E D.

From this it follows that the ſines of the angles of any oblique ſpherical triangle, as A B C, (fig. 10,) are to one other directly as the ſines of the oppoſite ſide; therefore, in every right angled ſpherical triangle, having the ſame perpendicular; the ſines of the baſes will be to each other inverſely as the tangents of the angles at the baſes.

Theorem II.

In every right angled ſpherical triangle, as A B C, (fig. 11,) the proportion is, as radius is to the coſine of one leg, ſo is the coſine of the other leg to the coſine of the hypothenuſe.

Therefore, if two right angled ſpherical triangles A B C, C B D, (fig. 10,) have the ſame perpendicular, B C the coſine of their hypothenuſes, will be to each other directly as the coſines of their baſes.

Theorem

Theorem III.

In any ſpherical triangle, the proportion is, as radius is to the ſine of either angle, ſo is the coſine of the adjacent leg to the coſine of the oppoſite angle.

Therefore, in right angled ſpherical triangles, having the ſame perpendicular, the coſines of the angles, at the baſe, will be to each other directly, as the ſines of the vertical angles.

Theorem IV.

In any right angled ſpherical triangles, the proportion is, as radius is to the coſine of the hypothenuſe, ſo is the tangent of either angle to the cotangent of the other angle.

Thus, as the ſum of the ſines of the two unequal arches is to their difference, ſo is the tangent of half the ſum of thoſe arches, to the tangent of half their difference; and as the ſum of the coſines is to their difference, ſo is the cotangent of half the ſum of the arches to the tangent of half the difference of the ſame arches.

Theorem V.

In many ſpherical triangles, A B C, (fig. 12 and 13,) the proportion is, as the cotangent of half the ſum of half the difference, ſo is the cotangent of half the baſe, to the tangent of the diſtance, (D E,) of the perpendicular, from the middle of the baſe.

Theorem VI.

Any ſpherical triangle A B C, (fig. 12,) as the cotangent of half the ſum of the angles at the baſe, is to the tangent of half their difference, ſo is the tangent of half the vertical angle, to the tangent of the angle which the perpendicular C D makes with the line C F, biſecting the vertical angle.

The

The Solution of the Cases of right angled spherical triangles, (Fig. 11.)

Cafe	Given.	Sought.	Solution.
1	The hypo. A C, & the angle A.	The oppofite leg B C.	As radius : fine hypothenufe A C :: fine A : fine B C.
2	The hypo. A C, & the angle A.	The adjacent leg A B.	As radius : cofine of A :: tangent A C : tangent A B.
3	The hypothenufe A C, and the angle A.	The other angle C.	As radius . cofine of A C :: tangent A : cotangent C.
4	The hypothenufe A C, and one leg A B.	The other leg B C.	As cofine A B : radius :: cofine A C : cofine B C.
5	The hypothenufe A C, and one leg A B.	The oppofite angle C.	As fine A C : radius :: fine A B : fine C.
6	The hypothenufe A C, and one leg A B.	The adjacent angle A.	As tangent A C : tangent A B :: radius : cofine A.
7	1 leg A B, and the adjacent angle A.	The other leg B C.	As radius : fine A B :: tangent A : tangent B C.
8	1 leg A B, and the adjacent angle A.	The oppofite angle C.	As radius : fine A :: cofine A B : cofine C.
9	1 leg A B, and the adjacent angle A.	The hypothenufe A C.	As cofine A : radius :: tangent A B : tangent A C.
10	1 leg B C, and the oppofite angle A.	The other leg A B.	As tangent A : tangent B C :: radius : fine A B.
11	1 Leg B C, and the oppofite angle A.	The adjacent angle C.	As cofine B C : radius :: cofine of A : fine C.
12	1 leg B C, and the oppofite angle A.	The hypothenufe A C.	As fine A : fine B C :: radius : fine A C.
13	Both legs A B and B C.	The hypothenufe A C.	As radius : cofine A B :: cofine B C : cofine A C.
14	Both legs A B and B C,	Any angle as A.	As fine A B : radius :: tangent B C : tangent A.
15	Both angles A and C.	Any leg as A B.	As fine A : cofine C :: radius : cofine A B.
16	Both angles A and C.	The hypothenufe A C.	As tangent A : cotangent C :: radius : cofine A C.

NOTE. The 10th, 11th, and 12th Cafes are ambiguous as it cannot be determined by the data, whether A, B, C, and A C be greater or lefs than 90 degrees each.

The Solution of the Cases of oblique spherical triangles. (Fig. 12 and 13.)

Cafe.	Given.	Sought.	Solution.
1	Two fides A C, B C, and the angle A.	The angle B	As fine B C : sine A :: sine A C : sine B Note. When B C is lefs than A C it cannot be determined whether B be acute or obtufe.
2	Two fides A C, B C, and the angle A.	The angle A C B.	Let fall the perpendicular C D upon A B, then radius : cosine A C :: tangent A : cotangent A C D.
3	Two fides A C, B C, and the angle A.	The other fide A B.	As radius : cosine A :: tangent A C : tangent A D : This and the laft cafe are both ambiguous, when the firft is fo.
4	Two fides A C, A B, and the angle A.	The other fide B C.	As radius : cosine A :: tangent A C : tangent A B. Whence A D is alfo known.
5	Two fides A C, A B, and the angle A.	Either of the other angles as B.	As radius : cosine A :: tangent A C : tangent A D. Whence B D is known ; then as sine B D : sine A D :: tangent A : tangent B.
6	Two angles A, A C B, and the fide A C.	The other angle B.	As radius : cosine A B :: tangent A : cotangent A C D. Whence B C D is alfo known ; then as sine A C D : sine B C D :: cosine A : cosine B.
7	Two angles A, A C B, and the fide A C.	Either of the other sides as B C.	As radius: cosine A C :: tangent A : cotangent A C D. Whence B C D is alfo known; then as cosine B C D : cosine A C D :: tangent A C : tangent B C.
8	Two angles A, B, and the fide A C.	The fide B C.	As sine B : sine A C :: sine A : sine B C.
9	Two angles A, B, and the fide A C.	The fide A B.	As radius : cosine A :: tangent A C : tangent A D; & as tangent B : tangent A :: sine A D : sine B D. whence A B is alfo known.
10	Two angles A, B, and the fide A C.	The other angle A C B.	As radius : cosine A C :: tangent A : cotangent A C D ; and as cosine A : cosine B :: sine A C D : sine B C D.
11	All the three fides A B, A C, and B C.	Any angle as A.	as tangent $\frac{1}{2}$ A B : tangent $\frac{AC+BC}{2}$ :: tangent $\frac{AC-BC}{2}$ tangent D E the diftance of the perpendicular from the middle of the bafe. whence A D is known.
12	All the three angles A, B, and A C B.	Any fide as A C.	As cotangent $\frac{ABC+A}{2}$: tangent $\frac{ABC-A}{2}$ :: tangent $\frac{ABC}{2}$: tangent of the angle between the perpendicular and a line befecting the vertical angles ; then as tangent A : to tangent A C D :: radius : cosine A C

The following propofitions, concerning fpherical triangles; will render them move intelligible.

1. A fpherical triangle is either equilateral, ifofcelar or fcalene, according as it has the three angles all equal, or two of them equal, or all three unequal, and *vice verfa.*

2. The greateft fide is always oppofite the greateft angle, and the fmalleft fide oppofite the fmalleft angle.

3. Any two fides, taken together, are greater than the third.

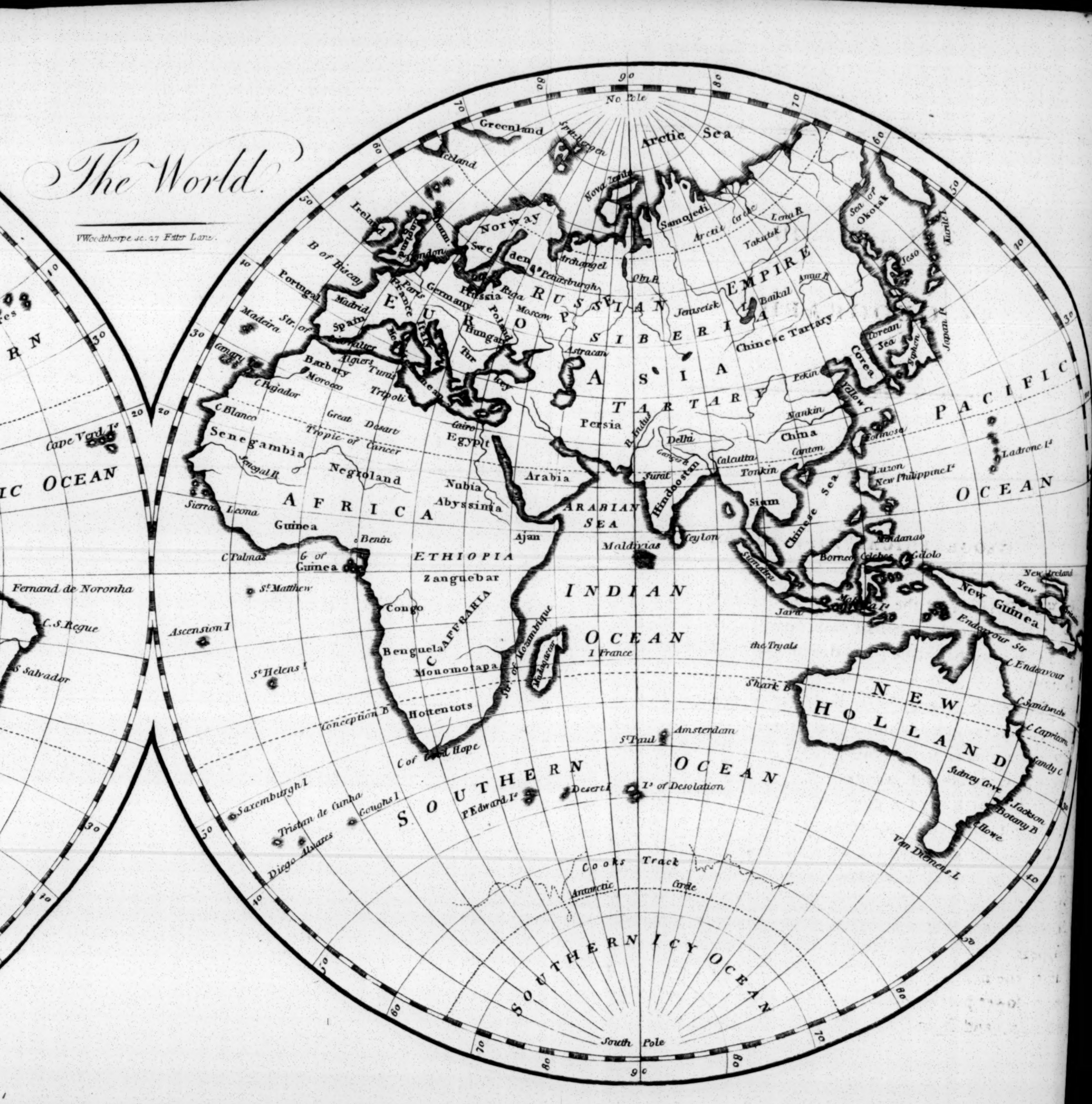
4. If the three angles of a fpherical triangle be all acute, or all right, or all obtufe angles, the three fides will be accordingly all lefs than 90 degrees or equal to 90 degrees, or greater than 90 degrees, and *vice verfa.*

5. If from the three angle A, B, C, (fig. 14,) of a fpherical triangle A B C as poles there be defcribed upon the furface oft he fphere, three arches of a great circle D E, D F, and F E, forming by their interfections another fpherical triangle D E F; each fide of this large triangle will be the fupplement of the angle at its pole; and each angle of the fame triangle will be the fupplement of the fide, oppofite to it in the triangle A B C.

6. In any triangle A B C or A b C, right angled at A, (fig. 15,) the angles at the hypothenufe are always of the fame kind as their oppofite fides. And the hypothenufe is greater or lefs than a quadrant, according as the fides, including the right angle, are of different kinds, or of the fame kind; that is to fay, according as thefe fame fides are either both acute, or both obtufe, or as one is acute, and the other obtufe, and *vice verfa.* Firft, the fides including the right angle, are always of the fame kind as their oppofite angles. Secondly, the fides including the right angle, will be of the fame or different kinds, according as the hypothenufe is lefs, or more than 90 degrees; but one of them at leaft will be of 90 degrees, if the hypothenufe be fo.

CHAP.

North Pole
Arctic Sea
Arctic Lands
GREENLAND
Baffins B
C Farewell
Kamtschatka
Behrings
Antarctic Circle
Cooks R
Alaska
Q Charlottes I
Nootka S
New Wales
Hudsons B
Labrador
Newfoundland
Fishing B
Azores
NORTH AMERICA
Canada
United States
Boston
New York
Philadelphia
Bermudas
Charleston
WESTERN
PACIFIC OCEAN
Drakes H
Louisiana
N Mexico
G of Mexico
Bahama I
Tropic of Cancer
Sandwich I
Owhyhee where Capt J. Cook was killed
West Indies
Jamaica
S. Domingo
Antigua
Barbadoes
ATLANTIC OC
Caribbean Sea
Mexico
Guatimala
Terra Firma
Surinam
Panama
Guiana
Oronoco R
Christmas I
Gallapagos
Equator or Equinoctial Line
Amazon R
Fernand de
C Blanco
SOUTH AMERICA
Lima
PERU
AMAZONIA
Brazil
Q Charlottes I
Marquesas
GREAT
Otaheite
S Salvador
Easter I
PARAGUAY
Juan Fernandes
N Caledonia
SOUTH SEA
Buenos ayres
Norfolk I
CHILI
R Plate
Chiloe I
C North
Patagonia
Str. of Magellan
C Horn
Banks I
Falklands I
Cooks Track
Antarctic Circle
C South
Sandwich
SOUTHERN ICY OCEAN
South Pole


The World.
P Woodthorpe sc. 17 Fetter Lane
No Pole
Arctic Sea
Greenland
Spitzbergen
Nova Zembla
Iceland
Norway
Sweden
Samojedi
Arctic Circle
Lena R
Takutsk
Sea of Okotsk
Kurile I
Jeso
London
Petersburgh
Archangel
B of Biscay
Portugal
Madrid
Spain
Paris
Germany
Prussia
Riga
Moscow
Poland
Hungary
Italy
Turkey
E U R O P E
R U S S I A N E M P I R E
S I B E R I A
Ob R
Jenseisk
L Baikal
Amur R
Chinese Tartary
Corean Sea
Corea
Japan I
Madeira
Canary
Str of Gibraltar
Algiers
Tunis
Tripoli
Barbary
Morocco
C Bajador
C Blanco
Astracan
A S I A
T A R T A R Y
Pekin
Yellow S
Nankin
China
Canton
Formosa
Ladrone Is
Luzon
New Philippine Is
P A C I F I C
O C E A N
Persia
R Indus
Delhi
Ganges
Calcutta
Tonkin
Surat
Hindoostan
Siam
Chinese Sea
Great Desart
Tropic of Cancer
Cairo
Egypt
Senegambia
Senegal R
Negroland
Nubia
Abyssinia
Arabia
ARABIAN SEA
Sierra Leona
A F R I C A
Guinea
Benin
Ajan
Maldivias
Ceylon
Mindanao
Borneo
Celebes
Gilolo
Sumatra
C Palmas
G of Guinea
E T H I O P I A
Zanguebar
St Matthew
I N D I A N
O C E A N
Java
Moluccas Is
New Ireland
New Britain
New Guinea
Endeavour Str
Congo
CAFFRARIA
Ascension I
Benguela
Monomotapa
Str of Mozambique
Madagascar
I France
the Tryals
C Endeavour
St Helens I
Shark B
N E W
H O L L A N D
C Sandwich
C Capricorn
Hottentots
Conception B
Sandy C
St Paul
Amsterdam
C of Good Hope
S O U T H E R N
O C E A N
Sidney Cove
P Edward Is
Desert I
Is of Desolation
Pt Jackson
Botany B
C Howe
Saxemburgh I
Tristan de Cunha
Goughs I
Diego Alvares
Van Diemens L
Cooks Track
Antarctic Circle
SOUTHERN ICY OCEAN
South Pole
Fernand de Noronha
C S Regue
S Salvador
Cape Verd Is
OCEAN

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CHAP. XIII.

OF GEOGRAPHY.

SECT. I.

GEOGRAPHICAL DIFINITIONS.

GEOGRAPHY is the knowledge of the earth, or a deſcription of the terreſtial globe, particularly of the ſurface and known habitable parts thereof, with all its different diviſions.

2. The earth is a globular body, as repreſented (fig. 1, plate 10,) ſurrounded with an atmoſphere of air, by which all terreſtial bodies are confined to its ſurface, being attracted thereto, by the laws of gravity.

That the earth is of a globular form, has been demonſtrated by a number of experiments, particularly by an obſervation on the eclipſe of the moon; in which it appears that the ſhadow of the earth always appears circular, which ever way it is projected. Alſo, by the obſervation of ſhips at ſea, which, after their departure from any coaſt, gradually diſappear to an obſerver on land, from the bottom upwards; that is, the firſt part which diſappears from the ſight, is the keel, or lower part of the ſhip; then thoſe parts which are higher up, and ſo on; the top of the maſt being the part

that is laſt ſeen; which is owing to the convexity of the waters which have the ſame globular figure as the earth*.

3. The earth alſo has a diurnal motion on its own axis, performing one revolution in 24 hours; thereby occaſioning the changes of the day and night, as will be ſeen in aſtronomy.

4. The circumference of the globe, is ſuppoſed to be divided into 360 parts, called degrees, and each degree ſubdivided into 60 minutes, and each minute into 60 ſeconds, &c. Every degree contains 60 geographic miles, conſequently, the circumference of the globe is 21600 ſuch miles; and the diameter 6900 miles. But as 60 geographical miles, are above 69 miles britiſh meaſure, the circuit of the globe is therefore 24840 Engliſh miles, and the diameter almoſt a third, or 7900 in round numbers.

5. The globe of the earth conſiſts of land and water; the proportion of the land to the water is not accurately known, but it is generally believed to be near one third.

6. The waters are divided into three oceans, (beſides the ſmaller ſeas:) viz.—The Atlantic, Pacific; and the Indian ocean. 1. The Atlantic or Weſtern ocean, divides Europe from America, and is 3000 miles wide. 2. The Pacific ocean divides America from Aſia and New Holland, and is 10.000 miles wide. 3. The Indian ocean lies between the Eaſt-Indies and Africa, and is 3,000 miles wide. The parts or branches

* The philoſophers of the laſt age differed greatly in their deſcriptions of the true ſpherical figure of the earth, of which there were two general opinions The one maintained, that the earth was a Prolate Spheriod, of this number was Caſſini. The other party maintained it to be an Oblate Spheriod, of this Sir Iſaac Newton was the chief, who inſiſted that the polar was ſhorter than the equational diameter by 36 miles; a party of philoſophers from France was ſent by the king of that country, to meaſure a degree on the polar circle, and alſo on the equator; the reſult of their experiment turned out exactly in favour of Sir Iſaac Newton's theory. But this unequality of the ſhape of the earth, as well as the unequalities occaſioned by the mountains, &c. make no ſenſible difference in the form of the earth.

branches of theſe oceans, called ſeas, as the Mediterranean ſea, &c. receive their names generally from the countries they border upon.

7. A Bay or Gulf, is a part of the ſea almoſt ſurrounded by land; as the gulf of Mexico, the bay of Biſcay, &c.

8. A Streight is a narrow paſſage out of one ſea into another; as the Streight of Gibraltar.

9. A Lake is a water ſurrounded by land: as the lake of Geneva.

10. Rivers, are ſtreams of water, iſſuing from ſprings in high grounds, and falling into the ſea or other rivers; and are wider near the mouth, than towards the head or ſpring: they are deſcribed in maps by black lines.

11. The Land, is divided into two great Continents: viz. The Eaſtern and Weſtern Continents, beſides Iſlands. The Eaſtern Continent is ſubdivided into three parts; viz.—Europe, which is the north weſt part; Aſia, the north eaſt; and Africa, the ſouth. The Weſtern Continent, conſiſts of America only, divided into North and South America.

12. An Iſland is a piece of land, entirely ſurrounded by water.

13. A Peninſula, is a country, or piece of land, ſurrounded by water, except on one ſide, where it joins to ſome other land.

14. An Iſthmus, is a narrow neck of land which joins a Peninſula to ſome other country; as the Iſthmus of Suez, which joins Africa to Aſia; the Iſthmus of Darien, which joins North and South America.

15. A Cape, called ſometimes a Promontory or Head-land is a point of land extending ſome way into the ſea.

16. The ſurface of the earth is ſuppoſed to be divided by ſeveral imaginary circles, for the better determining the ſituation and boundaries of the ſeveral countries and parts of the world, of which the moſt conſiderable circles are the following:—1. The equator, called alſo the equinoctial line, which divides the globe of the earth into two equal parts or hemiſpheres,

hemiſpheres, the one north and the other ſouth. This circle is every where equally diſtant from the two poles, and upon this circle, the degrees of longitude are marked. 2. The two tropical circles: viz.—The tropic of cancer, or the northern tropic, encompaſſes the globe, at the diſtance of $23\frac{1}{2}$ degrees from the equator. The tropic of Capricorn, or Southern tropic, encompaſſes the globe at the ſame diſtance on the ſouth ſide of the equator. The ſpace between theſe two tropics is called the *Torrid Zone*. 3. The two polar circles: viz.—The Artic circle which ſurrounds the north pole; and the Antartic circle which ſurrounds the ſouth pole; each at the diſtance of $23\frac{1}{2}$ degrees from the pole. The ſpace included between the tropic of Cancer and the Artic circle is called the northen *Temperate Zone*, and that ſpace between the artic circle and the north pole, is called the north *Frigid Zone*, and the correſponding ſpaces on the ſouthern hemiſphere, have ſimilar names, as the ſouthern *Temperate Zone*, and the ſouthern *Frigid Zone*. 4. The meridional lines which are drawn at right angles to the equator, coinciding at the poles. Theſe lines run directly north and ſouth, and when the ſun appears full ſouth of any place, he is then ſaid to be on the meridian of that place, and it is then 12 o'clock at noon, at that place. The latitude of places is always numbered on theſe lines.

17. Longitude, is the diſtance of one country from another, and is either eaſt or weſt, and meaſured on the equator. The longitude of a place is always taken from the capital of a kingdom, where the author or traveller is; thus, when a perſon in England mentions the longitude, it is always underſtood that the longitude is reckoned from London; that is, the degrees of longitude are meaſured on the equator, and from that part of the equator, where the meridian paſſing through London, cuts the equator, to that part of the equator, which is cut by the meridian of the other place meaſured to.

18. The

18. The Latitude of a place is the diſtance of that place from the equator meaſured on the meridional line, and is either north or ſouth.

19. The inhabitants of the earth, are diſtinguiſhed from each other, by their relative ſituations, of which there are reckoned three ſorts, *Periæci*, *Antaecis*, and *Antipodes* :—

1. The *Periæci*, are thoſe people who live at the ſame diſtance from the equator, but under oppoſite meridians; the length of their days, and ſeaſons are the ſame, but when it is mid-day with one, it is mid-night with the other.

2. The *Antaecii*, lies under the ſame meridian, but oppoſite parallels, or equally diſtant from the equator; the one being in the ſouth latitude, and the other in the north. Theſe have the ſun at the ſame hour at noon, but the longeſt day of the one is the ſhorteſt day of the other, and their ſeaſons of the year are different; for when it is ſummer in one, it is winter with the other.

3. The *Antipodes*, are ſituated directly on the oppoſite ſide of the globe to each other, the feet of the one being directly oppoſite the feet of the other, (fig. 1.) Theſe lie under, oppoſite meridians, and oppoſite parallels; it is noon-day with the one, when it is mid-night with the other; the longeſt day with the one, is the ſhorteſt day with the other; and when it is ſummer with the one, it is winter with the other.

20. The inhabitants of the earth are ſometimes diſtinguiſhed from each other, (in geography,) by the direction of their ſhadows at noon-day, and are either *Amphiſcii*, *Aſcii*, *Heteroſcii*, or *Periſcii*. 1. The *Amphiſcii*, are thoſe ſituated in the torrid zone, and have their ſhadows, one part of the year, directed towards the north at noon-day, and at another part of the year, towards the ſouth, at noon-day, according to the part of the ecliptic the ſun is in; conſequently, the ſun is vertical to theſe people twice a year. They are then called:—2. *Aſcii*, ſhewing no ſhadow at noon-day. 3. The

Heteroſcii,

Heterofcii, are thofe who inhabit the temperate zones, and whofe fhadows at mid-day, always fall one way: viz.—The fhadows of thofe in the northern temperate zone, falling always towards the north, at noon-day, and thofe in the fouthern zone, always fouth at the fame day. 4. The *Perifcii*, are thofe who inhabit either of the frigid zones. Thefe have their fhadows moved intirely round them, every 24 hours, when the fun is in their hemifpheres, and fo far decline towards their pole, as not to fet for feveral days.

21. The *Horizon*, is properly a double circle; one of the horizons being called the *Senfible*, and the other, the *Rational*. The former comprehends only that fpace which we can fee round us, upon any part of the earth; and is very different according to the difference of our fituation. The other called *rational*, is parallel to the former, but paffing through the centre of the earth, and fuppofed to be continued as far as the celeftial fphere itfelf; whereas, the former is fuppofed to pafs over the furface of the earth, where the fpectator ftands: but in geography, when the horizon is mentioned, the rational horizon is always underftood. By reafon of the round figure of the earth, every different part has a different horizon. The poles of the horizon, that is, the points directly above the head, and oppofite the feet of the obferver, are called the *zenith*, and *nadir*.

22. The *Zenith*, is that pole of the horizon, directly over the obferver's head.

23. The *Nadir*, is the oppofite pole of the horizon; or that directly under the obferver's feet.

A TABLE

A TABLE,

SHEWING

The number of Miles in a Degree of Longitude, in every Degree of Latitude, from the Equator.

Degrees of Latitude.	Miles.	Degrees of Latitude.	Miles.	Degrees of Latitude.	Miles.
1	59.96	31	51.43	61	29.04
2	59.94	32	50.88	62	28.17
3	59.92	33	50.32	63	27.24
4	59.86	34	49.74	64	26.30
5	59.77	35	49.15	65	23.36
6	59.67	36	48.54	66	24.41
7	59.56	37	47.92	67	23.45
8	59.40	38	47.28	68	22.48
9	59.20	39	46.62	69	21.51
10	59.08	40	46.00	70	20.52
11	58.89	41	45.28	71	19.54
12	58.68	42	44.95	72	18.55
13	58.46	43	43.88	73	17.54
14	58.22	44	43.16	74	16.53
15	58.00	45	42.43	75	15.52
16	57.60	46	41.68	76	14.51
17	57.30	47	41.00	77	13.50
18	57.04	48	40.15	78	12.48
19	56.73	49	39.36	79	11.45
20	56.38	50	38.57	80	10.42
21	56.00	51	37.73	81	9.38
22	55.63	52	37.00	82	8.35
23	55.23	53	36.18	83	7.32
24	54.81	54	35.26	84	6.28
25	54.38	55	34.41	85	5 23
26	54.00	56	33.55	86	4.18
27	53.44	57	32.67	87	3.14
28	53.00	58	31.70	88	2.09
29	52.48	59	30.90	89	1.05
30	51.96	60	30.00	90	0.00

Description, and use of the Globes, and Armillary Sphere.

By means of maps, the true ſituations of the different places of the earth, with regard to one another, and every other particular relative to them, may be eaſily known; conſequently, the hour of the day, ſeaſon of the year, &c. for any particular place may be diſcovered. But theſe problems, to be reſolved by maps, will be tedious and complex; therefore, thoſe machines, called the Celeſtial and Terreſtial globes, and the Armillary ſphere have been invented, by which all the calculations are ſaved, and every problem in geography may be ſolved mechanically, in the moſt eaſy and expeditious manner.

If a map of the World be accurately delineated on a ſpherical ball, the ſurface thereof, will repreſent the ſurface of the earth; for the higheſt hills bear no greater proportion to the bulk of the whole Earth, than ſo many grains of ſand, do to a common mathematical globe; the diameter of the earth being near 8,000 miles, and no hill upon its ſurface is above 3 miles in perpendicular height.

The Armillary ſphere is a large hollow ſphere of glaſs, having as many bright ſtuds fixed on its inſide, as there are viſible ſtars in the heavens, and of the ſame magnitude, and at the ſame angular diſtances from each other. This ſphere is a true repreſentation of the heavens, to an eye, ſuppoſed to be placed in the centre; for to an obſerver placed any where within the ſurface of an indefinite ſphere, all objects will appear equally diſtant, though ſome be much nearer than others; and if a ſmall globe, having the map of the earth upon it, be placed on an axis in the centre of this ſphere, and the ſphere be made to turn round its axis, it will repreſent the apparent motion of the heavens round the earth. And if the globe be turned round its axis while the ſphere remains fixed, it will repreſent the true motion of the earth.

If

If there be drawn a great circle upon this ſphere, equally diſtant from its poles, and having the plane of the circle perpendicular to the axis of the ſphere, it will repreſent the celeſtial equinox, which divides the heavens into two equal parts or hemiſpheres; and the two axis of the ſphere, will repreſent the two poles of the heavens.

If there be another great circle drawn upon the ſphere, cutting the equinoctial at an angle of 23½ degrees, in two oppoſite points; this circle will repreſent the ecliptic, or circle of the ſun's apparent annual motion; one half of which is on the north ſide, and the other half on the ſouth of the equinoctial.

If there be made a large ſtud to move eaſtward in this ecliptic, and with ſuch a motion as to go quite round it, in the time that the ſphere is turned round weſtward upon its axis 366 times; this ſtud will repreſent the ſun changing its place every day in the ecliplic, a three hundred and ſixty-fifth part, and going round weſtward in the ſame directions as the ſtars; but with a motion ſo much ſlower than that of the ſtars, that they will make 366 revolutions, in the time that the ſun makes only 365, about the axis of the ſphere.

If the terreſtial globe in this machine, be about one inch in diameter, and the diameter of the ſtarry ſphere about five or ſix feet; a ſmall inſect placed upon the globe, would ſee only a very ſmall portion of its ſurface; but it would ſee one half of the ſurface of the ſtarry ſphere, the convexity of the globe hiding the other half from its view. If the ſphere be ſet in motion as before directed, and the globe, alſo, revolving on its own axis, the inſect will ſee all the phenomena obſerved by the inhabitants of this world, in the diurnal rotation of the earth round its axis.

The exterior parts of this machine, are ſeveral braſs rings, which repreſent the principal circles in the heavens; viz.—1. The equinoctial. 2. The ecliptic, divided into the ſigns and degrees, and alſo, into the months and the days of the

 year.

year, to ſhew in what point of the ecliptic the ſun is in, on any given day in the year. 3. The two tropics. 4. The artic and antartic circles. 5. The equinoctial colure; which is a great circle paſſing through the north and ſouth poles of the heaven, and through the equinoctial circle at the points, where the equinoctial is cut by the ecliptic. 6. The ſolſtitial colure: which is the great circle paſſing through the poles of the heavens, and at right angles to the equinoctial colure. Hence the ſolſtitial colure paſſes through the equinoctial, at the points where the equinoctial is at the greateſt diſtance from the ecliptic. Theſe points in the equinoctial, are called the ſolſtitial points.

In the north pole of the ecliptic is a nut, to which is fixed one end of a quadrantal wire, having at the other end a ſmall ſun, which is carried round the ecliptic, by turning the nut; and in the ſouth pole of the ecliptic, another quadrantal wire is fixed, with a ſmall moon upon it, which may be moved round by the hand. There is, alſo, a particular contrivance, for cauſing the moon to move in her own orbit.

On the axis of the ſmall globe, is fixed a flat celeſtial meridian, which may be ſet directly over the meridian of any place on the globe; and then turned round with the globe, ſo as to keep over the ſame meridian. This globe has alſo a moveable horizon, which turns upon two wires, which proceeds from it eaſt and weſt points to the globe, and entering the globe at the oppoſite points in the equator, which is a moveable braſs ring, let into the globe in a groove. The whole fabric is ſupported on a pedeſtal, and may be elevated or depreſſed to any number of degrees, from 6 to 90.

Deſcription of the Terreſtial Globe.

On the terreſtial globe is drawn all the principal circles before mentioned, as the equator, ecliptic, tropics, polar circles, and meridians. The ecliptic is divided into twelve ſigns, and each ſign into thirty degrees. Each tropic is 23½ degrees from the

the equator ; and each polar circle $23\frac{1}{2}$ degrees from its respective pole. There are alſo circles drawn parallel to the equator, at every 10 degrees diſtance from it, on each ſide towards the poles ; theſe circles are called *parallels of latitude*. There are, alſo, ſeveral other circles, drawn perpendicularly through the equator, and interſecting each other at the poles ; theſe circles are called *meridians*, and ſometimes circles of *longitude*, *or hour circles* ; and on large globes they are drawn through every tenth degree of the equator ; but on globes of leſs than 12 inches diameter, are drawn through every fifteenth degree.

The globe is hung in a braſs ring, called the *brazen meridian*, turning upon a wire in each pole, ſunk into one ſide of the meridian ring. This ring is divided into 360 degrees ; one half of theſe degrees are numbered from the equator to the poles, to ſhew the latitude of places ; the other half are numbered from the poles to the equator, to ſhew how to elevate either of the poles above the horizon. This ring divides the globe into two equal parts, called the eaſtern and weſtern hemiſpheres ; as the equator divides it into the north- and ſouthern hemiſpheres.

The brazen meridian is let into two notches, made in a broad flat ring, called the wooden horizon ; the upper ſurface of which divides the globe into two equal parts, called the *upper and lower hemiſpheres*. This horizon correſponds to the true rational horizon ; and upon it, are ſeveral concentric circles, which contain the months of the year, the ſigns and degrees anſwering to the ſun's place for each month and day, the thirty-two points of the compaſs, and the circles of amplitude and azinuth, with ſome other circles.

There is a ſmall horary circle, fixed to the north part of the brazen meridian, and having the wire in the north pole of the globe in its centre ; on which wire is an index, which goes over all the twenty-four hours of the circle, as the *globe* is turned round its axis ; ſometimes there are two horary circles, one at each pole.

There

There is a thin ſlip of braſs, called the *quadrant of altitude*, divided into 90 degrees. This is occaſionally fixed to the uppermoſt point of the brazen meridian, by a nut and ſcrew, about which the quadrant turns round.

There is, alſo, to ſome globes, a magnetic needle, which moves over a circle, divided into 360 degrees; alſo over the thirty-two points of the compaſs. This needle ſerves to fix the globe according to the meridian of the place, as the needle makes nearer, a conſtant certain angle with the meridian, which angle is called the *variation;* and which being known, the globe may be rectified to the meridian of the place: thus, at London, the variation of the needle is 23 degrees northward; therefore, by moving the frame of the globe about, till the needle ſettles itſelf near the twenty-third degree, reckoning weſtward from the north point; we then have the braſs meridian of the globe, coinciding with the true meridian of the place.

SECT. II.

OF THE SOLUTION OF GEOGRAPHICAL PROBLEMS.

Directions for using the Globe.

IN uſing the globe, the eaſt ſide of the horizon muſt always be kept towards you, (except the globe require turning by the Problems) for then the graduated ſide of the meridian will be towards you, the quadrant of altitude before you, and the globe divided exactly into two hemiſpheres, by the graduated ſide of the meridian.

When

When it is required to turn the globe and frame entirely round, as is the caſe in working ſome Problems, the ball of the globe will ſometimes be moved from the degree, to which it is ſet by ſuch motion; to avoid this, the feather end of a quill may be thruſt between the ball of the globe and the meridian, which, without hurting the ball, will keep it from turning in the meridian, while you turn the horizon about.

PROBLEM I.

To find the Latitude and Longitude of any given place upon the Globe.

Turn the globe on its axis, until the given place comes exactly under the graduated ſide of the brazen meridian, and obſerve what degree of the meridian the place then lies under which is its *latitude*, and is either north or ſouth.

The globe remaining in this poſition, the degree of the equator, which is under the graduated ſide of the brazen meridian, is the *longitude* of the place; and is either eaſt or weſt, as the place lies on the eaſt or weſt ſide of that which is accounted the firſt meridian. Thus all the atlantic ocean, and the continent of America, is on the weſt ſide of the meridian of London; and the greateſt part of Europe and of Africa, together with all Aſia, is on the eaſt ſide of the meridian of London, which is reckoned the firſt meridian of the globe, by Britiſh geographers and aſtronomers; though the exact meridian for England, is that of Greenwich.

PROBLEM II.

To find any place on the globe, having the latitude and longitude.

This is the reverſe of the former problem, and is found by bringing the point of longitude in the equator to the brazen meridian; then count from the equator on the brazen meridian

meridian to the degree of the given latitude, and under that degree of latitude, will be the place required.

PROBLEM III.

To find the difference of longitude or difference of latitude between any two given places.

Bring each of theſe places to the brazen meridian, and mark their latitude; then if both places are on the ſame ſide of the equator, the leſſer latitude ſubtracted from the greater, will bring the difference; but if they are on different ſides of the equator, both latitudes muſt be added together. And the difference of longitude is found by bringing each place to the meridian, and reckoning on the equator, the difference of degrees between the meridians of the two places, if it be leſs than 180 degrees or half a circle; but if the difference be greater, it muſt be ſubtracted from 360, and the remainder is the difference of longitude.

PROBLEM IV.

To find all those places, that have the same latitude and longitude with any given place.

Bring the given place to the brazen meridian, then all thoſe places which lie under the ſame meridian will have the ſame longitude. Then turn the globe round on its axis, and all thoſe places which paſs under the ſame degree of latitude in the brazen meridian, that the given place does, have the ſame latitude.

PROBLEM V.

To find the distance between any two places on the globe.

Lay the graduated edge of the braſs quadrant of altitude, over both the places, and count the number of degrees intercepted

tercepted between them on the quadrant, which will be the diſtance in degrees; and which multiplied by 60, will give the diſtance in geographical miles; but multiplied by $69\frac{1}{2}$ gives the diſtance in Engliſh miles. Or, the diſtance between the two places may be taken with a pair of compaſſes, and that extent applied to the equator, will ſhew the number of degrees diſtance.

PROBLEM VI.

The hour of the day at any place being given, to find what is the hour at any other place.

Bring the given place to the brazen meridian, and ſet the index to the given hour; then turn the globe, until the place where the hour is required, comes to the meridian; and the index will point to the hour at that place.

PROBLEM VII.

To find the Sun's place in the Ecliptic and his declination, for any given day in the year.

Look on the wooden horizon for the given day, and againſt it there is placed the degree of the ſign, in which the ſun is, on that day at noon. Find the ſame degree of this ſign in the ecliptic line upon the globe, and having brought it to the brazen meridian, obſerve what degree of the meridian ſtands over it; and that is the ſun's declination, reckoned from the equator.

PROBLEM VIII.

To find all those places in the north frigid zone, where the sun begins to shine constantly, without setting on any given day: which must be between the 21st of March, and the 23d of September. (See fig. 2, plate 15.)

Having brought the ſun's place for the given day to the brazen meridian, and found his declination, (by Problem VII.)

count as many degrees on the meridian from the north pole, as are equal to the ſun's declination, and mark that degree from the pole; then turning the globe round its axis, obſerve what places in the north frigid zone paſs directly under that mark; for they are the places required.

The ſame problem may be reſolved for places within the ſouth frigid zone, for the other half of the year.

PROBLEM IX.

To find the place over which the sun is vertical, at any hour of a given day.

Find the ſun's declination for the given day, (by Problem VII.) which mark on the brazen meridian; then bring the place where you are, (ſuppoſe London,) to the brazen meridian, and ſet the index to the given hour; then turn the globe on its axis until the index point to 12 at noon; and the place on the globe which is directly under the point of the ſun's declination, marked upon the meridian has the ſun that moment in the zenith, or directly vertical.

PROBLEM X.

Having the day and hour of a lunar eclipse; to find all those places of the earth, to which it will be visible.

When the moon is eclipſed, ſhe is always at the full, and conſequently, oppoſite to the ſun; therefore, what ever part of the earth, the ſun is vertical to, the moon muſt be vertical to the antipodes of that part; conſequently, the ſun will be viſible then, to one half the earth, and the moon to the other half.

Therefore, find the place to which the ſun is vertical at the given hour, (by Problem IX.) elevate the pole to the latitude of that place, and bring the place to the upper part of the brazen meridian; then as the ſun will be viſible to all

all thoſe parts of the globe, which are above the horizon, the moon will be viſible to all thoſe parts below it, at the middle of the eclipſe.

PROBLEM XI.

To rectify the globe for the latitude, the zenith, and the sun's place.

Find the latitude of the place, (by Problem I.) and if the place be in north latitude, raiſe the north pole, as many degrees above the horizon, counting upon the meridian from the north pole to the horizon; but if the place be in ſouth latitude, raiſe the ſouth pole as many degrees: then turn the globe till the place comes under its latitude on the brazen meridian, and faſten the quadrant of altitude to the meridian, ſo that the chamfered edge of its nut may be joined to the zenith. Then bring the ſun's place in the ecliptic for the given day, to the graduated ſide of the brazen meridian, and ſet the hour index to 12 at noon, and the globe will be rectified.

NOTE. The hour 12 at noon on the hour circle, is the uppermoſt 12.

PROBLEM XII.

Having the latitude of any place between the two polar circles; to find the time of the ſun's riſing and ſetting, or the length of the day and night for any given day in the year.

Rectify the globe for the latitude, and the ſun's place, by the foregoing problem; then bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon, and the hour index will ſhew the time of ſun riſing; then turn the globe on its axis, until the ſun's place come to the weſtern ſide of the horizon, and the index will then ſhew the time of ſun ſetting.

The hour of ſun ſetting being doubled, gives the length of the day; and the hour of the ſun riſing doubled, gives the length of the night.

PROBLEM XIII.

Having the latitude of a place and the day of the month, to find when the morning twilight begins, and the evening twilight ends.

Rectify the globe, and bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon; then mark that point of the ecliptic, which is in the weſtern ſide of the horizon, which is the point oppoſite to the ſun's place; then lay the quadrant of altitude over the ſaid point, and turn the globe eaſtward, keeping the quadrant at the ſame mark, until the point on the ecliptic is 18° high on the quadrant, and then the index will point out the time when the morning twilight begins; for the ſun's place will be 18° below the eaſtern ſides of the horizon. Then, to find the time when the evening twilight ends, bring the ſun's place to the weſtern ſide of the horizon, and the point oppoſite to it, which was marked, will be riſing in the eaſt; bring the quadrant over that point, and keeping it thereon, turn the globe weſtward, until the ſaid point be 18° above the horizon on the quadrant, and the index will ſhew the time when the evening twilight ends, as the ſun's place will be then 18° below the weſtern ſide of the horizon.

When the ſun does not go 18° below the horizon of any place, the twilight continues the whole night in that place, and between 49° of latitude, and the polar circles, the twilight continues for ſeveral nights together in the ſummer ſeaſon: and the nearer the place is to the polar circle, the greater is the number of theſe nights.

PROBLEM

PROBLEM XIV.

To find what Day of the Year, the Sun begins to ſhine conſtantly without Setting, on any given Place in the frigid Zone, and how long it continues to do ſo.

Rectify the globe to the latitude of the place, and turn it about until ſome point of the ecliptic, between *aries* and *cancer*, (if the given place be in the north frigid zone) coincides with the north point of the horizon, where the brazen meridian cuts it. Then find on the wooden horizon, what day of the year, the ſun is in that point of the ecliptic; for that is the day on which the ſun begins to ſhine conſtantly on the given place, without ſetting. Then turn the globe, until ſome point of the ecliptic, between *cancer* and *libra*, coincide with the north point of the horizon, where the brazen meridian cuts it; and find on the wooden horizon, on what day the ſun is in that point of the ecliptic; which is the day the ſun leaves off conſtantly ſhining on the ſaid place, and riſes and ſets to it, as to other places on the globe. The number of natural days, or complete revolutions of the ſun about the earth, between the two days above found, is the time that the ſun keeps conſtantly above the horizon without ſetting; for all that portion of the ecliptic which lies between the two points, which interſect the horizon in the very north, never ſets below it; and there is juſt as much in the oppoſite part of the ecliptic that never riſes; therefore, the ſun will keep as long conſtantly below the horizon of every place upon the globe in winter, as he is above it in ſummer.

PROBLEM XV.

Having the Latitude, the Sun's place, and his Altitude, to find the Hour of the Day, and the Sun's azimuth, or number of Degrees that he is diſtant from the Meridian.

Having rectified the globe, and brought the ſun's place to the given heighth, upon the quadrant of altitude, which

muſt

muſt be on the eaſtern ſide of the horizon, if the time be in the forenoon, and on the weſtern ſide if it be afternoon, then the index will ſhew the hour of the day; and the number of degrees in the horizon, intercepted between the quadrant of altitude, and the ſouth point, will be the ſun's true azimuth at that time.

PROBLEM XVI.

To find what hour of the Day it is, in any Part of the World.

Rectify the globe for the latitude of the place; and having ſet the index to the hour of the day, turn the globe, and bring the places of which the hour is required, ſucceſſively to the brazen meridian, and the index will point to the ſeveral hours. For example:—if the place be London, and the hour 12 at noon, the globe being rectified for London, and London brought to the meridian, and the index ſet to the hour 12, turn the globe, till Naples comes to the brazen meridian, and the index then will point to the hour 1; Naples being 15° eaſtward of London. Then continue to turn the globe 15° further, and *Peterſburgh*, *Conſtantinople*, and *Grand Cairo*, will come under the brazen meridian, or very near it, then the index will point to the hour 2; theſe three cities having the noon-day ſun about two hours before us in London. And turning the globe 15° further, the index will point to the hour 3; and all the places under the meridian will have the ſun vertical to them. And thus for every 15° of longitude eaſtward, the inhabitants at thoſe places have the ſun, an hour ſooner, or before us at London. On the contrary, all the inhabitants ſituated to the weſtward of London, have the ſun later in the ſame proportion, that is, an hour later for every 15° of weſtern longitude.

Moſt of the foregoing Problems, may be reſolved by a map, as well as a globe, though the operation may be ſomewhat more

more tedious, particularly by plate 10, where the two hemispheres of the world, reprefent the furface of a terreftial globe in plano.

WINDS.

Winds are generally divided into two parts, according to the different parts of the earth on which they blow, being the tropical winds, or thofe which blow between the two tropics, and thofe which blow without the tropics.

The tropical winds generally extend to 30° on each fide the equator, and are of three kinds. 1. The general trade winds. 2. The monfoons. 3. The fea and land breezes.

1. The trade winds blow from north eaft on the north fide of the equator, and from the fouth eaft on the fouth fide of the equator, and near the equator almoft due eaft; but under the equator, and from 2 to 5° on each fide of the equator, the winds are variable, and fometimes it is calm for a month together.

2. The monfoons are periodical winds, which blow fix months is one direction, and the other fix months in the oppofite direction. At the change, or fhifting of the monfoons, are generally violent ftorms of wind, thunder, lightning, and rain, which always happen about the equinoxes. The monfoons extend about two hundred leagues from land, and are chiefly in the Indian feas.

3. The fea and land breezes are alfo periodical winds, which blow from the land in the night, and beft part of the morning; and from the fea, from about noon to midnight. Thefe do not extend above two or three leagues from fhore.

Near the coaft of Guinea, in Africa, the wind blows almoft conftantly from the weft.

On the coaft of Peru, in South America, the wind blows conftantly from the fouth weft.

Between the third and tenth degree of fouth latitude, the fouth eaft trade wind continues from April to October; but

during

during the reſt of the year, the wind blows from the north weſt. Between Sumatra and New Holland, this wind blows from the ſouth, from March to September; but from September to April, blows in the oppoſite direction. Between Africa and Madagaſcar, its direction is influenced by the coaſt, for it blows from the north eaſt, from October to April, and from the ſouth weſt, the reſt of the year.

In the Indian Ocean to the northward of the third degree of ſouth latitude, the north eaſt wind blows from October to April, and the oppoſite wind the reſt of the year. This wind blows nearly from the ſouth in the ſummer months, from the Iſle of Borneo, along the coaſt of Molacca, as far as China; and in the winter months, it blows from the ſouth.

In the temperate zones, the winds are very irregular, and no certain rule can be formed of their changes. But when winds are violent and continue long, it is generally found that they extend over a large tract of country; particularly, if they blow from the north or eaſt. By the multiplication and compariſons of meteorological tables, the following theorems have been deduced.

In Virginia, the prevailing winds are between the ſouth-weſt, weſt, north, and north-weſt; but the moſt frequent is the ſouth-weſt. At Ipſwich, in New England, the prevailing winds are the ſame, but the moſt frequent is the north-weſt. At Cambridge, in the ſame province, the moſt frequent is the ſouth-eaſt. The predominant winds of New York are the north and weſt. And in Nova-Scotia, north-weſt. And at Hudſon's Bay, weſt.

It appears from theſe obſervations that the weſterly winds are the moſt frequent over the whole eaſtern coaſts of North America; but in the ſouthern provinces, the ſouth-weſt wind is predominant, and the north weſt becomes gradually more frequent as we approach the frigid zone.

In

In Egypt from May to September, inclusive, the wind blows almost constantly from the north, varying in a few points from east to west, in the months of June and July.

In the mediterranean sea, the wind blows nearly 9 months of the year from the north; and at the equinoxes there is always an easterly wind in that sea. But in the streights of Gibraltar, the winds are either from east or west.

In Italy, the prevailing winds differ considerably, according to the situation of the places; at Rome and Padua, they are northerly; at Milan easterly.

The prevailing wind in Spain and Portugal, is the west; particularly on the western coasts of these countries; but at Madrid it is north-east.

In France, along the whole south coast of that country, the wind blows most frequently from the north, or north-east, to north-west. On the western coast of the netherlands, as far north as Rotterdam, the prevailing wind is the south-west.

From the register, kept by order of the Royal Society, at London; the average of the winds at that place, blow in the following order:—

Winds.	Days.	Winds.	Days.
South-west	112	South-east	32
North-east	58	East	26
North-west	50	South	18
West	53	North	16

It appears from this register, that the south-west wind blows at an average, more frequent than any other winds, during every month of the year; but particularly in July and August; and the north-east blows most constantly during the months January, March, April, May and June, and most seldom during February, July, September, and December; and the north-west blows more frequently, from November to March; and more seldom in September and October, than any other months.

Tides.

The tide is that riſe and fall of the water, obſerved on all maritime coaſts.

It is obſervable that on the ſhores of the ocean, and in all bays, creeks, harbours, &c. which have a free communication with the ocean, the waters riſe up, above a certain mean rate twice a day, and as often ſink below; this is what is called the *flood* and *ebb*, or an high and low water. The whole interval between high and low water, is called a *tide*; the water is ſaid to *flow*, and to *ebb*; and the riſing is called the *flood tide*, and the falling the *ebb tide*.

This riſe and fall of the waters, is very variable in quantity. Thus, at Plymouth, it is ſometimes twenty-one feet between the greateſt and leaſt depth of the water in one day, or between high and low water; and ſometimes it is only twelve feet.

The greateſt flow of tide in any place is called *a ſpring tide*, and the leaſt flow is called a *neap tide*; and the different heights of the tide, gradually increaſe every day, from a neap to a ſpring tide; and then gradually decreaſe from a ſpring to a neap tide.

The whole time between the ſpring and neap tide, is about fifteen days; and two of theſe intervals will make an exact lunation, or change of the moon. For the ſpring tide is obſerved to happen at a certain interval of time, (generally between two and three days,) after the new or full moon: and the neap tide at a certain interval, after the half moon. Thus, the high water happens at new and full moon, when the moon has a certain determined poſition with reſpect to the meridian of the place of obſervation, preceding or following the moons ſouthing a certain interval of time; which is always conſtant with reſpect to that place, but very different in different places.

The

The interval between two ſucceeding high waters, is very variable. It is leaſt of all about new and full moon, and greateſt when the moon is at her quadratures. As two high waters happen every day, we may call the double of their interval, a tide day. Now, this tide day is ſhorteſt about new and full moon, being then about 24 hours and 37 minutes; but longeſt at the moons quadratures, being then 25 hours and 37 minutes.

The tides, being in ſimilar circumſtances, are greateſt when the moon is at her leaſt diſtance from the earth; and leaſt, when ſhe is at her greateſt diſtance from the earth.

The ſame may be remarked with reſpect to the ſun's diſtance. Thus, the greateſt tides are obſerved during the winter months in Europe, or when the ſun is at his leaſt diſtance.

The tides in every part of the ocean increaſe, as the moon, by changing her declination, approaches the zenith of that place.

The tides which happen, while the moon is above the horizon, are greater than thoſe of the ſame day, when the moon is below the horizon.

Theſe are all the regular phenomena of the tides. They are of the utmoſt importance to all commercial nations, and have therefore been much attended to by all navigators, and aſtronomers.

SECT. III.

THE GRAND DIVISIONS OF THE EARTH.

THE principal diviſions of the earth (as before mentioned) are into land and water.

The land is divided into the two great continents, (besides Islands,) viz:—The eastern and western continents. The eastern continent is sub-divided into the following parts, viz:—Europe on the north west: Asia, on the north east, and Africa on the south, being joined to Asia by the Isthmus of Suez, which is 60 miles over. The western continent consists of North and South America, joined by the Isthmus of Darien, between 60 and 70 miles in breadth.

Europe, is again sub divided into the following principal parts, and is situated between the tenth degree. west longitude, and the sixty-fifth degree, east longitude; and between the thirty sixth and seventy-second degree of north latitude. It is bounded on the north, by the Frozen Ocean; on the east, by Asia; on the south, by the Mediterranean sea, which divides it from Africa; and on the west, by the Atlantic Ocean, which separates it from America, being 3,000 miles in length from Cape St. Vincent in the west, to the mouth of the river Oby in the north-east; and 2,500 miles broad, from north to south, from north cape in Norway, to Cape Cayha, in the Morea, the most southern point of Europe: It contains the following states and kingdoms.

A TABLE

A TABLE.

	Nations.	Long.	Broad.	Cities.	Distance bearing from Lon	Differ. of time from London.		Religions.
British Empire.	England	380	300	London		H.	M.	Proteſt. Pap.
	Scotland	300	150	Edinburgh	400 N.	0.	12 A	Calviniſts and
	Ireland	285	160	Dublin	270 N. W.	0.	26 A	Lutherans.
Netherlands.	Norway	1000	300	Bergen	540 N.	0.	24 B	Lutherans
	Denmark	240	180	Copenhagen	500 N. E.	0.	50 B	Ditto
	Sweden	800	500	Stockholm	750 N. E.	1.	10 B	Lutherans
	Ruſſia	1500	1100	Peterſburg	1140 N. E.	2	4 B	Greek Church
	Poland	700	680	Warſaw	760 E.	1.	24 B	Pap. luth. & cal.
	K. Pr. Dom	609	350	Berlin	540 E.	0	55 B	Luth. and Cal.
	Germany	600	500	Vienna	600 E.	1.	5 B	Pap. luth. & cal.
	Bohemia	300	250	Prague	600 E.	1.	4 B	Papiſts
	Holland	150	00	Amſterdam	180 E.	0.	18 B	Calviniſts
	Flanders	200	200	Bruſſels	180 S. E.	0	16 B	Papiſts
	France	600	500	Paris	200 S. E.	0.	9 B	Papiſts
	Spain	700	500	Madrid	800 S.	0	17 A	Papiſts
	Portugal	300	100	Liſbon	850 S. W.	0	38 A	Papiſts
	Switzerland	260	100	Bern coir &c	420 S. W.	0.	28 B	Cal. and Papiſts
Italy.	Several Small States.	Piedmont, Montſerrat, Milan, Parma, Modena, Mantua, Venice, Genoa, Tuſcany, &c. Turin, Caſſal, Milan, Parma, Moden, Mantua, Venice, Genoa, Florence.						
	Popedom	240	120	Rome	820 S. E.	0.	52 B	Papiſts
	Naples	280	120	Naples	870 S. E.	1.	0 B	Papiſts
	Hungary	300	200	Buda	780 S. E.	1.	17 B	Pap. & Proteſt.
Turkey in Eu.	Danubia	600	420	Conſtantino	1320 S. E.	1.	58 B	Mahometans and Greek Church.
	Little Tartary	380	240	Precop	1500 E.	2.	24 B	
	Greece	400	240	Athens	1360 S. E.	1.	37 B	

Beſides the foregoing ſtates, Europe contains ſeveral Iſlands of which the following are the principal:—

Islands.		Chief towns.	Subject to
In the Northern Ocean	Iceland - - -	Skalholt	Denmark
In the Baltic Sea.	Zealand, Funen, Alsen, Falster, Langland, Laland, Femeren, Mona, Bornholm.		Ditto
	Gothland, Aland Rugen		Sweden
	Osel Dagho,		Russia
	Usedom, Wollin		Prussia
Mediterranean Sea.	Ivaca - - - -	Ivaca	Spain
	Majorca - - -	Majorca	Spain
	Minorca - - -	Port Mahon	Spain
	Corsica - - -	Bastia	England
	Sardinia - - -	Cagliaria	K. Sardinia
	Sicily - - -	Palermo	K of 2 Sici.
Gulf of venice	Lusiena, Crfu Cephalonia, Zaut, Leucadia.		Venice
Archipelago & Levant seas	Candia, Rhodes, Negropoint, Lemnos, Tenedos, Scyros, Mytelene, Scio, Samos, Patmus, Paros, Serigo, Santorin, &c. being part of ancient and modern Greece.		Turkey

The continent of Asia, is situated between 25 and 180 degrees of east longitude; and between the equator 80 degrees noth latitude; being about 4740 miles in length, from the Dardanelles on the west to the easter shore of Tartary, and about 4380 in breadth, from the most southern part of Malacca, to the most northern cape of Nova Zembla. It is bounded by the frozen ocean on the north; on the west, it is separated from Africa, by the Red Sea, and from Europe, by the Levant or Mediterranean, the Archipelago, the Hellespoint, the sea of Marmora, the Bosphorus, the Black Sea, the River Don, and a line drawn from it to the River Tobel, and

and from thence to the River Oby, which falls into the frozen ocean; on the east, it is bounded by the Pacific Ocean or South Sea, which separates it from America; and on the south, by the Indian Ocean; thus, it is almost surrounded by the sea. The principal divisions are as follows:—

	Nations.	long	broad	chief cities	Distance & bear. from London.	Diff. of time from London.	Religions.
						H. M.	
Tartary.	Russian	the bounds of these parts are variable: each power extending his own conquest.		Tobolsk	2160 N. E.	4. 10 B	Chris. & Pag.
Tartary.	Chinese			Chinian	4480 N. E.	8. 4 B	Pagans
Tartary.	Mogulean			Tibit	3780 E.	5. 40 B	Pagans
Tartary.	Independent			Samarcand	2800 E.	4. 36 B	Pagans
	China	1440	1000	Pekin	4320 S. E.	7. 24 B	Pagans
	Moguls	2000	1500	Delhi	3720 S. E.	5. 16 B	Maho. & Pag.
	India	2000	1000	Siam, Pegu	5050 S. E.	6. 44 B	Maho. & Pag.
	Persia	1300	1100	Ispahan	2460 S. E.	3. 20 B	Mahometans
Turkey in Asia.	part of Arabia	1300	1200	Mecca	2640 S. E.	2. 52 B	Mahometans
Turkey in Asia.	Syria	270	160	Aleppo	1800 S. E.	2. 30 B	Chris. & Maho.
Turkey in Asia.	Holy Land	210	90	Jerusalem	1920 S. E.	2. 21 O	Chris. & Maho.
Turkey in Asia.	Natolia	750	300	Bursa or Smyrna	1410 S. E.	1. 48 B	Mahometans
Turkey in Asia.	Diarbeck or Mesopotamia	240	210	Diarbeck	2060 S. E.	2. 56 B	Mahometans and some Christians
Turkey in Asia.	Irack or Chaldea	420	240	Bagdad	2240 S. E.	3. 30 B	
Turkey in Asia.	Aronenia	360	300	Erzerums	1860 S. E.	2. 44 B	
Turkey in Asia.	Georgia	240	180	Teflies	1920 E.	3. 10 B	
	Curdistan or Assyria	240	205	Scherazer	2220 E.	3. 0 B	Mahometans

All the Islands of Asia, (except Cyprus, in the Levant sea, belonging to the Turks,) lie in the Pacific Ocean, of which the principal are the following:—

Islands.	Towns.	Belonging to.
The Japanese Isles	Jeddo & Meaco	Dutch
The Ladrones	Guem	Spain
Formosa	Tai-ouan-fou	China
Anian	Kiont-cheow	China
The Phillippines	Manilla	Spain
The Molucca or Clove Isles	Victoria Fort	Dutch
The Banda or Nutmeg Isles	Lantor	Dutch
Amboyna	Amboyna	Dutch
Celebes	Macasser	Dutch
Gilolo	Gilolo	Dutch
The Sunda Isles, Borneo,	Borneo Caytonged	Several Nations
Sumatra,	Achen & Bencoolen	Eng. & Dutch
Java, &c.	Batavia & Bantam	Dutch
The Andaman and Nicobar Isles	Andaman Nicobar	Several Nations
Ceylon	Candia	English
The Maldives	Caridon	English
Bombay	Bombay	English
The Kurile Isles in the Sea of Kamschatka, discovered by the Russians.		Russia

Africa, the third grand division of the globe, is generally represented as bearing some resemblance to the form of a pyramid, whose vertex or point is the Cape of Good Hope, and its base the shores of the Mediterranean Sea. It is a Peninsula of great extent, joined to Asia by the Isthmus of Suez its greatest length from north to south from Cape Bona in the Mediterranean, to the Cape of Good Hope, is 4300 miles, and the breadth from Cape Verd, to Cape Guardafic is 3500 miles, it is bounded on the north, by the Mediterranean sea, which separates it from Europe; and the east by the

Isthmus

Iſthmus of Suez, the Red Sea and the Indian Ocean, which divides it from Aſia; on the ſouth, by the Southern Ocean; and on the weſt, by the great Atlantic Ocean, which ſeparates it from America.

Very few travellers have penetrated into the interior part of this quarter of the world; conſequently, we ſtill remain ignorant of the bounds, and even of the names of many of the inland parts, but according to the beſt accounts, it is divided according to the following Table:—

	Nations.	length	broad.	Cities.	Diſtance bearing from London.	Differ of time from London H. M.	Religions.
Barbary.	Morocco, Tafilet, &c.	500	480	Fez	1080 S.	0. 24 A	Mahom.
	Algiers	480	100	Algiers	920 S.	0. 13 B	Mahom.
	Tunis	220	170	Tunis	990 S. E.	0. 39 B	Mahom.
	Tripoli	700	240	Tripoli	1260 S. E.	0. 56 B	Mahom.
	Barca	400	303	Polemeta	1440 S. E.	1. 26 B	Mahom.
	Egypt	600	250	Grand cairo	1920 S. E	2. 21 B	Mahom.
	Bil-dulgerid	2500	350	Dara	1560 S.	0. 32 A	Pagans
	Zaara	3400	660	Tegeſa	1800 S.	0. 24 A	Pagans
	Negroland	2200	840	Madinga	2500 S.	0. 28 A	Pagans
	Guinea	1800	300	Benin	2700 S.	0. 20 B	Pagans
Up. Ethiopia.	Nubia	940	600	Nubia	2418 S. E.	2. 12 B	Maho. pag.
	Abyſſinia	900	800	Gondar	2880 S. E.	2. 20 B	Chriſtians
	Abex	540	130	Doncala	3580 S. E.	2. 36 B	Chr. & pag.

The middle parts called Lower Ethiopia, are very little known to Europeans, but are computed at one million, two hundred thouſand miles.

Loango	410	300	Loango	3300 S.	0 44 B	Chr. & pag.
Congo	540	420	St. Salvador	3480 S.	1. 1 B	Ditto
Angola	360	250	Loando	3750 S.	0. 58 B	Ditto
Benguela	430	100	Benguela	3900 S.	0. 58 B	Pagans
Mataman	450	240	No Towns	*	*	——
Ajan	900	300	Brava	3732 S. E.	2. 40 B	——
Zanguebar	1400	350	Melinda	4440 S. E.	2. 38 B	——
Monomotapa	960	660	Monomotape	4500 S.	1. 18 B	——
Monemugi	900	660	Chicona	4260 S.	1. 44 B	——
Sofola	480	300	Sofola	4600 S E.	1. 18 B	——
terra-de-nata	600	350	No Towns	*	*	——
Caffaria or Hottentots	708	660	Cape of Good Hope	5200 S.	1. 4 B	——

Iſlands.	Towns.	Belong to
Babel Mandel	Bab. Man.	
Zocotra in the Indian Ocean	Calauſia	
The Comora Iſles ditto.	Joanna	
Madagaſcar ditto.	St. Auſtin	
Mauritius ditto.	Mauritius	French.
Bourbon ditto.	Bourbon	Ditto.
St. Helena in Atlan. Ocean	St. Helena	Engliſh.
Aſcenſion Iſle ditto.		Uninhabited.
St. Matthew ditto.		Uninhabited.
St, Thomas, Anaboa, Princeſs Iſlands - Fernandopo ditto.	St. Thomas Anaboa.	Portugueſe.
Cape Verd Iſlands ditto.	St. Domingo	Portugueſe.
Goree ditto.	Fort. St. Mich.	French.
Canaries ditto.	Palma St. - Chriſtopher	Spain.
Madeiras ditto.	Santa Cruz	Portugueſe.
The Azores, or Weſtern Iſles, which are at an equal diſtance from Europe, Africa, and America, ditto.	Angra, St. Michael,	Portugueſe.

America, the greateſt weſtern continent, called the new world, runs north and ſouth through every habitable climate upon the earth, extending from the eightieth degree of north latitude to the fifty-ſixth degree of ſouth latitude, and its breadth where it is known, extends from the thirty-fifth to the one hundred and thirty-ſixth degree of weſt longitude, from London, being near 9000 miles in length, and 3690 in breadth. Extending into both the Hemiſpheres, it has conſequently two ſummers, and two winters, and has all the variety of climates to be met with on the face of the earth. On the eaſt it is bounded by the great Atlantic Ocean, which divides it from the eaſtern continent. On the weſt it has the Pacific Ocean, or great South Sea, which ſeparates it from Aſia. It is compoſed of two

two parts, called North and South America, joined together by a narrow neck of land, called the Isthmus of Darien, in the kingdom of Mexico, 1500 miles long, and at one part being only 60 miles in breadth, so that the communication between the two oceans is by no means difficult. In the great gulph, which is formed between the Isthmus, and the northern and southern continents, lie a multitude of Islands, denominated the West Indies, in distinction to the Islands of Asia, beyond the Cape of Good Hope, which are called the East Indies. The grand divisions of North America are as follows:

Colonies.	leng	Brdth	Chief Towns	Distance and bearing from London.	Belongs to
New Britain	850	750			G. Britain.
Prov. of Quebec	600	203	Quebec		Ditto.
New Scotland & New Brunswick	350	250	Halifax Shelburne		Ditto.
New England	550	200	Boston	2760 W.	Un. States.
New York	300	150	New York		Ditto.
New Jersey	160	60	Perth Amboy		Ditto.
Pennsylvania	300	240	Philadelphia		Ditto.
Maryland	140	135	Annapolis		Ditto.
Virginia	750	240	Wilamsburg		Ditto.
North Carolina			Edenton		Ditto.
South Carolina	700	380	Charles Town		Ditto.
Georgia			Savannah		Ditto.
East Florida West Florida	500	440	St. Augustine Pensacola		Spain.
Louisiana	1200	645	New Orleans	4084 S. W.	Ditto.
New Mexico & California	2000	1000	St. Fee St. Juan	4420 S. W.	Ditto.
Mexico, or New Spain	2000	600	Mexico	4900 S. W.	Ditto.

Grand Divisions of South America.

Nations.	leng.	Brdth.	Chief Towns	Diſtance of bearing from London.	Belongs to
Terra Firma	1400	700	Panama	4650 S. W.	Spain.
Peru	1800	600	Lima	5520 S. W.	Spain.
Amazonia	1200	960			
Guiana	780	680	Surinam Cayenne	3840 S. W.	Dutch. French.
Braſil	2500	700	St Sebaſtian	6000 S. W.	Portugal.
La Plata	1500	1000	Buan'Ayres	6040 S. W.	Spain.
Chili	1200	500	St. Jago	6600 S. W.	Ditto.
Terra Magellanica, or Patagonia	1400	460			

Th

The Principal Islands in North America belonging to the Europeans.

	Islands.	Length	Brdth	Chief Tns.	Belongs to
In the gulph of St. Lawrence.	Newfoundland	350	200	Placentia	Great Britain.
	Cape Breton	110	80	Louisburg	Ditto.
	St. John	60	30	Charlotte	Ditto.
					Ditto.
In the Atlantic.	The Bermuda Isles	20000		St. George	Ditto.
	The Bahama Isls			Nassau	Ditto.
The West India Islands lying in the Atlantic, between N. & S. America.	Jamaica	140	60	Kingston	Ditto.
	Barbadoes	21	14	Bridgetow	
	St. Christophers	20	7	Base Terre	Ditto.
	Antigua	20	20	St. John's	Ditto.
	Nevis	6	3	Charlestn.	Ditto.
	Montserrat	5	4	Plymouth	Ditto.
	Barbuda	20	12		Ditto.
	Anguilla	30	10		Ditto.
	Dominica	28	13		Ditto.
	St. Vincent	24	18	Kingston	Ditto.
	Granada	30	15	St. George	Ditto.
	Cuba	700	90	Havannah	Spain.
	Hispaniola	450	150	St. Domin	Spain & France
	Porto Rico	100	49	Porto Rico	Spain.
	Trinidad	90	60	St. Joseph	Spain.
	Margarita	40	23		Ditto.
	Martinico	60	30	St. Peter's	England.
	Gaudaloupe	45	38	Basse Terre	Ditto.
	St. Lucia	23	12		Ditto.
	Tobago	32	9		England.
	St. Bartholomew				Sweden.
	Deseada				France.
	Marigalanta				Ditto.
	St. Eustatia	7	4	The Bay	Dutch.
	Curasson	30	10		Ditto.
	St. Thomas	5	3		Denmark.
	St. Croix	30	10	Basse End	Ditto.

I shall

I shall here subjoin a table of the superficial content of the several parts of the globe in square miles, accounting 60 miles to a degree on the equator.

	Square Miles.	Islands.	Square Miles.	Islands.	S.M.
The globe	199,512,595	Cuba	38,400	Funen	768
Seas and un-		Java	38,250	Yvica	625
known parts	160,522,026	Hispaniola	36,000	Minorca	520
The habita-		Newfoundland	35,500	Rhodes	480
ble world	38,990,567	Ceylon	27,730	Cephalonia	420
Europe	4,456,065	Ireland	27,457	Amboyna	400
Asia	10,768,822	Formosa	17,000	Orkney Po-	
Africa	9,654,807	Anian	11,900	mona	324
America	14,110,874	Gilola	10,400	Scio	300
Persian Emp		Sicily	9,400	Martinico	260
under Darius	1,650,800	Timor	7,800	Lemnos	220
Roman Emp		Sardinia	6,[illegible]	Corfu	194
in its great-	1,610,000	Cyprus	6,300	Providence	168
est height		Jamaica	6,000	Man	160
Russian	4,161,685	Flores	6,000	Bornholm	160
Chinese	1,749,000	Ceram	5,400	Wight	150
Great Mogul	1,116,000	Breton	4,000	Malta	150
Turkish	950,057	Socatia	3,600	Barbadoes	140
Present Persia	800,000	Candia	3,220	Zant	120
ISLANDS.		Porto Rico	3,200	Antiqua	100
Borneo	228,000	Corsica	2,520	St. Christo-	
Madagascar	168,000	Zealand	1,935	pher	80
Sumatra	129,000	Majorca	1,406	St. Helena	80
Japan	118,000	St. Jago	1,400	Guernsey	50
Great Britain	72,926	Negropont	1,300	Jersey	43
Celebes	68,400	Teneriff	1,272	Bermudas	40
Manilla	58,500	Gothland	1,000	Rahodes	36
Iceland	46,000	Madeira	950		
Terra, Del		St. Michael	920		
Fuego	42,075	Skye	900		
Mindiano	39,200	Lewis	880		

There are also several other considerable Islands chiefly in the South Seas, the exact dimensions of which are not certainly known, but they may be ranged according to their magnitude in the following order. New Holland being nearly equal in size to the whole continent of Europe.

New

New Holland.	Otaheite, or King George's Island.
New Guinea.	Friendly Islands.
New Zealand.	Marquesses.
New Caladonia.	Easter, or Davis's Island.
New Hebrides.	

SECT. IV.

OF THE DIFFERENT GOVERNMENTS OF THE WORLD.

MANKIND were no sooner united into civil societies, than they discovered an inclination to oppress each other. That system of equality, in which they were left by nature, gave the strongest and the most crafty the advantage over his weaker and undesigning neighbours. From hence arose the necessity of forming conjunctions of several individuals or families together; who should implicitly follow the dictates or commands of some chosen superior, or leader. And to prevent the altercations, strife, and consequently bloodshed, that inevitably followed the nomination of every new leader, or prince, they caused the office to be made hereditary. Consequently, absolute, and hereditary monarchy, was the first original form of government, as appears from sacred writ; where Nimrod is represented by his courage and dexterity to have acquired a superiority of fame and power above his contemporaries; and he founded at Babylon the first monarchy whose origin is mentioned in history.

In the 1496 before Christ, the Greeks were the first people who by the advice and public spirited endeavours of Cecrops and Cranaus his successor formed a regular counsel. For Amphictyon, one of those disinterested characters

characters who live for the good of the community, of which he is a member, endeavoured to find an expedient to unite the ſeveral independent kingdoms of Greece into one body; and thus to put a ſtop to thoſe fatal conſequences of inteſtine diviſion, and civil diſcord, which rendered them a prey to each other, and an eaſy conqueſt to the invader. He therefore engaged the kings or leaders of twelve different cities to unite together for their mutual ſecurity and welfare. Two deputies from each city aſſembled twice a year at Thermopylac, and formed the Amphictyonic Counſel. In this aſſembly the general intereſts of the ſtates was diſcuſſed. Amphictyon, in order to render thoſe ſeveral connections more durable, connected them with religious charge, intruſting the care of the temple at Delphi, with the riches that accrued to this place from thoſe who conſulted the oracles, to the care alſo of theſe deputies. This aſſembly was the firſt political eſtabliſhment of a plurality of power, that we have any authentic account of in hiſtory; and gave an energy of action to Greece, which enabled them to defend their liberty and independance againſt the great force of the Perſians.

This was the firſt deviation from abſolute monarchy, recorded in profane hiſtory; from that time, various have been the modes and forms of government in different nations; though if we except ſome part of the Romiſh Hiſtory, Greece, and a few nations of leſs note, the monarchial form of government was the moſt prevailing for the next two thouſand years.

Athens is an inſtance of the pernicious effects of diviſion in a ſtate; and alſo diſplays the benefits of unanimity. Theſeus king of Attica, about the year before Chriſt 1234, perceiving the danger to which his country was expoſed by this twelvefold diviſion, endeavoured to form a conjunction of the ſtates; for this purpoſe, he detached the leaders of the different tribes as much as poſſible from the people they governed; he aboliſhed the different courts eſtabliſhed in different parts of Attica,

Attica, and appointed one Council Hall common to all the Athenians. He eſtabliſhed a common form of religion, with certain religious ceremonies to be performed at Athens, the more effectually to ſtrengthen civil allegiance; and by inviting ſtrangers from all parts of the world, by the promiſe of privileges and protection, he raiſed the city to the higheſt pitch of fame and popularity. The ſplendor of Athens eclipſed that of all the other States of Greece.

This is a brief outline of the origin of the firſt monarchy of which we have any authentic account in profane hiſtory; but this monarchy ſoon gave place to an over-bearing influence. Theſeus had formed his kingdom into three diſtinct claſſes: the nobles, the artiſans, and the huſbandmen. And to prevent the increaſing power of the nobles, he granted many immunities and privileges to the two other claſſes. This ſyſtem of politics in a few years gave the two inferior claſſes an opportunity of acquiring conſiderable property, and conſequently, they became conſiderable members of the ſtate. And by their riches and independence, upon the death of Codius, a prince of great merit, in the year B. C. 1070, they had power and influence enough to aboliſh the regal power, under pretence of finding no one worthy of filling the throne of Codius, who had devoted himſelf to death for the ſafety of his people. Thus, they proclaimed Jupiter king, declaring none elſe was fit to govern Athens. This was the firſt inſtance of a perfect republican form of government.

From this period ſo various have been the modes and forms of government, that it is impoſſible to diſtinguiſh them all. Governments are generally divided into three diſtinct forms, each of which has its partizans, viz. the Monarchial, Ariſtocratical, and Democratical.

The monarchial form of government, is, where a nation is governed by a king, or monarch, and is divided into two parts called abſolute and limited monarchy. Abſolute monarchy, is where the ſovereign is entirely unreſtrained, having the legiſlative, as well as the executive power. A limited monarchy

is where the ſovereign is reſtrained by certain laws, beyond which he cannot paſs.

An ariſtocracy, is where the legiſlative and executive authority is veſted in the hands of a ſelect number of perſons, genererally titled nobility, and in whom the office is moſtly hereditary.

A democracy, is that government in which the legiſlative and executive authority is veſted in a certain number of individuals, who hold their office by election; and generally elected by the majority of the nation at large.

From the various modifications of theſe different forms of government, all the governments of the earth are formed; ſome approaching nearer to one, and ſome to another form. For there is hardly a government exiſting, that is entirely either an abſolute monarchy, a perfect ariſtocracy, or a complete republic.

SECT. V.

OF RELIGION.

RELIGION is coeval with the origin of mankind: without it the preſent order of the univerſe would be entirely overturned, and mankind, from his natural depravity, be rendered worſe than the moſt voracious of the brute creation.

The diſtinguiſhing religions in antiquity, were Judaiſm, and Polytheiſm, or Pagan.

But in modern times, the prevailing religions may be divided into the four following, viz. the Jewiſh, Chriſtian, Mahometan, and Pagan.

Before

Before treating of the four foregoing ſyſtems, it may be neceſſary to premiſe the following general axiom, viz. That all ſyſtems of religion, whether true or falſe, contribute more or leſs to the welfare of ſociety. From hence we deduce the following theorem; that all religion muſt have ſomewhat in its origin of a divine nature, however it may be transformed, corrupted, or miſapplied, by the ignorance or artifice of its propagators.

In conſidering the Jewiſh code of religion, it does not appear as a complete ſyſtem of religion, adapted to all countries and ages, but ſeems particularly deſigned by the all-wiſe Creator for the people to whom it was ſent; for the age they lived in, being over-run by idolatry; the circumſtances in which they had lived in Egypt; and the means by which they were to form their new ſettlement in the land of Canaan.

From hence they were enjoined the obſervation of the ſabbath, in honour to that Being who created the heavens and the earth, with all the hoſt of heaven; which hoſt, ſun, moon, ſtars, &c. &c. were worſhipped by the Egyptians as eternal beings. To prevent their communication with the neighbouring idolatrous nations, they were proſcribed the uſe of certain animals for food, and permitted others; that by being forbidden the uſe of thoſe animals for food, ſuch as the hog, &c, which the Gentile nations conſidered as the greateſt luxury, a perpetual bar might be kept up between the Jews and Gentiles. And by being allowed other animals for food, ſuch as goats, ſheep, oxen, &c. which were worſhipped in Egypt, and from which the Egyptians religiouſly withheld all violence, the Jews would ſoon overcome any religious prejudices they might have acquired from the Egyptian idolatry. The reſtitution of property, in the year of Jubilee, which would anſwer no purpoſe in another ſtate, was deſigned to preſerve the order of rank, and that diviſion of property, originally eſtabliſhed.

In condeſcenſion to their rude and groſs notions of Deity,

 the

the Creator permitted them in their wanderings through the wildernefs to have a tabernacle, or portable temple, in which he fometimes condefcended to difplay fome rays of his glory.

From the general view of the Jewifh religion, it appears happily adapted to promote the welfare of its followers. In comparing it with other religions, it is neceffary to reflect on the peculiar purpofes for which it is eftablifhed, which were principally two, firft, to preferve the Jews a feparate people; and fecondly, to guard them from the idolatry with which they were every where furrounded. The religion of the Jews was not formed nor defigned to be propagated through all the earth; that would have been inconfiftent with the purpofes for which it was inftituted; therefore, we fee the Jewifh religion, though near four thoufand years old, wants that effential attribute for propagation to be found in all other religions, viz. a difference of fentiment, and, confequently, a divifion, and fubdivifion into different fects.

The Chriftian religion is to be confidered as an improvement of the Jewifh. The effects of the Jewifh religion were indeed beneficial, but were confined almoft to them alone; whereas, the effects of the Chriftian religion are extended to all mankind, reprefenting them with true philanthropy as children of the fame God, and heirs of the fame falvation. It levels all diftinctions of rich and poor, native and foreigner, as accidental and infignificant diftinctions with that impartial Being, who rewards or punifhes, according to the demerits of his creatures.

The precepts of the Chriftian religion are more happily calculated to promote the happinefs of mankind, than thofe of any other religion. Its whole defign is to infpire mankind with mild, benevolent, and peaceable difpofitions. Its diftinguifhing rule, by which it excels all other religions is, *to do unto others as we would they fhould do unto us.* And fuch is its purity, that it does not allow an impure thought; it requires its followers to abandon their vices, however dear; and

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to join the cautious wisdom of the serpent with the innocent simplicity of the dove. And to prevent perseverance in immorality, it offers a pardon for the past, provided the offender forsake his vicious practices. The practice and belief of this gospel have a peculiar tendency to raise the mind of man above the trifling pursuits of time, and to render its followers incorruptible, by wealth, honours, or pleasures. It not only requires the Christian to abstain from injuring his neighbour, but even enjoins him to forgive any unmerited injuries, which he himself suffers, upon the principle of his being forgiven by his offended Creator. It represents the Deity and his attributes in the fairest light, so as to render our ideas of him consistent with the correct principles of reason and philosophy. The rites of this gospel are few and simple; easy to perform, expressive and edifying. It inculcates no duties, but what are founded on the principles of human nature, and on the relation in which man stands to God, as his Creator, redeemer and sanctifier. The assistance of the spirit of God is there promised to those who labour to discharge the duties which it enjoins. It teaches us that worldly afflictions are casual accidents; incident to both bad and good men: a doctrine highly encouraging to virtue, consoling in affliction, preventing despair, and encouraging in difficulty.

Such are the precepts and spirit of the Christian religion. And even those who have refused to give credit to its history, and follow its doctrines, have acknowledged the excellency of its precepts. Bolingbroke, one of its most zealous opposers, says, that "no religion ever yet appeared in the world, of which the natural tendency was so much directed to promote the peace and happiness of mankind as the Christian; and that the gospel of Christ is one continued lesson of the strictest morality, of justice, benevolence and universal charity." Thus we can pronounce with confidence, that the precepts of a religion, which is so happily formed to promote

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all that is juſt and beneficial to mankind, cannot but be in the higheſt degree divine. By reviewing the effects which it has produced, we ſhall be more confirmed in our aſſertion.

Chriſtianity has produced the moſt beneficial change in the circumſtances of domeſtic life: it has greatly contributed towards the abolition of ſlavery, and towards the mitigation of the rigours of ſervitude. We meet with no laws in Chriſtian countries, ſo inhuman, as thoſe practiſed at Rome; where maſters were allowed to remove their ſick or infirm ſlaves to an iſland in the Tiber, there to periſh without any aſſiſtance. The rigours of ſlavery are eaſed and aboliſhed; not by any particular precept of the goſpel, but by the gentle and human ſpirit which breathe through the general tenor of the whole ſyſtem. And though it may be objected, that a trade in ſlaves is at preſent carried on by people who preſume to call themſelves Chriſtians, and ſanctioned by the legiſlature of ſome Chriſtian ſtates; yet it muſt be remembered, that the ſpirit of the Chriſtian code condemns the practice, and the true Chriſtian will not engage in it.

Chriſtianity is alſo gradually ſoftening barbarous nations into humanity; the influence of ſelfiſhneſs has been checked and reſtrained; and even war, with all the pernicious improvements, by which mankind has ſought to render it more terrible, has aſſumed much more the ſpirit of mildneſs and peace, than ever entered into it under the influence of Paganiſm.

Theſe are a few of the excellencies of the Chriſtian ſyſtem. Its laſt diſtinction I ſhall mention, is that of its extending its benefits to thoſe nations who have not received its doctrines and precepts. The virtues aſcribed to Julian the apoſtate, are in no doubt owing to his acquaintance with Chriſtianity; and after the propagation of Chriſtianity through th Roman Empire, even while the purity of its doctrine was deſpiſed, it had a remarkable effect on the manners of thoſe unconverted

unconverted Pagans, who, in their religious doctrines and worſhip, became leſs immoral and abſurd.

Upon the whole, we muſt conclude, that Chriſtianity is infinitely ſuperior to every other religious ſyſtem, both in point of its religious doctrines, and the effects it has produced upon ſociety. It is an univerſal religion, formed to exert its happy influence in all ages, and among all nations; and has a tendency to diſpel the ſhades of barbarity and ignorance; to promote the cultivation of the powers of the human underſtanding, and to encourage every virtuous refinement in manners.

As the Chriſtian religion is deſtined to be of an univerſal nature, and to be diſſeminated into all parts of the world; ſo, in order for its more effectual propagation, its all-wiſe founder has ordained that it ſhall be divided into different ſects and parties; that the leaders of each being governed by a mutual emulation, might endeavour to propagate their reſpective opinions, and thereby form a grand junction for propagating a religion, the fundamentals of which would be invariably the ſame.

The two principal ſects into which the Chriſtian religion is divided, are the Proteſtant and Romiſh churches.

The Proteſtant church has already been deſcribed under the Chriſtian religion. The Romiſh church differs from the Proteſtant, chiefly in the following particulars: 1. In believing every thing that was defined by the Counſel of Trent, concerning original ſin and juſtification. 2. In believing tranſubſtantiation, or the converſion of the material bread and wine, given at the ſacrament, into the real body and blood of Jeſus Chriſt. 3. In the belief of a purgatory; and that ſouls are kept priſoners there after their departure from the body, and that they receive help by the prayers of the faithful. 4. That the ſaints reign together with Chriſt, and are to be worſhipped as mediators for man. 5. That the images of Chriſt, the Virgin Mary, and other Saints, ſhall be retained, and due honour and veneration be given unto them.

them. 6. That the power of indulgencies was left by Chriſt to the church. 7. That the holy church of Rome is the mother and miſtreſs of all churches; and that the Biſhop of Rome, or Pope, is the ſucceſſor of St. Peter, the Prince of the apoſtles, and vicar of Jeſus Chriſt on earth, and that he is infallible and invincible.

Theſe are the chief tenets which diſtinguiſh the church of Rome from the Proteſtant church. The implicit obedience which the followers of this church pay to their leaders, has been a ſource of very black corruption and error. Of which their numerous perſecutions of the Proteſtants are an ample proof. But on the other hand, it muſt be allowed, that there is no religion ſo zealous of propagating its doctrines. Their miſſionaries have been ſent to all parts of the earth, ſome of whom, by their perſeverance and abſtemiouſneſs, were as great an honour, as others by their profligacy were a diſgrace, to the cauſe in which they were concerned.

The church of Rome is now divided into two ſects. That already deſcribed, which prevails over moſt parts of Italy, Spain, France, Ruſſia, and ſeveral other parts of the continent of Europe; and the Greek Church, which differs from the former, in not allowing the Pope's ſupremacy, not worſhipping idols, though they have many in their churches, and in not enjoining their prieſts to celibacy.

The Proteſtant religion is divided into numerous ſects and parties. The two principal of which are Lutherians and Calviniſts.

The Lutherians maintain, that man is a free agent, perfectly capable of performing good or evil, that according to his actions he ſhall be rewarded or puniſhed hereafter; and that he is left at perfect liberty to chuſe the good or evil; and that God has no predeliction for any particular perſons. That the ſacrament of the Lord's Supper is nothing but a mere ordinance.

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The Calvinifts, on the contrary, affert, that man is not a free agent, that he has no power to perform any good action without the fpirit of God, affifting him. That God, according to his fore-knowledge, has elected a certain number of individuals to be faved. That he is the former of every good thought; and faves the elect not from any goodnefs in themfelves, but merely from his own unmerited mercy, confequently, that Chrift did not die for all the world. That the facrament is a fpiritual rite, that the bread and wine is confubftantiated (not tranfubftantiated) in a fpiritual manner into the body and blood of Chrift.

Befides thefe two divifions of the Proteftant religion, one or both of which prevail in moft Proteftant countries, on the continent of Europe, there are a great number of inferior fects in England, America, Holland, Germany, and other parts, and fome of them very numerous, as the Quakers, Baptifts, Diffenters, Methodifts, &c. which are too well known to need any defcription. Suffice it to fay, each of them differs from the eftablifhed church, on account of fome trifling errors which they imagine to have detected in the national church.

The next grand religion that deferves our notice is that of Mahomet, which ftill makes fuch a confpicuous figure in the world, extends over a large tract of country, and is profeffed by very powerful nations. Like the Jewifh religion, it is not merely a fyftem of religious doctrines and moral precepts; but it forms both the civil legiflature and religious fyftems of the nations by which it is profeffed. It alfo appears to be rather calculated for one particular period, in the progrefs of mankind from barbarity to refinement, than for all ages, and all parts of the world.

In viewing this fyftem of religion, there are many parts of it which feem copied from the Chriftian, the Jewifh, and the Pagan religions. It is difficult to tell which is the greater,

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the purity of ſome parts of this doctrine, or the abſurdity of other parts.

The greateſt abſurdity, or that which tends moſt effectually to promote impurity of manners, are the Prophet's ideas of heaven and hell. Paradiſe, or the place of future rewards, he makes to abound with rivers, trees, fruits, and ſhady groves; wine, without its intoxicating quality, was to be there ſerved out to believers, who, as they enjoy perpetual youth, their powers of enjoyment are to be enlarged and invigorated according to the delights they were to enjoy. Mahomet celebrates the pearls and diamonds, robes of ſilk, palaces of marble, diſhes of gold, numerous attendants, wines and dainties, with the whole train of ſenſual luxury, reſerved for the faithful in theſe regions. Seventy-two black eyed damſels of reſplendent beauty, blooming youth, virgin purity, and exquiſite ſenſibility, will be created for the uſe of the meaneſt believer. A moment of pleaſure will be prolonged to one thouſand years, and the faculties will be encreaſed a hundred fold, to render him worthy of his felicity. There are alſo certain more refined enjoyments; as, believers are to ſee the face of God morning and evening; a pleaſure which is to exceed all the other pleaſures of paradiſe.

In Hell, the place of future puniſhments, the wicked are to drink nothing but boiling, ſtinking water; eat nothing but briars and thorns, and the fruit of a tree that grows in the bottom of hell, whoſe branches reſemble the heads of devils, and whoſe fruit ſhall be in their bellies like burning pitch; they are to breath nothing but hot winds, and dwell for ever in continual burning, fire and ſmoke.

From a view of Mahometiſm it appears to be a ſtrange mixture of abſurdities, with a few truths and valuable precepts incongruouſly intermixed. A great part of it is incompatible with virtue, and the progreſs of knowledge and refinement. It ſubſtitutes trifling ſuperſtitious ceremonies in the

the room of genuine piety and virtue; and presents such a prospect of futurity as renders purity of heart no necessary qualification for seeing God.

However, Mahometism forms in some measure a regular system of religion, as it has borrowed many of its precepts and doctrines from both Judaism and Christianity, which are however, greatly degraded and corrupted. It has, however, considerably contributed towards the support of civil government in those countries in which it is established.

It is divided into a numerous party of sects, which, however, differ so little from each other as scarcely to deserve mention in this place.

Paganism was the most prevailing religion of antiquity, and may be principally divided into two parts: 1. The Pagan religion of the antient barbarous nations; and 2. The Polytheism of the more civilized Greeks and Romans.

The Paganism of the ancient nations, present us with a most shocking picture of ignorance, superstition and absurdity. We there are presented with the most absurd doctrines concerning the future state. Various nations have imagined that the scenes and objects of the world of spirits are only a shadowy representation of the things of the present world: according to them, not only the souls of men inhabit those regions, but all the inferior animals and vegetables, and even inanimate bodies that are killed or destroyed here, are supposed to pass into that visionary world, and existing there in unsubstantial forms, to execute the same functions or serve the same purposes, as on earth. These are the ideas of futurity, entertained by the inhabitants of Guinea; and by these ideas they were stimulated on the death of a king or other great personage, to provide for his accommodation in the world of spirits, by burying with his corpse, meat and drink for his subsistance, slaves for his attendance, and wives for his enjoyment. His faithful subjects vied with each other in their offerings, upon this occasion; one brought a servant, another a wife, a third, a son or daughter, to accompany

their monarch in his future ſtate. Similar practices, on the ſame occaſion, prevailed in New Spain, in the iſland of Java, in the kingdom of Benen, and among the inhabitants of Hindoſtan. Similar belief alſo prevailed among the Japaneſe. They not only bribed their prieſts to ſolicit for them a place in the bliſsful manſions of futurity, but looking upon the preſent life with diſguſt and contempt, when ſet in competition with the joys of futurity, they uſed to daſh themſelves from precipices, or cut their throats, in order to get to Paradiſe as ſoon as poſſible. Various other ſuperſtitions ſubſiſting among rude nations, might here be adduced, as inſtances of the perverſion of the religious principles of the human heart, which render them injurious to virtue and happineſs. Innumerable are the ways of torture, which have been invented and practiſed on themſelves, by men, ignorantly ſtriving to obtain the favour of heaven. Theſe are ſufficient proofs of religious ſentiments having been ſo ill directed by the influence of imagination, and unenlightened erring nature, aided by the corrupt deſigns of artful prieſts.

The Polytheiſm of the Greeks and Romans, though more favourable to virtue and civilization, than the Pagan notions of antiquity, (are yet a very imperfect, not to ſay a pernicious, code of religion,) the vicious characters of their deities, the abſurd notions they entertained, concerning the government of the univerſe, and a future retribution, the abſurdities of their religious rites and ceremonies, the frivolous practices with which they were intermixed, muſt altogether have a great tendency to pervert both the reaſoning and moral principles of the human mind; however, it cannot be denied, that this ſyſtem was friendly to the encouragement of arts; particularly, to ſuch as depend on the vigorous exertion of a fine imagination, as muſic, poetry, ſculpture, architecture and painting; all theſe arts appear to have been conſiderably indebted for that perfection to which they attained to the ſplended and fanciful ſyſtem of mythology, which was received by thoſe people, particularly by the Greeks.

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The effect of this religion, to reform the lives of its votaries, was very imperfect. Sacrifices and prayers, temples and festivals, not purity of heart, and integrity of life, were the means prescribed for obtaining the favour of the deities. There were also other means of gaining admission into the Elysian fields, or the seat of the counsel of Gods: but none of these means appear to have been those commanded by the Christian Religion. And whatever might be the effects of the religion of Greece and Rome in general, upon the civil and political establishments, and on the manners of the people, yet it must be confessed, to have been but ill adaptedt impress the heart with such principles as might in all circumstances direct to a firm, uniform tenor of righteous conduct.

From this view of religion, it appears, that though some particular forms, such as those of the christian, have had a greater influence in reforming the manners of their followers; yet as they have often contributed to form the mind to virtue, it must be acknowledged, that they have always, and under all their forms, been infinitely more beneficial than hurtful to mankind.

When we view the different systems in a comparative light, with respect to their influence, on the welfare of society, no one will hesitate to prefer the Polytheism of the Greeks and Romans, to the ruder ideas of the more ancient Pagans; and Mahometism to the Polytheism of the Greeks and Romans; Judaism is however, greatly preferable to Mahometism, and Christianity to all of them.

SECT.

SECT. VI.

OF EUROPE.

EUROPE, though the leaſt quarter of the World, is by far the moſt eminent in modern hiſtory. And is at preſent the moſt diſtinguiſhed part of the globe for the literature, arts, and ſciences, which it has produced and encouraged, and for the learned men it has produced. It is alſo the moſt civilized quarter of the globe. Here are no public marts, for buying and ſelling the human ſpecies, as are found in Aſia and Africa. The Chriſtian Religion alſo prevails here almoſt univerſally. Its languages are as mixed as its inhabitants, but all derived from the ſix following, viz.—The Celtic, Sclavonic, Tentonic, Greek, Latin, and Gothic. It extends about 3,000 miles in length, and 2,500 in breadth, and divided into ſeveral kingdoms and ſtates, as ſeen in the Table, page 133.

The British Isles, lying on the weſtern part of Europe, conſiſt of England, Wales, and Scotland, which together is called Great Britain, Ireland, and the Iſles of Man, Jerſey, Guernſey, Alderney, Sark, and Wight. England lies between 50 and 56 degrees north latitude, and between 2 degrees eaſt, and 6 degrees 20 minutes weſt longitude; and is divided into forty counties. Its conſtitution is that of a limited monarchy, conſiſting of king, lords, and commons, with certain prerogatives and privileges annexed to each.

The legiſlative authority, or power of making laws and raiſing money, is veſted in theſe three branches of the government; and each branch has a negative voice.

The

NORTHERN
ATLANTIC OCEAN
Arctic Circle
ICELAND
North Cape
Luffoden Isles
Mahlstrom
Faro Isles
Shetland Isles
Orkney Isles
Western Isles
BRITISH ISLES
BRITISH OCEAN
SCOTLAND
IRELAND
ENGLAND
WALES
Edinburgh
Aberdeen
Dublin
York
Bristol
London
Dover
Lands End
St. George
English Chan.
Bergen
Christiania
Stockholm
SWEDEN
FINLAND
G. of BOTHNIA
Tornea
Ulea
Wasa
Abo
Wyborg
Upsal
Gothland I.
Riga
Narva
Revel
Pernau
Memel
Koningsberg
Dantzic
Thorn
PRUSSIA
Copenhagen
Bornholm
BALTIC
Hamburg
Amsterdam
Hanover
Berlin
Cologne
Frankfort
Leipsic
Dresden
Prague
Mentz
Strasburg
Ratisbon
Vienna
Munich
GERMANY
POLAND
Warsaw
Cracow
Lemburg
Wilna
Grodno
Mohilow
HUNGARY
Buda
Presburg
Belgrade
TURKEY
Bucharest
Ragusi
Scutari
Philippopoli
Adrianople
Salonichi
GREECE
ARCHIPELAGO
Morea
Cerigo I.
Candia
Paris
Rouen
Rheims
Nancy
Orleans
Nantes
Tours
Rochelle
Bourdeaux
Toulouse
Lyons
Marseille
Geneva
Berne
SWITZERLAND
FRANCE
BAY OF BISCAY
Bayonne
Bilboa
Oviedo
Leon
Burgos
Madrid
Toledo
Saragossa
Barcelona
Valencia
Murcia
Carthagena
Granada
Malaga
Gibraltar
Cadiz
Seville
Lisbon
Oporto
SPAIN
PORTUGAL
Minorca
Majorca
Corsica
Sardinia
Turin
Genoa
Milan
Venice
Mantua
Parma
Florence
Leghorn
Rome
Naples
SICILY
Palermo
Messina
Syracuse
Malta I.
Lipari I.
G. of VENICE
MEDITERRANEAN
Algiers
Oran
Tunis
Gabes
Sallee
Fez
Tangier
AFRICA


EUROPE,
from the best
AUTHORITIES.
Roper, Engraver, New St. Square Shoe Lane.
Longit. E from London.
II Hours East from London.
III
OCEAN
WHITE SEA
BLACK SEA
CASPIAN SEA
LITTLE TARTARY
North Cape
Kola
Archangel
Petersburg
Moscow
Constantinople
Astrachan
Asoph
Smyrna
Cyprus
Rhodes
Aleppo
Tripoli
Damascus
Trebizond
Erzerum
Sinope
Samsoun
Bursa
Adana
Tarku

The crown is made hereditary in the Hanover line, by several acts of parliament, provided they do not profess Popery, marry Papists, or subvert the Constitution.

The Peers are created by the Crown, but their honours are hereditary, and cannot be taken from them any more than their lives or estates; unless forfeited by the commission of high treason; and they can only be tried by the House of Peers, being subject to no other jurisprudence. This House is the last resort in all civil cases, and the highest Court in the kingdom.

Any bill for making a new law, or altering an old law, may be brought in first in the House of Peers; but, a bill relating to the revenues, or public taxes, must be brought into the House of Commons first; and it cannot be *altered* by the Peers, though it may be rejected.

The House of Peers can apprehend and commit any man for a reflection on their judicature.

The Commons are composed of five hundred and fifty eight members, viz. eighty knights, every county in England sending two, and elected by the freeholders; fifty citizens, two being sent from each of the twenty five cities in England, (London sending four and Ely none): three hundred and thirty four burgesses, from one hundred and sixty seven boroughs, sending two each; five burgesses, from the boroughs of Abingdon, Banbury, Bewdley, Highamferrers, and Monmouth; four representatives from the two Universities; sixteen barons from the five cinque ports, Hastings, Dover, Sandwich, Romney, and Hythe, and their three dependants, Rye, Winchelsea, and Seaford; twelve knights, from the twelve counties of Wales: twelve burgesses from the twelve boroughs in Wales, (Pembroke sending two and Monmouth none); thirty knights, from the Shires of Scotland, and fifeeen burgesses from the Scotch boroughs.

WALES is situated on the west, and north west of England, to which it joins, and is divided into twelve counties; it is a principallity,

a principallity, and always considered as the right of the king's eldest son, who is therefore titled Prince of Wales. It was populated in the year 410, by the ancient inhabitants of England, who fled thither from the persecution of the Picts and Scots.

Ireland, is situated between 6 and 10 degrees of west longitude, and between 51 and 55 degrees of north latitude. Bounded on the east by St. George's Channel, or the Irish Sea, which divides it from Great Britain; it is divided principally into four provinces; viz. Ulster on the north, Leinster on the east, Munster on the south, and Connaught on the west.

The climate of these Islands is in general mild for the latitude, but very changeable; the weather never continuing a month the same, owing to the exhalations from the surrounding sea, which renders the air humid. But the soil is in general fruitful, and has been of late years greatly improved.

These Islands have several very good mines of tin, copper, iron, and lead; gold has also been found in Scotland, in solid pieces in the brooks, after a great torrent.

The manufactures of England is chiefly woollen-cloth, which is accounted the staple trade of the kingdom; as linen cloth is that of Ireland.

Denmark, including Norway, is the most northern kingdom of Europe, and includes Denmark proper, territories in Germany, Norway, part of Lapland, and several islands in the Baltic Sea, and in the German Ocean, and extends from fifty-two degrees of north latitude, to the farthest habitable part of the Artic Circle. Denmark proper, is bounded on the north by the Cattegeate, or Schaggerac, on the the south by Germany, on the west by the German Ocean, and on the east by the Sound.

The established religion is Lutherism. The king is absolute, though in general mild in his government. It is divided

vided into two parts, called North Jutland, and South Jutland.

The air in this country is ſharp, but the exhalations from the ſea abate its ſeverity. The ſummers are very ſhort and hot, but the ſoil in general fruitful for the northern latitude, except on the tops of mountains. The manufactures of this country are chiefly hardware, and their artiſts and mechanics, in every branch, are generally ſkilful.

NORWAY, is bounded on the ſouth by the Cattegate; on the weſt and north by the Northern Ocean; and on the eaſt by the mountains which ſeparate it from Sweden; and is divided into the north, ſouth, and middle diviſions. The air in Norway is generally healthy and dry in the inland parts of the country, but on the ſea coaſt it is moiſt. In winter it is exceſſive cold, the whole country being covered with ſnow; it is alſo very hot in the ſummer. Their trade conſiſts of copper, timber, iron, marble, mill-ſtones, fiſh, fowls, tallow, tar, oil, allum, vitriol, &c. Their language is the ſame as that uſed in Denmark, and their religion is that of Lutheriſm.

ICELAND, is ſituated in the Northern or Atlantic Ocean, being 726 miles in length from eaſt to weſt, and 300 in breadth; extending from ſixty-three to ſixty-eight degrees of north latitude, and from fourteen to twenty nine degrees of weſt longitude; and has a milder climate than any other country in the ſame latitude. It is a very mountainous country, but well watered, with ſeveral large rivers. In this country there are ſeveral large ſprings of boiling hot water, the principal of which is Geyſer, near Skalholt. The water iſſues from this ſpring ſeveral times a day, with a violent noiſe, like that of a great torrent, ſometimes raiſing to the heighth of ſixty fathom, and ſeldom leſs than 90 feet.

There are alſo ſeveral burning mountains in this country, of which the moſt remarkable are Hecla, Kotlegau, and Oraiſe, the eruptions of which have ſometimes done conſiderable damage.

damage. The inhabitants live chiefly by fiſhing and breeding cattle, attending very little to agriculture. Their commerce is monopolized by a Daniſh company of merchants, and conſiſts chiefly of ſalt-meat, butter, tallow, oil, wool, ſkins, furs, and feathers. The revenue ariſing from this country to the king of Denmark, amounts to 30,000 crowns per annum.

Greenland, is the moſt northern boundary of the king of Denmark's dominions; and is the fartheſt part of the globe northward which has been diſcovered. Eaſt Greenland extends beyond 76 degrees of north latitude; and between 10 and 11 degrees of eaſt longitude. There are no inhabitants here but a few convicts tranſported from Ruſſia, and who gain their liberty by procuring ſkins, furs, tuſks of morſe, &c. for the Sovereign of Ruſſia.

Weſt Greenland extends beyond 60 degrees of north latitude; and between 5 and 50 degrees of weſt longitude. There are a few natives who inhabit this country, and many of whom have lately been converted to Chriſtianity, by the Daniſh and Moravian miſſionaries.

Sweden, extends from 55 degrees 20 minutes, to 69 degrees 30 minutes north latitude; and from the twelfth to the thirtieth degree of eaſt longitude. It is bounded on the ſouth by the Baltic, the Sound, and the Cattegate Sea; on the north by Daniſh Lapland; by Ruſſia on the eaſt; and by the mountains of Norway on the weſt; and principally divided into ſeven provinces: viz. 1. Sweden, properly ſo called, lying between Norway and the gulph of Bothnia; 2. Gothnia, or Gothland; 3. Livonia, on the ſouth of Finland gulph; 4. Ingria, on the north-eaſt of Livonia; 5. Finland on the eaſt ſide of the gulph of Bothnia; 6. Swediſh Lapland, in the northern parts; 7. The iſlands of Gothland, Oeland, Aland, Hogland, and Rugen.

Note. The Provinces of Livonia and Ingria, with Kexholm

holm and Carelia in Finland, and the Islands of Dago and Osel, are under the government of Russia.

The natural soil of this country is in general barren, but has been greatly improved of late years, by the industry of the inhabitants, assisted by the affluent part of the nation, so that they have now fruitful harvests. Their manufactures are chiefly in silver, copper, and iron: and vast quantities of these metals, with timber, tar, hemp, flax, hides, furs, fish, &c. constitute the chief articles of their trade.

Their religion is the same as that of Denmark and Norway. The language is also partly the same, being only a dialect of the Teutonic language. The government of this country is a limited monarchy.

Russia, the largest Empire upon the globe, and greater than all the rest of Europe besides, extends in length from the Baltic Sea on the west, to within a few miles of America on the east, upwards of 6000 miles; and above 2400 miles in breadth from north to south. It is bounded on the west by Sweden and the Baltic; on the east by China, and the Pacific Ocean, which separates Asia from America; on the north by the Frozen Ocean; and on the South by Prussia, Poland, Turkey, Persia, and Tartary. Its measured length from the Isle of Dagho to its eastern bounds is near 170 degrees. Thus it contains several different climates; on the southern parts, the longest day is scarcely 16 hours, while on the northern parts it is nearly three months. The climate therefore of this country is very various. In the southern provinces it is very hot, and extremely cold in the northern parts. The soil beyond the sixtieth degree of north latitude, scarcely ever produces corn to any perfection; and beyond the seventieth degree scarcely any specie of fruit is found; but in the middle provinces the soil is fruitful, and produces good pasture for cattle, and excellent grain; the southern provinces being hot, has all the fertility of a hot country, where there is a sufficient depth of soil. There is a great variety of in-

habitants in this extenſive country, viz. The Tartars, Kamſchatdales, Samoeides, Laplanders, &c. But there is very little variety in the manners of theſe different countries. In ſome parts of the north they live in caverns, not five foot in height. In ſome parts they are given to robbing, and rambling from one place to another; in other parts they are more harmleſs; and in other parts, again, they practice agriculture, while in other parts they live on the ſpontaneous productions of the ſoil.

The religion in ſome parts, is next to Paganiſm, idolizing inanimate objects, as a ſheep's ſkin; but in ſome parts they make no public profeſſion of religion. The eſtabliſhed religion of Ruſſia, is the Greek Church.

The European part of Ruſſia, called Muſcovy, is divided into the following provinces, viv.—

(In the Northern Diviſion;) Lapland, Samoieda, Bellamornſckoy, Meſeen, Dwina, Syrianes, Permia, Rubeninſki, Belacſeda (In the Middle Diviſion;) Pereſlaf, Belozero, Wologda, Jereſlaf, Tweer, Moſcow, Belgorod. (In the Eaſtern Diviſion;) Bulgar, Kanſan, Lit, Novogorod, Don Caſſacks. (In the Weſtern Diviſion;) Great Novogorod, Rus, Findland, Kexholm, Karelia, Ingria. (In the Southern Diviſion;) Livonia, Smolenſko, Zernigof, Seeſſk, Ukrian. Their articles of commerce and manufacture are the ſame as thoſe of Sweden and Denmark; they have moreover, ſilk, cotton, teas, gold, &c. which they bring from China and India, in caravans, by way of the Caſſian Sea.

The language, is derived from the Sclavonian, to which is added many words from the Greek; their alphabet conſiſts of forty-two characters, which are principally Greek. The people of higher rank generally ſpeak French and high Dutch, but their prieſts ſpeak the modern Greek.

Poland, before its late diſmemberment, was bounded on the north, by Livonia, Muſcovy, and the Baltic Sea; on the eaſt, by Muſcovy; on the ſouth, by Hungary, Turkey and Little Tartary; and on the weſt, by Germany; extending from 47 degrees, 40 minutes, to 56 degrees, 30 minutes, north

north latitude; and from 16 to 34 degrees eaſt longitude. It was divided into the provinces of great and little Poland, Poliſh, Pruſſia, Samogitia, Courland, Lithunia, Maſovia, Podolachia, Poleſia, Red Ruſſia, Podolia, and Volhinia. The ſoil of Poland, is in general very fruitful, and the air moſtly temperate, except in the northern parts, where it is very cold. Their paſture land is ſo fruitful, that the heighth of the graſs often conceals the cattle from the view of a paſſenger at two hundred yards diſtance. Great number of beaſt, as horſes, aſſes, oxen, buffaloes, bears, foxes, wolves, &c. run wild in the foreſt. There are ſeveral mines in the country, of gold, ſilver, copper, lead, iron, &c.

The greateſt curioſities in this country, is the ſalt mines, of which, that of Wielitſka, is the largeſt in the World, and has been wrought above ſix hundred years. It is 743 feet below the ſurface of the ground; and 1115 feet in breadth, and 6691 in length, and appears like a ſpacious plain, with vaulted roofs, ſupported by columns of ſalt, which have been left ſtanding. Many public lights are placed in this mine, for general uſe, which reflect a moſt luminous appearance from every part of the mine. Here are alſo great numbers of huts for the accommodation of the miners, and their families; many of whom, are born and ſpend their lives in this place, without ever making their appearance on the ſurface of the earth. Through the midſt of the mine, is the great road, which paſſes to the mouth of the mine, which is generally crowded with carriages full of ſalt. A ſtream of freſh water alſo runs through the mine.

The wild men which have been ſeen of late years in the woods of Poland, form another curioſity.

The Poles at preſent ſeem ſomewhat oppreſſed by the powers of Ruſſia, Auſtria, and Pruſſia; but the Ottoman Port, has ſhewn them conſiderable favour.

Prussia, is bounded on the north, by Samogitia; on the ſouth, by Poland proper and Maſovia; on the eaſt, by part of Lithunia; and on the weſt, by Poliſh Pruſſia, and the Baltic; but if we take it in its full extent, this kingdom conſiſts

cofifts of various territories, different parts of Germany, Poland, Switzerland, and other northern countries.

The principal divifions of this kingdom, are regal Pruffia, fituated in Poland; and Upper Saxony, containing Brandenburg, Pruffian Pomerania, and Swedifh Pomerania, Magdeburg, and Halberftadt in Lower Saxony; Glatz in Bohemia; Minden, Ravenfburg, Lingen, Cleves, Meures and Mark, in the Dutchy of Weftphalia; Eaft Friefland, Lippe, Gulick, and Tacklenburgh, in the circle of Weftphalia; the Margraviate of Anfpach, in the circle of Franconia; Gelder in the Netherlands; Neufchatel in Switzerland; and part of Silefia.

Prussia, carries on a confiderable trade, and the balance in favour of Ruffia, is reckoned greater than that of any other European ftate: great quantities of glafs, iron works, cloth, camblet, filk, linen, paper, powder, copper and brafs, are annually exported.

Amber, is found in great quantities here, from which the crown receives 26000 dollars annually, alfo great fums from the Bitumen of which feveral kinds, is found in the Baltic Sea.

The religions of Pruffia, are thofe of the Lutherans, and Calvinifts; but all religions are tolerated. His Pruffian majefty is abfolute, through all his dominions. The Pruffian army, even in times of peace, confifts of 180,000 men, which are reckoned the beft difciplined troops in the world; but in time of war, it has been augmented to between 3 and 400,000 men.

Germany, is bounded on the north, by the German Ocean, Denmark and the Baltic; on the eaft, by Poland, Hungary, and Bohemia; on the fouth, by Switzerland and the Alps; and on weft, by France and the low countries. It extends from 45 degrees, 4 minutes, to 54 degrees, 40 minutes, north latitude; and from 6 degrees to 19 degrees, 45 minutes eaft longitude. Germany, is a great empire, having feveral dependant fovereignties under it, under different modifications of government, fome of whom, fcarcely exceed an Englifh manor in extent. It is divided into nine circles, three of which

which lie in the north, three in the middle, and three in the ſouth, viz.—Upper Saxony, Lower Saxony, Weſtphalia; Upper Rhine, Lower Rhine, Franconia; Auſtria, Bavairia, and Suabia. Theſe circles are ſub-divided into principalities, dutchies, marquiſates electorates, palatinates, counties, baronie's, abbies, biſhoprics, &c.

The climate of Germany is in general healthy and agreeable, except in the moſt northern and ſouthern parts. And the ſoil is particularly fruitful; for though only a ſmall proportion of the country is cultivated, yet proviſions are in general cheaper, than in moſt other countries of Europe. They have alſo a greater quantity of domeſtic animals and wild beaſts, as boars, hares, rabbits, foxes, badgers, goats, &c. &c. than other European countries. They alſo abound in moſt of the ſpecie of tame fowl, as well as wild fowl.

There are ſeveral mines in Germany of ſilver, copper, iron, lead, quickſilver, ſulphur, nitre, &c. and coal pits are found in every part of the Empire.

Germany is alſo in great eſteem in all other European countries for its mineral ſprings and baths, the moſt remarkable of which are thoſe of Aix-la-Chapelle, Spa, Pyrmont, Embs, Wiſbaden, Schwalbach, Wildungen and Brakel, which laſt is incloſed as the waters are ſo ſtrong as to be capable of intoxication.

The manufactures of Germany conſiſts of velvets, ſilks, cotton, and woollen ſtuffs, linen, fuſtian, ribbands, lace, tapeſtry, &c. They alſo make beautiful porcelain, and lacquered ware, and every kind of hard ware.

The Germans have a conſiderable commerce, owing to their centrical ſituation, and the balance of trade is greatly in their favour. The eſtabliſhed religion is either Romiſh, Lutheran, or Calviniſt, being different in the different parts of the Empire; but moſt other religions are tolerated at preſent.

The

The German language is a dialect of the Teutonic, and is called the high Dutch, being the mother tongue of the whole Empire; but every different province has a different dialect.

The government of Germany is in the hands of about three hundred civil and ecclesiastical princes, each of whom is absolute in the government of his own state, and the whole of them form a great confederacy, governed by political laws, at the head of which is the Emperor, whose power in the collective body is only executive. The Emperor is elected; but the Empire for some centuries has belonged to the House of Austria, as being the most powerful of the German princes. The nine electors of the Empire have each a particular office in the Imperial court: and they have the sole election of the Emperor, and are as follows: 1. The Archbishop of Mentz, who is High Chancellor of the Empire, when in Germany. 2. The Archbishop of Treves, who is High Chancellor of France and Arelat, (a dignity merely nominal.) 3. The Archbishop of Cologne. 4. The king of Bohemia, who is cupbearer. 5. The Elector of Bavaria, who is grand sewer. 6. The Elector of Saxony, who is great Marshal of the Empire. 7. The Elector of Brandenburgh, (now king of Prussia), who is great Chamberlain. 8. The Elector of Palatine. 9. The Elector of Hanover, (King of Great Britain) who claims the post of Arch-treasurer.

The revenue of the Emperor, as such, is about 5 or 6000 pounds sterling, per annum, arising from the fiefs in the black forest. The Austrian revenues are immense, amounting to one hundred florens, or twelve million pounds sterling.

The military force of this country amounts to near half a million of men; the secular Princes bringing upwards of 379,000, and the ecclesiastical 7450, and the Emperor, as the head of the House of Austria, 90,000.

Hungary, Bohemia, and the provinces of Transylvania, Scylavoni, Croatia, and Morlatia, may be considered as part of

of the German Empire, having been brought under the dominion of the House of Auſtria. The eſtabliſhed religion of theſe countries is the church of Rome. Bohemia lies between 48 and 52 degrees north latitude; and between 12 and 19 degrees eaſt longitude. Bounded on the north by Saxony and Brandenburg; on the eaſt by Poland and Hungary; on the ſouth by Auſtria and Bavaria; and on the weſt by Bavaria. Tranſylvania lies between 45 and 48 degrees north latitude; and between 22 and 25 degrees eaſt longitude. Bounded on the north by Hungary and Poland; on the ſouth by Walachia; on the eaſt by Moldavia, and on the weſt by Hungary. Sclavonia lies between 45 and 47 degrees north latitude; and between 16 and 22 degrees eaſt longitude. Bounded by the river Drave on the north; by Auſtria on the ſouth; by the Danube on the eaſt; and by the Save on the weſt. Croatia lies between 44 and 47 degrees north latitude, and between 15 and 17 degrees eaſt longitude. Bounded on the north by the Save; on the South by Morlachia; on the eaſt by Boſnia; and on the weſt by Carniola. Morlachia lies between 44 and 46 degrees north latitude, and between 16 and 17 degrees eaſt longitude. Bounded on the north by Carniola; on the ſouth by Dalmatia; on the eaſt by Boſnia; and on the weſt by the gulph of Venice.

SWISSERLAND is bounded on the north by Swabia; on the eaſt by the Lake of Conſtance, Tirol and Trent; on the ſouth by Italy; and on the weſt by France; extending from 45 to 48 degrees north latitude, and from 6 to 11 degrees eaſt longitude, and is divided into thirteen cantons: viz. Bern, Fribourg, Baſil, Lucern, Soloturn, in the weſt diviſion; Schaffhauſen, Zurick, Appenzel, in the eaſt diviſion; and Zug, Swiſs, Glaris, Uri, and Underwald, in the middle diviſion. Seven of theſe Cantons profeſs the Romiſh religion: Fribourg, Lucern, Soloturn, Zug, Swiſs, Uri, and Underwald; the other ſix are Proteſtants. The climate of this country is very various on account of the inequality of the ſurface of

the ground, being ſituated among the alps, the higheſt mountains in Europe: ſo that it is common for the inhabitants to be reaping on one ſide of a mountain, while thoſe on the other ſide of the ſame mountain are ſowing. The froſts in winter are very ſevere; and in the ſummer, the heat is in ſome parts very intenſe.

The commerce of Swiſſerland, conſiſts of their cattle, horſes, cheeſe, butter, hides, ſkins, and the productions of their own manufacture, the principal of which, are ſilks, brocades, linen, lace, woollens, ſtuffs, hats, paper, leather, poreclain, toys, watches, and clocks.

Each canton forms a ſeparate republic; but when any controverſy ariſes, it is referred to the general diet which ſits at Baden, where each canton has a vote, and ſends two deputies.

The Netherlands lie between 50 and 54 degrees, north latitude, and between 2 and 7 degrees eaſt longitude. They are bounded on the north, by the German Ocean; on the eaſt, by Germany; on the weſt, by the Britiſh Channel; and on the ſouth, by France and Lorrain. The Netherlands are divided into ſeventeen provinces; the ſeven northerly ones are called Holland, or the United Provinces, and the other ten, are called Flanders, or the Auſtrian and French Netherlands.

The provinces of Holland, are Holland, Zealand, Friezland, Groningen, Overyſſel, Gelderland, Zutphen, and Utrecht.

The air of theſe provinces, is very moiſt and foggy, and their harbours are generally frozen up four months in the year, and the ſoil is very unfavourable for vegetation; but the induſtry of the inhabitants has greatly improved it, by making canals and ditches to drain the land.

Their commerce is carried on to ſuch an extent, that there is hardly a commodity of traffick on the face of the globe, but

but may be bought here, and almoſt as cheap as in the places where they were produced.

The religion of this country, is calviniſm, but all profeſſions and ſocieties are tolerated, of which there are great numbers.

The government of this country, is a democracy, and has ſo continued for upwards of two hundred years; notwithſtanding they had a prince under the title of a ſtadtholder, whoſe powers had very little of the regal nature.

The ten other provinces of the Netherlands, called Flanders, have been divided among the Auſtrians, French, and Dutch, and contain the ten following provinces: viz.—Brabant, Antwerp, Malines, Limburgh, Luxemburg, Namur, Hainault, Cambreſis, Artois, and Flanders.

The ſoil in moſt of theſe provinces is extremely fruitful, and the air generally healthy, except in Brabant, and ſome parts of the ſea coaſts.

The commerce of theſe provinces, conſiſts chiefly of their own manufacture, viz.—Fine linens, cambricks, laces and woollen manufacture.

France, extends from 42 to 51 degrees, north latitude, and from 5 degrees weſt, to 8 degrees eaſt longitude. It is bounded on the north, by the netherlands, and the Engliſh Channel; on the eaſt, by Germany, Swiſſerland and Italy; on the ſouth, by the Mediterranean Sea and Pyrenean mountains; and on the weſt, by the Bay of Biſcay. France was formerly divided into 12 provinces; but at the late revolution it was divided into 84 departments, each department being divided into diſtricts, and each diſtrict into cantons. The eighty-four departments are as follows:—1. Streights of Calais. 2. North. 3. Lower Seine. 4. Somme. 5. Aiſne. 6. Ardennes. 7. Channel. 8. Calvados. 9. Eure. 10. Oiſe. 11. Marne. 12. Meuſe. 13. Moſelle. 14. Lower Rhine. 15. Finiſterre. 16. North coaſt. 17. Iſle and Vilaine. 18. Mayenne. 19. Orne. 20. Eure and Loire. 21, Seine and

Oiſe. 22, Paris. 23, Seine and Marne. 24, Aube 25, Upper Marne. 26, Meurte. 27, Voſges. 28, Upper Rhine. 29, Morbihan. 30, Lower Loire. 31, Mayenne and Loire. 32. Sarte. 33, Loire and Cher. 34, Loirete. 35, Yonne. 36, Cote d'Or. 37, Upper Soanne. 38, Doubes. 39, Vendée. 40, Two Sevres. 41, Vienne. 42, Indre. and Loire. 43, Indre. 44, Cher. 45, Nievre. 46, Soanne and Loire. 47, Jura. 48, Lower Charente. 49, Charente. 50, Upper Vienne. 51, Creuze. 52, Allier. 53, Rhone and Loire. 54. Ain. 55, Gironde. 56, Dordogne. 57, Correze. 58, Puy De Dome. 59. Upper Loire. 60, Iſere. 61, Landes. 62, Lot and Garonne. 63, Lot. 64, Cantal. 65, Lozere. 66, Ardeche. 67, Drome. 68, Upper Alps. 69, Lower Pyrenees. 70, Gers. 71, Upper Garonne. 72, Tarne 73, Aveiron. 74, Herault. 75, Gard. 76 Lower Alps. 77, Upper Pyrenees. 78, Arriege. 79, Aude, 80, Eaſt Pyrenees. 81, Mouths of Rhone. 82, Var. 83, Corſica. 84, Mount Blanc.

The climate of France, is reckoned upon the whole, to be more ſettled than that of any other country in Europe. In the north, the winters are very cold ; but in the interior parts, the air is very temperate and healthy; and in the ſouth it is ſo mild, that invalids retire thither from all the northern countries, to avoid the rigour of their own climates.

The commerce of France, conſiſts of wines, brandy, vinegar, drugs, oils, fruits, of which they have great variety, ſilk, cambricks, laces, paper, parchment, hardware, toys, &c. and their trade is very conſiderable and lucrative both to the Eaſt and Weſt Indies: but particularly to the European countries.

The national religion was always Romiſh. And their monarchs were always limited till the three laſt ſovereigns of France. What their religion and government is, or rather, their want of religion and government, ſince the late revolution, is too well known, to need a deſcription.

SPAIN,

SPAIN, lies between 36 and 44 degrees, north latitude, and between 10 degrees, weſt, and 3 degrees, eaſt longitude. It is bounded on the north, by the Bay of Biſcay, and the Pyrenean Mountains; on the ſouth, by Gibraltar Streights; on the eaſt, by the Mediterranean Sea; and on the weſt, by Portugal and the Atlantic Ocean; It is divided into the following kingdoms or provinces:—Galicia, Aſturia, Biſcay, Navarre, Arragon, Catalonia, Valencia, Murcia, Granada, Andaluſia, Old Caſtile, New Caſtile, Leon, and Eſtremadura.

Spain, enjoys a dry, clear, temperate air, except during the equinoctial rains; and in the ſouthern provinces, during the ſummer months, where it is very hot. The ſoil is as fruitful as the ſoil of any part of Europe; but the natives are very indolent. In many parts the choiceſt fruits grow ſpontaneouſly. They alſo have a great variety of aromatic herbs. Seville, is celebrated for its oranges, and Murcia produces mulberry trees in ſuch abundance, that the ſilk exported from this part, amounts to 200,000 pounds per annum.

The chief articles of commerce in Spain, are gold and ſilver, which they derive from their ſettlements in South America. The chief manufactures are ſilk, wool, iron, copper and hardware.

The national religion of Spain, is the profeſſion of the Church of Rome. The Inquiſition always reigned in this country, till of very late, when by a late edict it is under ſome reſtrictions.

The conſtitution of Spain, is the moſt abſolute monarchy in Europe. And the revenue from old Spain only, amounts to upwards of 6,000,000 ſterling, what the exact amount of the whole revenue is, is not accurately known.

The military force of Spain, is never leſs than 70,000 in time of peace; and in time of war, the king has raiſed near 200,000.

PORTUGAL, joins to Spain, and is bounded by it on the north and eaſt; and on the ſouth and weſt, by the Atlantic

Ocean.

Ocean. It extends from 36 degrees 50 minutes, to 43 degrees, north latitude, and from 7 to 10 degrees, west longitude.

The climate of Portugal, is more temperate than that of Spain, on account of its vicinity to the sea. Their commerce, consists chiefly of wines, fruits, salt, linen, woollen, and some coarse silk. Their religion is that of the Church of Rome, and the inquisition has greater power here, than in any other country. The constitution is like that of Spain, absolute monarchy.

ITALY, including Sicily, lies between 37 and 47 degrees, north latitude, and between 7 and 19 degrees, east longitude. On the east, south, and west, it is washed by the Adriatic and Mediterranean Seas; and on the north, it is separated from the rest of Europe, by the Alps. It contains the following countries: Piedmont, Montferrat, part of Milan, Sardinia Isle, Naples, Sicily, Milanese, Mantua, Tuscany, the Duke of Parma's territories, Genoese territories, Oneglia, the Duke of Modena's territories, Venetian territories, Pope's dominions, Corsica Isle, Malta Isle, and some other small islands. Each of these countries are distinct from each other; having different forms of government, different trade, and separate interests.

Italy has a fine soil, and temperate but warm climate; the soil however is greatly neglected, owing to the indolence of the inhabitants.

The religion, universally professed throughout Italy, is that of the Church of Rome, but people of all other religions, generally live unmolested in most parts of Italy. The commerce and manufactures are various, according to the different states; but wines, fruits, and oil, constitute the chief articles. The curiosities to be met with in this extensive tract of country, are almost innumerable, being the residence of so many ancient nations, particularly of ancient Rome; hence, there are innumerable remains of the ancient arts;

arts; the works of ancient artifts; the burning mountains, alfo conftitute one of their greateft natural curiofities. The Italian language is derived from the Latin; with an intermixture of words from the Goths, and other barbarous nations; but every feparate ftate has a different dialect.

To defcribe the different forms of government, would be to enter into too minute a detail, as they are different in every eftate.

TURKEY, extends into both Europe and Afia.

European Turkey, extends from 17 to 40 degrees eaft longitude, and between 37 and 49 degrees, north latitude. It is bounded on the north, by Ruffia, Poland, and Sclavonia; on the eaft, by the Black Sea, the Hellefpont, and the Archipelago; on the fouth, by the Mediterranean; and on the weft, by the Mediterranean, and Venetian and Auftrian territories.

Turkey in Europe, contains fome of the moft genial climates in the world; and is divided into the following provinces:—Crim and Little Tartary, Budziac Tartary, Beffarabia, Moldavia, Wallachia, Bulgaria, Servia, Bofnia, Romania, Macedonia, Janna, Livadia, Epirus, Albania, Dalmatia, Ragufa, Corinthia, Argos, Sparta, Olympia, Arcadia, Elis.

The foil of Turkey is extremely fruitful, where the leaft induftry has been employed. And all the fruits common to all the warm climates, are produced here in great perfection. And many valuable drugs are natives of this country.

The commerce and manufactures of this country, are chiefly filks, drugs, dying ftuffs, in their natural ftate; with cottons, carpets, leather, velvets, foap, &c. but though the Turks are fituated in the moft advantageous part of both Europe and Afia for traffic, yet they fhamefully neglect it.

The religion which the Turks univerfally profefs, is Mahometifm; but they are divided into as many fects as the profeffors

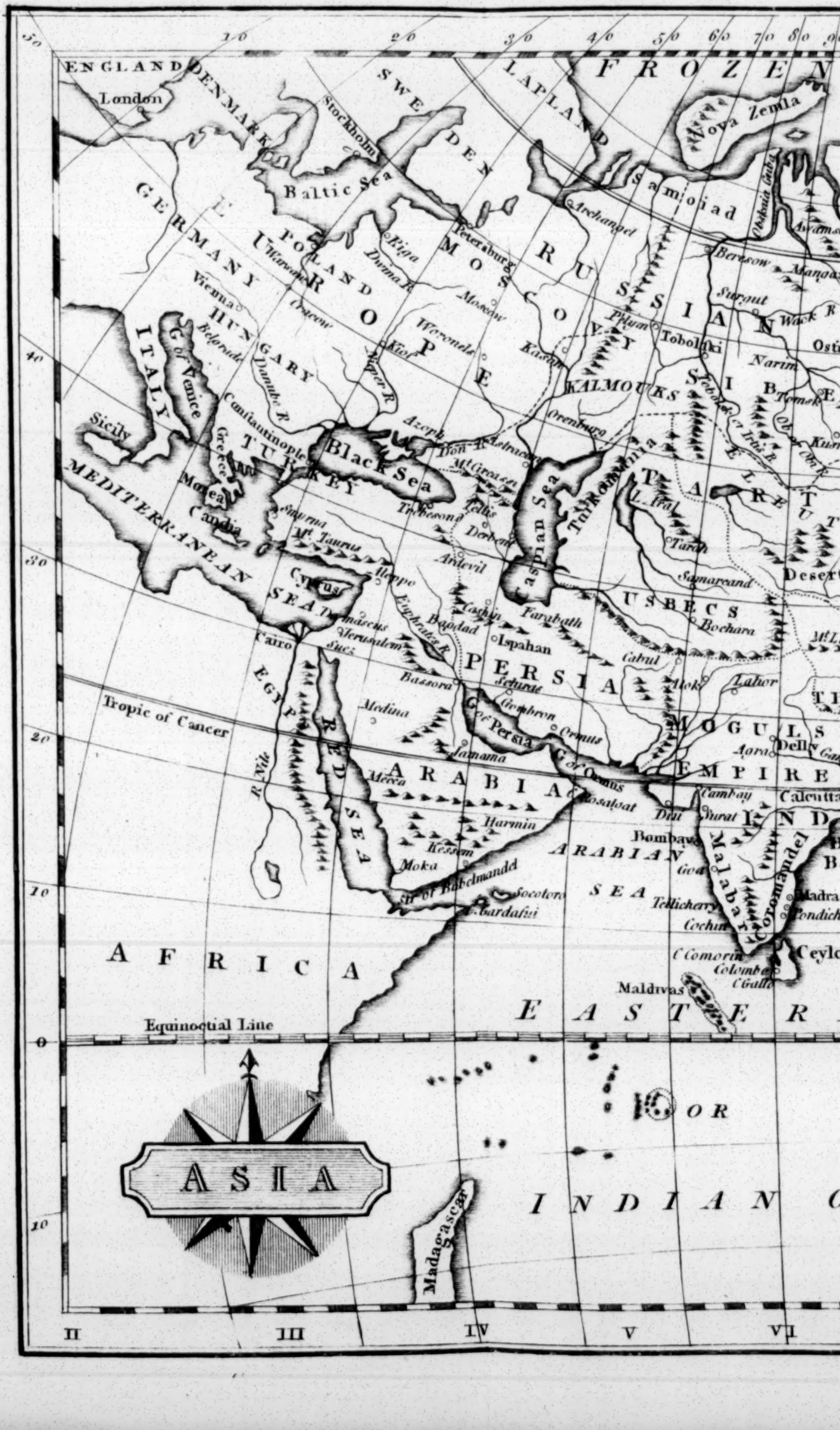
professors of christianity. The high Priest, or Mufti, is a place of such honour, that whenever he comes into Court, the Grand Seignior rises from his seat and meets him. Most other religions are tolerated here by paying an annual tax.

The government of Turkey is that of an absolute monarchy; and in this Empire there is no hereditary succession by law to any property; yet the rights of individuals are rendered secure by being annexed to the church, by which means even Jews and Christians may secure their property in lands to the latest posterity. The revenue of Turkey amounts to upwards of twenty five millions per annum, but does not produce four millions to the Emperor's treasury. The rest being expended in collecting, &c. The forces of the Turkish Empire is of two parts; the one has certain lands for their maintenance, and the other is paid out of the treasury. The former amount to 268,000 troopers; the latter, called the Horse Guards, are about 12,000; and the Janizaries, or foot guards, 25,000. Besides 100,000 foot soldiers in different parts of the Empire.

SECT. VII.

OF ASIA.

ASIA forms the most remarkable quarter of the globe in antient history. It was here that the first man was created—and here the Patriarch Noah was preserved during the flood—and from this quarter the world was re-peopled a second time. In this quarter lived all the patriarchs recorded in scripture—and


ASIA
ENGLAND
London
DENMARK
SWEDEN
Stockholm
Baltic Sea
LAPLAND
FROZEN
Nova Zemla
Samoiad
Archangel
GERMANY
POLAND
EUROPE
Riga
Peterburg
MOSCOVY
RUSSIAN
Vienna
HUNGARY
Moscow
Tobolski
ITALY
G. of Venice
Sicily
Constantinople
Greece
TURKEY
Black Sea
Azoph
KALMOUKS
SIBERIA
Tomsk
MEDITERRANEAN SEA
Morea
Candia
Caspian Sea
Trebesond
Mt. Taurus
Aleppo
Cyprus
Samarcand
USBECS
Bochara
Damascus
Jerusalem
Cairo
Suez
EGYPT
Ispahan
PERSIA
Bassora
Cabul
Lahor
Tropic of Cancer
Medina
G. of Persia
Ormus
MOGULS EMPIRE
Agra
Delly
RED SEA
ARABIA
Mecca
Cambay
Surat
Calcutta
Bombay
Goa
Malabar
Coromandel
Madras
Moka
Str. of Babelmandel
Socotora
ARABIAN SEA
Tellicherry
Cochin
C. Comorin
Colombo
C. Gallo
Ceylon
Maldivas
AFRICA
Equinoctial Line
EASTERN
INDIAN
Madagascar
II
III
IV
V
VI

V. Woodthorpe sc. Fetter Lane.

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and this was the ſcene of all the tranſactions recorded in holy writ—and finally, it was here Jeſus Chriſt appeared, and wrought the ſalvation of mankind—and from hence the Chriſtian religion was propagated.

This quarter of the globe enjoys the moſt ſerene air and fruitful ſoil of any of the other quarters, and produces the moſt delicious fruits, odoriferous ſhrubs, ſpices, and valuable drugs, gums, &c.

Idolatry and Mahometiſm is almoſt univerſal in this quarter of the globe, except in a few European ſettlements. The languages in uſe in this quarter are chiefly the Arabic, Perſian, Malayan, Chineſe, Japaneſe, Tartarian, Ruſſian, and Turkiſh.

Aſia is bounded on the weſt by the Red Sea, the Mediterranean, the Archipelago, the Black Sea, and Europe; on the north by the Frozen Ocean; on the eaſt and ſouth by the Pacific and Indian Oceans. It is ſituated between the equator and the frigid zone, and between 25 and 180 degrees of eaſt longitude; it is about 4800 miles in length, and 4300 in breadth, and contains the following countries.

Turkey in Aſia, being the other part of the Turkiſh Empire, and is about 1000 miles in length from eaſt to weſt, and 800 in breadth from the northern parts to the deſerts of Arabia. It is bounded on the north by the Black Sea, and Circaſſia; on the eaſt by Perſia; on the ſouth by Arabia and the Levant Sea; and on the weſt by the Archipelago and the Helleſpont.

This part of Turkey was the principal ſcene of all the tranſactions recorded in ancient, ſacred, and profane writ.

Tartary is an extenſive country, taken in its full extent, and extends from Muſcovy on the weſt, to the Pacific Ocean on the eaſt; and from the nations of China, India, Perſia, and Turkey on the ſouth, to the impenetrable regions of the north. It extends from the thirtieth degree of north latitude to the frozen regions of the north pole; and from 50 to 190

degrees eaſt longitude; and contains Ruſſian, Chineſe, Mogulean, and Independent Tartary, which are its four grand diviſions, 4000 miles in length and 2400 in breadth.

Through ſuch an extenſive tract of country the ſoil and climate muſt neceſſarily partake of a great variety.

Their manners, language, &c. muſt alſo be as various.

CHINA lies on the eaſtern borders of the continent of Aſia, and is divided from Chineſe Tartary, on the north, by a prodigious wall, and in ſome places by inacceſſable mountains; on the eaſt, it is bounded by the Yellow Sea and Pacific Ocean, which ſeparate it from America; on the ſouth by the Chineſe Sea, and the kingdom of Tonquin; on the weſt by Tibet. It extends from 21 to 24 degrees north latitude, and from 94 to 133 degrees eaſt longitude.

In ſuch an extenſive country there muſt no doubt be a variety of climates. The ſouthern parts are very hot, and have violent rains, while the northern parts are very cold, and their rivers frozen for ſome months during the winter; but the middle parts are temperate and pleaſant. The ſoil alſo partakes of a great variety, though there is no part of this extenſive country but is fruitful, either from nature or art; for ſuch is the induſtry of the Chineſe, that they ſuffer very little, if any land, to lie uncultivated.

The Chineſe have a conſiderable trade with every European nation, exporting ſilks, cotton, gold and ſilver ſtuffs, painted gauzes, teas, china-ware, paper, and Indian ink, for which they receive ready money; deſpiſing the manufactures of every other country but their own.

There are a great number of natural and artificial curioſities in China. Among the latter are reckoned the famous wall which divides China from Tartary, extending over mountains and vallies: of 1500 miles in length, and from 20 to 25 feet in height, and broad enough for ſix horſemen to travel abreaſt. It has ſtood near 1800 years, and is now almoſt entire. 2. Their canals are works of great magnitude, infinitely

infinitely exceeding thofe in Europe. 3. The bridge over the river Saffrany, which confifts of a fingle arch, whofe fpan is 400 cubits, and its height 500. 4. The Cientao, or road of pillars, which is a road broad enough for four horfes to travel abreaft, and near 4 miles in length, defended by an iron railing; and unites the fummits of feveral mountains, in order to avoid the winding of the roads. It refts upon ftrong ftone pillars for the moft part. 5. The bridge of chains, which is a bridge built upon a number of ftrong iron chains, and hangs over a very deep valley, in the neighbourhood of King-Tung. 6. The triumphal arches of China, of which there are above 1100; 200 of them are very magnificent; they were erected in memory of their great princes, legiflators, &c. 7. The Tower of Nan King, called the Porcelain Tower, being wholly covered with the moft beautiful China; upwards of three hundred feet in height, nine ftory high; each ftory decreafing gradually to the top; the whole forms the moft correct and grand piece of architecture, to be met with in the eaft.

Among the natural curiofities, may be reckoned their water falls, and Volcanos.

Their religion is that of Paganifm; the deities are men that have been eminent in arts and fciences. They alfo worfhip inanimate beings, as mountains, woods, and rivers; but they acknowledge but one fupreme being.

INDIA, or HINDOSTAN, is an extenfive country in its full extent. Bounded on the north, by Tibet and Ufbeck Tartary; on the fouth, by the Indian ocean; on the eaft, by China and the Pacific; and on the weft, by Perfia and the Indian ocean. It extends from 1 degree to 40 degrees north latitude, and from 66 to 109 degrees eaft longitude: and is principally divided into three parts. 1. The Peninfula of India beyond the Ganges, on the eaft. 2. The Main land, or Empire of the Great Mogul, on the north. 3. The Peninfula within the Ganges, or on this fide of it, on the weft.

A great part of the ſea coaſt of India, belong to the Engliſh Eaſt India Company, where there are many large and rich ſettlements, from which we receive great quantities of Eaſt India commodities.

As the country extends through ſo many degrees of latitude, there is a great difference of the climates of the different parts. In the northern parts, the air is very dry and healthy; but in the ſouthern parts, near the ſea in low lands, the air is very hot and moiſt: they divide the year into the dry and wet ſeaſons.

The ſoil in general, throughout the whole country, is very fruitful, producing all the variety of plants, drugs, and fruits, to be met with in the other tropical climates. There are alſo mines of gold, diamonds, rubies, topazes, and other precious ſtones.

In the European ſettlements, the religion is Chriſtianity; but in the northern and inland parts, they are either Mahometans or Pagans: and divided into ſeveral kingdoms, each of which is governed by one or more abſolute monarchs.

Persia extends from 25 to 45 degrees north latitude, and from 45 to 67 degrees eaſt longitude. It is bounded on the eaſt, by the Mogul's dominions; on the north, by Uſbeck Tartary, the Caſpian ſea, and Circaſſia; on the ſouth, by the Indian ocean, and Gulf of Perſia; and on the weſt, by Arabia, and the Turkiſh empire.

The climates of this country are very various. In the northern parts, and near the mountains, which are covered with ſnow, the air is very cold; in the midland parts, it is ſerene, pure, and healthy; but towards the ſouthern parts, there are ſometimes hot ſuffocating winds, which blow over a ſandy deſert from ſouth and eaſt; a blaſt of which has ſometimes ſtruck the unwary traveller with death in an inſtant. The ſoil is very various alſo, being in ſome parts very barren, but where it is well watered, it is very fruitful.

The

The principal commodities of traffic, are ſilks, camblets, carpets, leather, embroidery, gold and ſilver, threads, mohair, &c.

The national religion of Perſia, is that of Mahometiſm, and the ſect of Ali.

ARABIA extends from 35 to 60 degrees eaſt longitude, and from 12 degrees 30 minutes to 30 degrees north latitude. It is bounded on the north, by Aſiatic Turkey; on the ſouth, by the Indian ocean; on the eaſt, by the Euphrates and gulf of Baſſora; and on the weſt, by the Red ſea.

Arabia is divided into three parts, viz. Arabia Petraea, or the Stony; Arabia Deſerta, or the Deſert; and Arabia Felix, or the Happy.

Arabia the Stony, is the wilderneſs in which the children of Iſrael ſojourned forty years: and in it may be ſeen the mountains of Horeb and Sinai, mentioned in ſacred writ.

Arabia the Deſert, principally conſiſts of a large ſandy deſert; it has, however, a few ſpots of fruitful land, covered with verdure, which are interſperſed in different parts of the deſert. It is over this deſert, that ſome of the eaſtern nations bring their commodities of traffic from the eaſt, travelling in large caravans.

Arabia the Happy, is in general barren; but ſome of the vallies between the mountains, and thoſe plains, which are well ſupplied with water, are very fruitful. From this part great quantities of drugs are exported to Europe, and alſo Turkey coffee.

The Arabs are in general a wandering people: many of their tribes live wholly in tents, and ſubſiſt partly by robbing the caravans, which travel through the deſert, and partly by the produce of their country, and the fleſh of their cattle; raiſing no grain of any kind for domeſtic uſe.

Their religion is that of Mahometiſm; but many of the tribes are ſtill Pagans. Their language is ſaid to exceed even the Greek itſelf in copiouſneſs. The Arabians have never yet been ſubdued by any military force, though ſeveral attempts have been made for that purpoſe.

SECT.

SECT. VIII.

OF AFRICA.

THE Continent of Africa is in the form of a Peninſula, ſurrounded on each ſide by water, except where it joins to Aſia, by the Iſthmus of Suez. Several countries, famous in antiquity for the arts and ſciences, were ſituated in the northern parts of this quarter. And in the early days of Chriſtianity, ſeveral Chriſtian churches were founded here; but at the preſent period, Mahometiſm and Idolatry degrade this moſt fertile quarter of the globe. That moſt inhuman commerce, trafficking in men, alſo is carried on here by the European nations.

The ancients believed the greateſt part of this quarter of the globe to be uninhabited, as alſo the greateſt part of Aſia, and, indeed, all that part of the globe, lying between the tropics; but modern travellers have diſcovered, that the tropical countries are in general the moſt fertile, and beſt populated; and of theſe the ſouthern and interior parts of Africa, are found the moſt eligible, both for vegetation and population. Its ſea coaſts, are the only parts of which we are particularly acquainted: but travellers are now buſily employed, in making diſcoveries in the internal parts.

Africa is bounded on the weſt, by the Atlantic ocean; on the north, by the Mediterranean; on the eaſt, by the Red ſea; and on the ſouth, by the Southern ocean. It lies between 37 degrees north, and 36 degrees ſouth latitude; the equator running nearly through the middle thereof; and between 17 degrees weſt, and 51 degrees eaſt longitude. In length, from north to ſouth, it is about 4600 miles; and in breadth, from eaſt to weſt, 3500 miles.

EGYPT,


AFRICA
Pico
Tercera
St. Michael
Azores or Western Isles
St. Mary
Lisbon
PORTUGAL
Madrid
SPAIN
Corsica
Sardinia
Str. of Gibraltar
Tangier
Sala
Tetuan
Oran
Constantina
Bona
ALGIERS
TUNIS
Madeira I.
C. Cantin
Morocco
Fez
MOROCCO
Suz
Taradant
Canary Islands
Forteventura
Canary
Ferro
Tenerif
C. Bajador
Daro
Tafilet
Teguat
Techort
Fezzan
Tessel
Tropic of Cancer
C. das Barbas
ZAARA OR THE D
St. Anthony
St. Lucia
St. Nicholas
Sal
Bonavista
Cape Verd Islands
S. Jago
C. Blanco
Carogoli
Tocror
Senegal R.
Agades
Germia
Cano
NEGROLAND
C. Verd
Gambia R.
Tombut
James F.
Muni
Mandinga
R. Grande
Guber
Gaga
Zegzeg
GUINEA
Sierra de Leon
C. St. Ann
Sherbra F.
C. Coast Cast.
Valta R.
R. Formosa
Benin
Old Callebar
C. Formosa
I. Fernando Po
Princes I.
I. St. Thomas
C. Palmas
C. 3 Points
ATLANTIC
OCEAN
Equinoctial Line
Anaboa
C. Lapas
St. Matthew
SOUTH AMERICA
Ascension
Loando
Old Benguela
St. Helena
Meridian of London
C of G
30
20
10
0
10
20
30
20
10
0
II
0

60 Degrees East from London
10
20
30
40
50
ASIA
TURKEY
BLACK SEA
CASPIAN SEA
Constantinople
Corsica
Sardinia
Sicily
ITALY
Rome
GREECE
Smyrna
Morea
Candia
Cyprus
Aleppo
Ispahan
Malta
TUNIS
MEDITERRANEAN SEA
Tripoli
TRIPOLI
Alexandria
BARCA
Jerusalem
Suez
Cairo
BARBARY
Fezzan
BILEDULGERID
Garam
Techort
Elocath
Bassora
G. of Persia
Jamana
G. of Ormus
C. Razalgat
Medina
ARABIA
Mecca
Harmin
EGYPT
RED SEA
R. Nile
Cataracts
THE DESART
Agades
Germia
Cano
Kaugo
Borno L.
Sangea
Nubia
NUBIA
Sira R.
Coast of Abex
Caresen
Snuhare
Sibith
Ambamarjam
Tana L.
Mocca
Str. of Babelmandel
Socotora I.
C. Guardafui
Zegzeg
Zanfara
Niger
Syre
Medra
Nile Head
Zeyla
Formosa
Benin
ABYSSINIA
Sangara
Hawash R.
Brava
Median
Biafar
Old Callebar
Barca
Nigar L.
ETHIOPIA
Adea
AJAN
Opin
C. d. Bassas
EASTERN
Princes I.
Mt. St. Esprit
Magadoxa
LOANGO
Zara R.
C. Lapas
Loango
CONGO
S. Salvador
Congo
MACACO
Tocui
Zaflan
Zaire
Marais
Bog of Zaflan
Luna Mt.
Patte
Melinda
Mombaze
Almirante Is.
7 Irmanos
3 Irmanos
Cross I.
St. Francisco
Gratio
Loando I.
ANGOLA
Coanza R.
Old Benguela
S. Philip or Benguela
Zembre
ZANGUEBAR
C. del Gado
Gallega
OR
Diego
la Malha
Comarra
Comarra I.
Joanna
Zanga
Roquepiz
C. Ambra
BENGUELA
MONOMUGI
Zachai L.
Merango
Zambeze R.
Sena
Mosambique
C. S. Andrew
MADAGASCAR
St. Mary
INDIAN
C. Negro
MATAMAN
R. Angra
Amara
SOFALA
Sofala
R. St. Maria
C. St. Sebastian
Mosambique Channel
Mauritius
MONOMOTOPA
Manbone
Bourbon
Mogar
Porto de Ilheos
R. St. Esprit
Monomotopa
TERRA DE NATAL
C. St. Mary
Bay St. Augustin
C. S. Mary
Ft. Dauphin
C. St. Sebastian
Tropic of Capricorn
OCEAN
CAFFRARIA
Camissa R.
HOTTENTOTS
R. St. Christopher
B. St. Helena
Saldanha B.
Table Bay
Cape Town
Hermosa B.
C. of Good Hope
Bd. St. Brass
Agulhas
II
IV Hours East from London

Egypt, is bounded on the north, by the Isthmus of Suez; on the east, by the Red Sea; on the south, by Nubia; and on the west, by the interior parts of Africa. It lies between 30 and 36 degrees east longitude; and between 20 and 32 degrees north latitude; and is divided into Upper and Lower Egypt.

The climate during the summer season, is excessive hot; when the south winds often raise such a cloud of sand as to obscure the light of the sun, and cause epidemical diseases.

The soil is exceeding fruitful, owing to the annual overflowing of the Nile. This river so famous in ancient history, has its rise in Abyssinia, at between 11 and 12 degrees of north latitude, and pursues a northern course, for above 1500 miles; when it divides into two branches, about 6 miles below Grand Cairo; one branch extending eastward, and the other westward. This river begins to rise in the beginning of summer, and increases 3 or 4 inches in height each day, for the first week: the next fortnight it increases in a still greater proportion, and it is near four months before it is reduced into its channel again. The principal cities and towns are built on eminencies on the banks of the Nile, and during the inundation correspond with each other by means of boats. When the Nile rises to the height of 49 feet, it produces a plentiful season, but if it exceed that height, it is productive of great mischief, sweeping away both houses and cattle.

In Egypt, they generally have three crops in a year; the first of lettuces and cucumbers; the second of corn; the third of melons; and all the fruits common to hot climates.

Their pastures, are the richest in the world; the grass being usually as high as the cattle.

Their trade consists of great quantities of flax and cotton, both prepared and unmanufactured; leather of different kinds, also a great variety of drugs, and roots for dying.

The common language spoken here, is the vulgar Arabic, as it is under the dominion of the Turks.

Barbary

Barbary, extends from Egypt, to the Atlantic Ocean, and from the Mediterranean Sea, to the Lybian Deſerts, being 750 miles in breadth, and near 2000 in length. Containing the countries of Morocco and Fez, which form one diſtinct empire, and the ſtates of Algiers, Tunis, Tripoli, and Barca, which form ſeveral diſtinct ſtates, united together in confederacy, under the Turkiſh government.

Its ſoil is exceeding fruitful, producing excellent corn, cattle and paſture, and all the variety of tropical fruits; and great quantities of fiſh and fowl; alſo a great variety of tame and wild animals.

The commerce of this country, is chiefly carried on by caravans; their exports conſiſt of leather, mats, handkerchiefs, carpets, elephants' teeth, oſtrich feathers, copper, tin, wool, fruits, gums, drugs, &c. for which they receive timber, artillery, gunpowder, &c.

Their religion is that of Mahometiſm. Their language varies according to the different parts of the country. That ſpoken in the inland parts, is either an African language, or a corrupt Arabic. The latter is alſo ſpoken in moſt of the Sea-port towns; but in ſome parts they uſe a mixed language, ſuch as is ſpoken in moſt of the Mediterranean Ports.

Moſt of the Barbary ſtates, ſubſiſt by piracy. And their ſailors fight deſperately, when they meet any European veſſel.

The government is that of an abſolute monarchy. The emperor is in general, both judge and executioner; and he acknowledges the Grand Signior of Turkey, to be his ſuperior. When there is a vacancy in the government, every ſoldier in the army has a vote in chuſing a new Emperor, which is often attended with great bloodſhed.

The parts of Africa, from the Tropic of Cancer to the Cape of Good Hope, is very little known; except the Sea coaſt thereof. The natives are in general all black, except theſe

thoſe of Abyſſinia, who are of a tawny complexion, and are a mixture of Jews, Chriſtians and Pagans. The religion of the other countries in this part, is generally that of Paganiſm, and the form of government every where monarchial, except in a few ſettlements formed by the Europeans, on the Sea coaſt. Few of their princes, however, poſſeſs an extenſive degree of territory. As the natives are ignorant of all the arts, utility, and refinement, the different kingdoms are therefore unconnected with each other; and are generally at war.

The ſoil of Africa is in general very fruitful: though in ſome parts it is perfectly barren, particularly in thoſe parts where there is very little water; the heat of the ſun reducing the ſoil to a perfect ſand; ſuch are the countries of Anian and Zaara; but the countries of Mandingo, Ethiopia, Congo, Angola, Batua, Truticui, Monomotapa, Caſati, and Melunenrugi, are extremely fruitful, and very rich in gold and ſilver.

On the weſtern Coaſt, the Engliſh trade is carried on at James's Fort, and other ſettlements, near and up the River Gambia, where woollen and linen cloths, hardware and ſpirituous liquors are exchanged for the perſons of the natives. Many of the negroes will ſell their own family for theſe ſuperfluities. Many of the natives are trepanned by foreigners, or their own countrymen, and then ſold to the Europeans: and many more are ſold by the princes of the different ſtates, being captives taken in war. Gold and ivory form the principal branches of commerce next to that of the ſlaves.

The Portugueſe poſſeſs the greateſt part of the Eaſt and Weſt Coaſt of Africa, from the Tropic of Capricorn to the Equator. The Dutch have ſome ſettlements towards the ſouthern parts of the Continent; and the Engliſh poſſeſs Cape Town, at the Cape of Good Hope, which is well fortified, and where the ſhips bound for India, uſually put in, and trade with the natives or Hottentots, for their cattle and

other provisions for which they give them spirituous liquors. There are several Islands near the coast of Africa, lying in the Eastern or Indian Ocean, or in the Western or Atlantic Ocean, of which the chief are:

1. Zocotra, situated in 53 degrees, east longitude, and 12 degrees north latitude: 30 leagues, east of Cape Gardefoi, on the continent of Africa: being 80 miles in length, and 54 in breadth, and has two good harbours. It is a populous, plentiful country, governed by a prince who is tributary to the port.

2. Babelmandel, situated in the strait of the same name, at the entrance of the Red-Sea, in 44 degrees, 30 minutes, east longitude, and 12 degrees, north latitude, being a small sandy Island, not 5 miles round.

3. The Islands of Joanna, Mayotta, Mohilla, Angezeia, and Comora, situated between 41 and 46 degrees, east longitude, and between 10 and 14 degrees, south latitude: the chief of these is Joanna, to which the others are tributary, being 30 miles long, and 15 broad: affording excellent fruits and provisions. The natives are a friendly set of people, and profess the Mahometan religion.

4. Madagascar, the largest of the African Islands, situated between 43 and 51 degrees, east longitude, and between 10 and 26 degrees, south latitude. Three hundred miles south-east of the continent of Africa, being near 1000 miles in length, from north to south; and between 2 and 300 miles in breadth. Between this Island and the Cape of Good Hope, or the Continent of Africa, the Sea rolls with great force, and is exceeding rough. In this channel, all European ships pass in their voyage to and from India, except the water be too rough. Madagascar is a fertile country, abounding in all the variety of fruits and vegetables to be met with in the same climate. The air is temperate also, and healthy. It is inhabited by both blacks and whites, professing different religions; but principally Mahometism and Paganism, and governed by several petty princes.

5. Mau-

5. MAURITIUS, or Maurice, ſituated in 56 degrees, eaſt longitude, and 20 degrees ſouth latitude; about 400 miles, eaſt of Madagaſcar. It is of an oval form, and about 150 miles in circumference, with a large fine harbour. The climate is healthy and pleaſant. And the Iſland is well watered with ſeveral rivers: though the ſoil is not ſo fruitful as that of the former, it neverthelefs, feeds a great number of cattle, ſheep, deer, and goats.

6. BOURBON, ſituated in 54 degrees, eaſt longitude; and 21 degrees ſouth latitude, about 300 miles eaſt of Madagaſcar, and about 90 miles in circumference. Surrounded for the moſt part with blind rocks, a few feet under water. The climate is in general healthy, though hot. It affords very good paſture and cattle.

There are ſeveral other ſmall Iſlands about Madagaſcar, and on the eaſtern coaſt of Africa.

7. ST. HELENA, ſituated in 6 degrees weſt longitude, and 16 degrees ſouth latitude, being 1200 miles, weſt from the Continent of Africa, and 1800 eaſt from ſouth America. The whole Iſland is ſituated on a rock, and is about 21 miles in circumference. There is but one landing place in the Iſland, at the eaſt ſide of the rock. It is a fine fruitful Iſland, diverſified by hills and vallies. It abounds in all the conveniences and comforts of life. There are about 200 families, moſtly deſcended from Engliſh parents.

8. ASCENSION, ſituated in 7 degrees 40 minutes, ſouth latitude, and 600 miles, north-weſt of St. Helena. It is a mountainous barren Iſland, and uninhabited; about 20 miles round.

9. St. MATTHEW, ſituated in 6 degrees 1 minute, weſt longitude; and 1 degree 30 minutes, ſouth latitude; and uninhabited.

10. CAPE VERD ISLANDS are ſituated between 23 and 26 degrees, weſt longitude; and between 14 and 18 degrees north latitude. They are about twenty in number; but the principal are St. Jago, Bravo, Fogo, Mayo, Bonoviſta, Sal,

St. Nicholas, St. Vincent, Santa Cruz, and St. Antonio. They moſtly belong to the Portugueſe and Spaniards. The air in general is very hot, and in ſome very unwholeſome. They are inhabited by Europeans and their deſcendants.

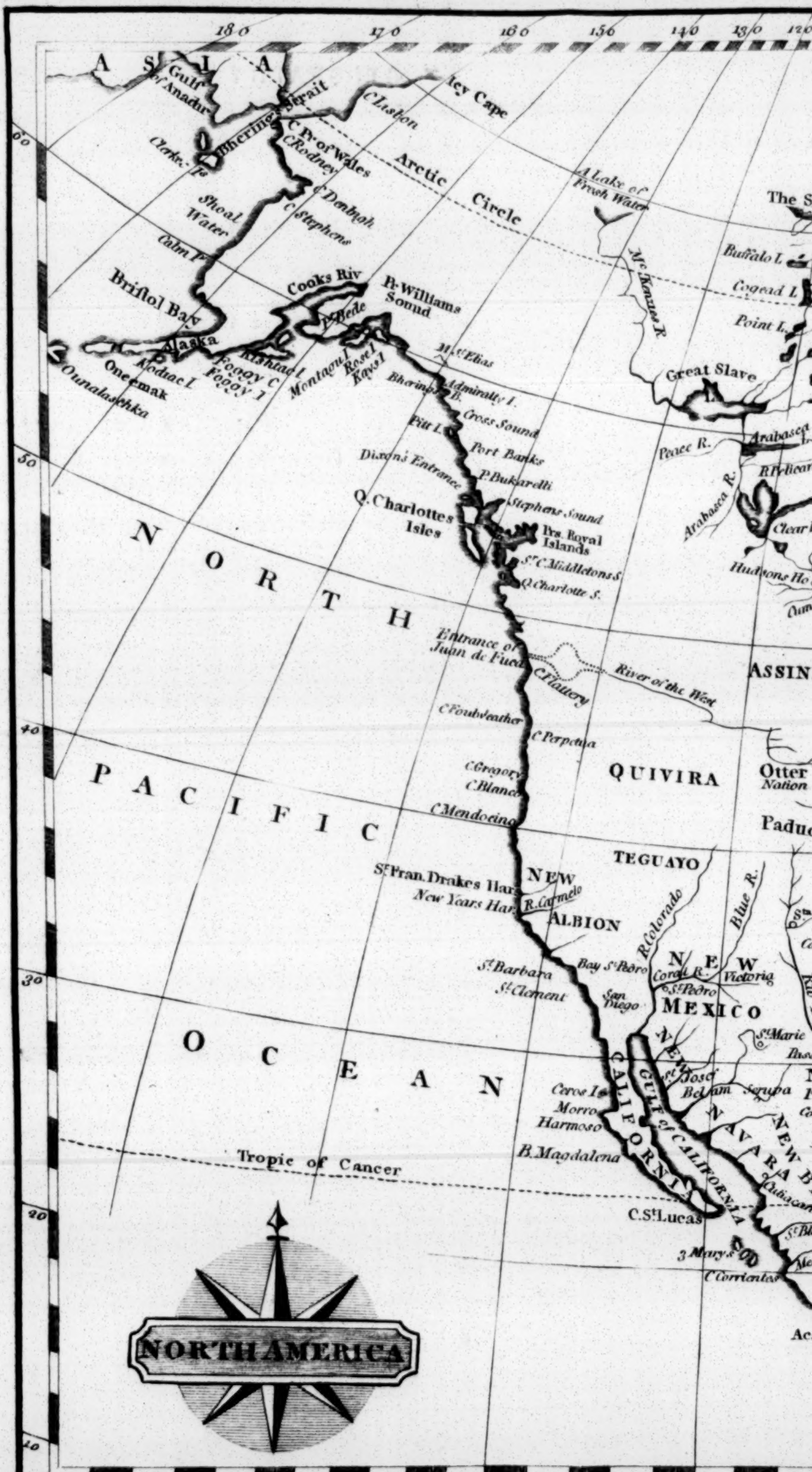
11. Goree, ſituated in 14 degrees 43 minutes north latitude, and 17 degrees 20 minutes, weſt longitude. It is a ſmall ſpot not exceeding two miles in circumference; but an important ſituation for trade.

12. The Canaries, or Fortunate Islands, are ſeven in number, and ſituated between 12 and 19 degrees, weſt longitude; and between 27 and 29 degrees, north latitude. Theſe Iſlands have a pure temperate air, and abound in moſt delicious fruits, from whence they have thoſe rich wines, called Canary, of which they export 10,000 hogſheads annually.

13. The Madeiras, are three Iſlands, ſituated in 32 degrees 27 minutes, north latitude; and between 18 degrees 30 minutes, and 19 degrees, weſt longitude. Theſe Iſlands are moſtly famous for producing the Madeira wine, of which no leſs than 20,000 hogſheads are annually exported.

14. The Azores, or Western Islands, are ſituated between 25 and 32 degrees, weſt longitude; and between 37 degrees and 40 degrees, north latitude. Being 900 miles weſt of Portugal, and lying in the Mid-way between Europe and America. Of theſe St. Michael is the largeſt, being near 100 miles in circumference, and containing 50,000 inhabitants. Tercera is the moſt important of theſe Iſlands on account of its harbour, which is very ſpacious, and affords good anchorage. There are ſeven other of theſe Iſlands, their names are Santa Maria, St. George, Gracioſa, Fayal, Pica, Flores, and Corvo.

SECT.


NORTH AMERICA
ASIA
Bhering Strait
Arctic Circle
NORTH PACIFIC OCEAN
Tropic of Cancer
QUIVIRA
TEGUAYO
NEW ALBION
NEW MEXICO
CALIFORNIA
Gulf of CALIFORNIA
Icy Cape
Cooks Riv
P. Williams Sound
Bristol Bay
Alaska
Kodiac I.
Oonemak
Ounalashka
Q. Charlottes Isles
Entrance of Juan de Fuca
River of the West
C. Mendocino
St. Fran. Drakes Har.
New Years Har.
St. Barbara
St. Clement
San Diego
B. Magdalena
C. St. Lucas
C. Corrientes
3 Marys
Great Slave L.
Peace R.
Arabasca R.
Buffalo L.
Cogead L.
Point L.
Otter Nation
R. Colorado
Blue R.
Longitude West 130 from London
V. Woodthorpe sc. 27 Fetter Lane.

BAFFINS BAY
GREENLAND
DAVIS'S STRAIT
Hudsons Str.
HUDSONS BAY
James Bay
LABRADOR
NORTH ATLANTIC OCEAN
CANADA
UNITED STATES
LOUSIANA
New found land
Great Bank of Newfoundland
Gulf of St. Laurence
Nova Scotia
New Brunswick
N. England
N. York
Penfilvania
Philadelphia
Maryland
Virginia
N. Carolina
S. Carolina
Georgia
Charles Town
Savanna
St. Augustin
East Florida
W. Florida
New Orleans
BAHAMA Is.
GULF of MEXICO
CUBA
Havannah
JAMAICA
Kingston
St. DOMINGO
Porto Rico
CARIBBEAN SEA
Carthagena
MEXICO or NEW SPAIN
Mexico
Acapulco
Yucatan
Bay of Honduras
NEW LEON
NEW BISCAY
Texas
Cenis
Sioux
Paducas
ASSINIPOELS
Superior
Michigan
Huron
Ontario L.
L. Erie
Quebec
Boston
Halifax
Superior
C. Farewell
C. Cod
C. Hatteras
Chesterfield Inlet
York Fort
Severn R.
Albany Fr.
Mistassins L.
100
90
80
70

SECT. XI.

OF AMERICA.

AMERICA, or the GREAT WESTERN CONTINENT, frequently called the New World, extends from the eightieth of north latitude, to the fifty-sixth degree of south latitude; and where the breadth is known, from the thirty-fifth degree to the one hundred and thirty-sixth degree, west longitude; extending near 9000 miles in length, and 3690 in breadth. As it extends into both hemispheres, it has two summers and two winters. It is washed by the two great oceans, the Atlantic and Pacific; having the former on the east, and the latter on the west: by these seas it has a direct communication with the other three quarters of the world. It is composed of two great continents, the North, and South America, connected together by the kingdom of Mexico, which is an Isthmus of 1500 miles long, and in one part, only 60 miles broad.

AMERICA, is the best watered of any part of the globe: even those vast tracts of country, situated beyond the Aplachian Mountains, at an immense distance from the Ocean, are watered by inland Seas, as the Lakes of Canada, which give rise to several large rivers, as the Mississippi, the Misaures, the Ohio, and on the north, the river St. Lawrence, all of them being navigable to their heads, which is a great advantage for commerce.

SOUTH AMERICA is better watered (if possible,) than North America, having the two largest rivers in the World, viz:—The river of Amazons, and the River of La Plata; the former having a course of about 3000 miles.

A country

A country of ſuch vaſt extent on both ſides of the equator muſt neceſſarily have all the varieties of ſoils and climates to be met with in every other part of the globe. It alſo produces moſt of the metals, minerals, plants, fruits, trees, and wood, to be met with in the other parts of the world, and many of them in greater quantities and higher perfection. The gold and ſilver of America has ſupplied Europe with ſuch quantities, that their value is greatly diminiſhed.

This country alſo produces diamonds, pearls, emeralds, amethyſts, and other precious ſtones, alſo cochineal, indigo, anatto, logwood, brazil, fuſtic, pimento, lignum-vitæ, rice, ginger, cocoa or chocolate, ſugar, cotton, tobacco, the balſams of Peru, tolu, and chili, Jeſuit's bark, mechoacan, ſaſſafras, ſarſaparilla, caſſia, tamarinds, hides, furs, ambergreaſe, and a great variety of other woods, roots, and plants, many of which were not known before the diſcovering of America.

Though the Indians ſtill live in quiet poſſeſſion of many large tracts of country, in the inland parts, yet America, ſo far as is known, is generally claimed by four powers, viz:—The Spaniards, Engliſh, Portugueſe, and American ſettlers, being the deſcendants of Europeans, and who have the largeſt ſhare of country, except the Spaniards, who poſſeſs the largeſt and moſt extenſive portion of all, extending from New Mexico Louiſiana, in North America, to the ſtraits of Magellan, except the large province of Brazil, which belongs to Portugal. The thirteen United States of America, poſſeſs all that tract of country, which is bounded by the Miſſiſſippi, the river St. Lawrence, and the Lakes of Canada on the north and weſt; and waſhed by the Atlantic Ocean on the eaſt; and on the ſouth by the Gulf of Mexico.

The American Islands, commonly called the Weſt-Indies, was the firſt of America diſcovered by the Europeans, and are ſituated in the Gulf, called the Caribean Sea, between the Continents of North and South America, and extend

tend from the coaſt of Florida, to the river Oronooko. And are divided between five European nations, viz:—The Engliſh, French, Spaniards, Dutch and Danes.

As all theſe Iſlands lie between the Tropics, their climates and ſoil are pretty much alike. The heat would be intolerable, if it were not for the trade winds which blow the fore part of the day, and the ſea and land breezes. Their ſeaſons are divided into the wet and dry: in the wet ſeaſons, the rain pours down with ſuch impetuoſity as to overflow the rivers, and lay the low country under water.

The principal trade of the Weſt Indies, conſiſts of ſugar and rum. They alſo export cotton, indigo, chocolate, coffee, and dying and phyſical drugs, ſpices, and hard woods; for which they receive from Europe, manufactures; from the African iſlands, wine; and from the neighbouring continent, lumber and proviſions.

The Bahama Iſlands, which are ſaid to be five hundred in number, lie to the ſouth of Carolina, between 21 and 27 degrees north latitude, and between 73 and 81 weſt longitude. There are, however, not above twelve of them of any magnitude, the reſt being little better than rocks or banks, and almoſt uninhabited, except Providence Iſland.

The BERMUDAS, or SUMMER ISLANDS, lie in the Atlantic Ocean, about 300 leagues eaſt from Carolina, in 32 degrees north latitude, and in 65 degrees weſt longitude. Theſe are ſaid to be about 400 in number; but containing not more than 20,000 acres.

The ISLANDS of NEWFOUNDLAND, CAPE BRETON, and St. JOHN, lie at the mouth of the River St. Lawrence: and are celebrated for the quantity of fiſh found on their coaſts, which is ſuppoſed to increaſe the national ſtock, upwards of three hundred thouſand pounds annually. In this branch of commerce, upwards of 3000 ſmall craft are employed, and 10,000 hands.

BRITISH AMERICA, or the territories on the Continent, belonging to the Engliſh, are New Britain, Canada, or the

Province

Province of Quebec, and Nova Scotia, or Arcadia; bounded on the eaſt and ſouth, by the Atlantic Ocean and the American States; on the north and weſt, their boundaries have never been defined, but are blended with the lands of the Indian nations. New Britain contains Labrador, and New North and South Wales. Canada contains the towns of Quebec, Trois, Ricreres, and Montreal, all ſituated on the River St. Lawrence.

Nova Scotia contains the towns of Hallifax, Annapolis, and St. John's.

The UNITED STATES of AMERICA are bounded on the weſt, by the Indian nations; on the north, by Britiſh America; on the weſt, by the Atlantic; and on the ſouth, by Spaniſh America; containing the following States or Colonies: New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, New York, New Jerſey, Pennſylvania, Delaware, Maryland, Virginia, North Carolina, South Carolina, Georgia, Vermont, Weſtern Territory, and Kentucky.

The United States in the year 1776, were only thirteen in number: Vermont, Kentucky, and the Weſtern Territory have been added ſince. The Weſtern Territory is of ſuch extent, that the Congreſs have determined to divide it into ten new States.

From the lateſt accounts it appears, that the population of the United States, amount to upwards of three million, eighty three thouſand, and ſix hundred perſons, who are compoſed of almoſt all nations, languages, characters, and religions. The greater part, however, have deſcended from the Engliſh.

The language univerſally ſpoken through all theſe States, is the Engliſh, in which all their civil and eccleſiaſtical matters are performed, and their records kept.

There are, however, great numbers of Dutch, French, Germans, Spaniards, Jews, and Swedes, who retain in a great degree, each their native language, and have their re-

ſpective

ſpective places of worſhip; and in general, live comfortably and unmoleſted, as to principles of conſcience.

NEW ENGLAND is bounded on the north, by Canada; on the eaſt, by Nova Scotia, and the Atlantic Ocean; on the ſouth, by the Atlantic; and on the weſt, by New York; and is divided into five States; viz. New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, and Vermont, which are ſubdivided into counties, and thoſe counties again ſubdivided into townſhips.

New England is a fine country for paſture; the valleys are generally interſected with brooks of water, the banks of which are covered with a track of rich meadow land.

The State of NEW YORK is bounded on the ſouth eaſt, by the Atlantic Ocean; on the eaſt, by Connecticut, Maſſachuſetts, and Vermont; on the north, by Canada; on the ſouth and ſouth weſt, by Pennſylvania and New-Jerſey; being 350 miles in length, and 300 in breadth; and containing about 44,000 ſquare miles, equal to 28,160,000 acres. The River St. Lawrence divides this ſtate from Canada. The ſettlements formed in this State, are chiefly upon two oblongs, extending from the city of New York, eaſt and north. The eaſt is Long Iſland, which is 140 miles in length; the other extending north, is about 40 miles in breadth. This ſtate exports to the Weſt-Indies, biſcuits, peas, Indian corn, apples, onions, boards, ſtaves, horſes, ſheep, butter, cheeſe, pickled oyſters, beef and pork; but the principal part of their trade is wheat, of which in the year 1775, they exported 677,700 buſhels, and 2555 tons of bread, beſides 2828 tons of flour. For which they receive in exchange, the commodoties of the Weſt India Iſlands.

NEW JERSEY is 160 miles in length, and 52 in breadth. Bounded on the eaſt by Hudſon's River and the Sea; on the ſouth, by the Sea; on the weſt, by Delaware Bay and River, which divides it from Pennſylvania; and on the north, by a line drawn from the mouth of Mahakkamak river, in latitude 41 degrees, 24 minutes to a point in Hudſon's River,

in latitude 41 degrees; containing about 8320 ſquare miles.

This ſtate has a great variety of ſoil, from the worſt to the beſt. But it contains a greater proportion of barren land than any other ſtate, there being nearly one fourth of this ſtate unfit for cultivation. But thoſe parts which are fruitful are equal in fertility to any parts of the United States.

The STATE of PENNSYLVANIA, is bounded by Delaware River on the eaſt, by the ſtate of New York on the north; by the parallel of latitude, 39 degrees, 43 minutes, 18 ſeconds, on the ſouth; and by a Meridian line drawn from the ſaid parallel, at 5 degrees of longitude, from a point on Delaware River. This ſtate lies in the form of a parallellogram. The north ſide of Pennſylvania is the beſt ſoil, and moſt populated, owing to the great quantity of new roads, which have lately been made.

The STATE of DELAWARE, is bounded on the north, by the territorial line, (which is a circle,) deſcribed with a radius of 12 Engliſh miles, and whoſe centre is in the middle of the town of Newcaſtle, which divides it from Pennſylvania; on the eaſt, by Delaware River and Bay; on the ſouth, by a line drawn due eaſt and weſt from Cape Henlopen in 38 degrees, 30 minutes, north latitude, to the middle of the Peninſula, which line divides the ſtate from Worceſter county, in Maryland; and on the weſt, by Maryland; containing about 1400 ſquare miles; being 92 miles in length, and 16 in breadth.

The STATE of MARYLAND, is bounded on the north, by Pennſylvania; on the eaſt by the Delaware State; on the ſouth, and ſouth-eaſt, by a line drawn from the Ocean over the Peninſula, (dividing it from Accomac county in Virginia) to the mouth of Patomac River, and from thence up the river to its firſt ſource; from thence by a due north line, till it interſects the ſouthern boundary at Pennſylvania; being 134 miles in length, and 110 in breadth. The ſoil of Maryland

where

where it is good, will produce from 12 to 16 bushels of wheat, or from 20 to 30 bushels of Indian corn, per acre. Their trade is chiefly with the other states, the West-Indies, and some parts of Europe; to which places they annually export 30,000 hogsheads of tobacco, besides great quantities of pig-iron, lumber, flax seed, and provisions.

The STATE of VIRGINIA, is bounded on the east by the Atlantic; on the north by Pennsylvania, and the River Ohio; on the west by the Mississippi; and on the south by North Carolina; being 758 miles in length, and 224 in breadth.

The STATE of KENTUCKY, is bounded on the north-west by the River Ohio; on the west by Cumberland River; on the south by North Carolina; and on the east by Sandy River, and a line drawn full south from its source to the northern boundary of North Carolina: being 250 miles in length, and 200 in breadth. The fertility of the soil is such, that the land, in common, will produce 30 bushels of wheat or rye an acre. And the best lands are too rich for wheat, and will produce from 50 to 80 bushels of good corn per acre, and few soils yield more and better tobacco.

The STATE of NORTH CAROLINA, is bounded on the north by Virginia: on the east by the Atlantic: on the south by South Carolina and Georgia: and on the west by the Mississippi: being 758 miles in length, and 110 in breadth.

The STATE of SOUTH CAROLINA, is bounded on the east by the Atlantic: on the north by North Carolina: on the south and south-west by the Savannah River. The western boundary is not ascertained. It is reckoned 200 miles in length, and 125 in breadth.

The STATE of GEORGIA is bounded on the east by the Atlantic; on the south by Florida; on the west by the river Mississippi; on the north and north-east by South Carolina: being 600 miles in length, and 250 in breadth.

The WESTERN TERRITORY includes all that part of the United States on the north-weſt of the river Ohio; bounded on the weſt by the Miſſiſſippi river; on the north by the Lakes; on the eaſt by Pennſylvania; and on the ſouth and ſouth-eaſt by the river Ohio; containing, 411,000 ſquare miles, equal to 263,040,000 acres, from which deducting 43,040,000 acres, occupied by the water, there remains 220,000,000 acres, which is to be ſold by congreſs, for the diſcharge of the national debt.

Territories of Spain in North America.

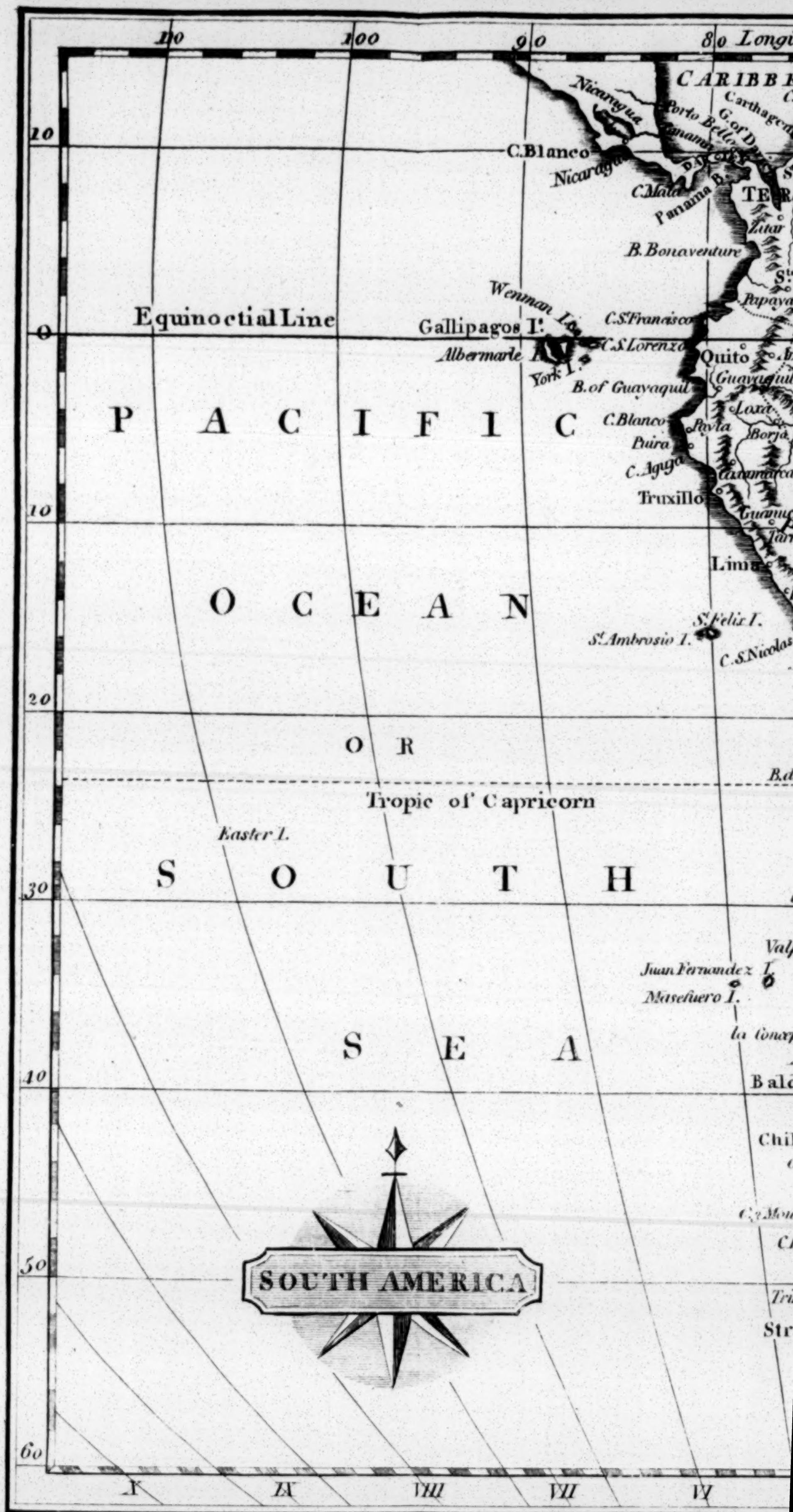
The dominions of Spain in North America, extends from 81 degrees to 120 degrees, weſt longitude, and from 8 to 43 degrees, north latitude. Bounded on the north by the United States, and the Indian nations; on the weſt by the Pacific Ocean; on the eaſt by the Gulf of Mexico and the Atlantic; and on the ſouth, terminating in the Iſthmus of Darien. They contain the following counties, viz.—Eaſt Florida, Weſt Florida, Louſiana, New Mexico, California, and Old Mexico.

The ſoil of this extenſive tract of country, is very various; but in general very fertile: and it is in the moſt mountainous parts that the mines of gold and ſilver are found. The air is in general warm and pleaſant; but the north parts have very cold winds; and the ſouthern parts, lying within the torrid zone, are exceeding hot.

South America.

SOUTH AMERICA, from the Northern coaſt of Terra Firma, and the Iſthmus of Darien, to the Straits of Magellan, belong to the Spaniards, except the province of Brazil, which belongs to the Portugueſe; and the ſettlements of the Dutch in Surinam; and thoſe of the French, in Cayenne.

BRAZIL,



SOUTH AMERICA
PACIFIC OCEAN OR SOUTH SEA
Equinoctial Line
Tropic of Capricorn
CARIBBE
Nicaragua
Porto Bello
Panama
C.Blanco
C.Mala
Panama B.
B.Bonaventure
Wenman I.
C.S.Francisco
Gallipagos I.
C.S.Lorenzo
Albermarle
York I.
Quito
B.of Guayaquil
C.Blanco
Payta
Puira
C.Aguja
Truxillo
Lima
S.Felix I.
St.Ambrosio I.
C.S.Nicolas
Easter I.
Juan Fernandez I.
Masefuero I.
110
100
90
80 Longi
10
0
10
20
30
40
50
60
X
IX
VIII
VII
VI

Roper Engraver, 20 New Str. Square Shoe Lane

Brazil, belonging to the Portugueſe, extends from the equator, to 35 degrees ſouth latitude, and from 35 to 60 degrees, weſt longitude. Bounded on the north by the mouth of the River Amazons and the Atlantic; on the eaſt by the Atlantic; on the ſouth by the mouth of the River Plata; and on the weſt by the unknown country of the Amazons.

Cayenne, is the only ſettlement in the ſouthern Continent of America, retained by the French, and is ſituated between the equator and 5 degrees north latitude; and between 52 degrees 15 minutes, and 55 degrees 30 minutes, weſt longitude. It extends 240 miles along the coaſt of Guiana, and near 300 miles inland, Bounded by Surinam on the north; by the Atlantic on the eaſt; by Amazonia on the ſouth; and by the territories of the Indians on the weſt.

Surinam, or Dutch America, lies between 5 and 7 degrees north latitude; and is bounded on the north by Cayenne; on the weſt by Terra Firma; on the ſouth by the Indian nations; and on the eaſt by the River Oronooko.

The dominions of Spain, in South America, contain the following extenſive countries, viz:—Terra Firma, Peru, Chili, Paraguay, Amazonia and Patagonia; extending as before obſerved, through the whole Continent of South America; and, in fact, including that Continent, except Brazil, Cayenne, and Surinam.

The climate and ſoil of Spaniſh America, vary greatly, from the hot burning ſand, and ſmoaking ſwamp, in the northern parts, in the torrid zone, to the cold region of the ſouthern part, near Cape Horn.

The Iſlands of South America, are Terra-Del-Fuego, the Falkland Iſlands, and the Iſland of Juan Fernandes; the latter of which gave riſe to the famous ſtory of Robinſon Cruſoe, from one Alexander Selkirk, Mariner, native of Scotland, who was put a ſhore in this Iſland, by his captain, in the year 1697, and diſcovered by Woodes Rogers, in the year 1709, who took him on board, and brought him to Europe;

Europe; after having been on this uninhabited Iſland for twelve years.

The number of inhabitants in the known parts of the World, computed at a medium from the beſt calculations are about nine hundred and fifty-three millions, viz:—

Europe contains	153	millions
Aſia - - -	500	
Africa - - -	150	
America - -	150	
	953	

CHAP. XIV.

OF ASTRONOMY.

SECT. I.

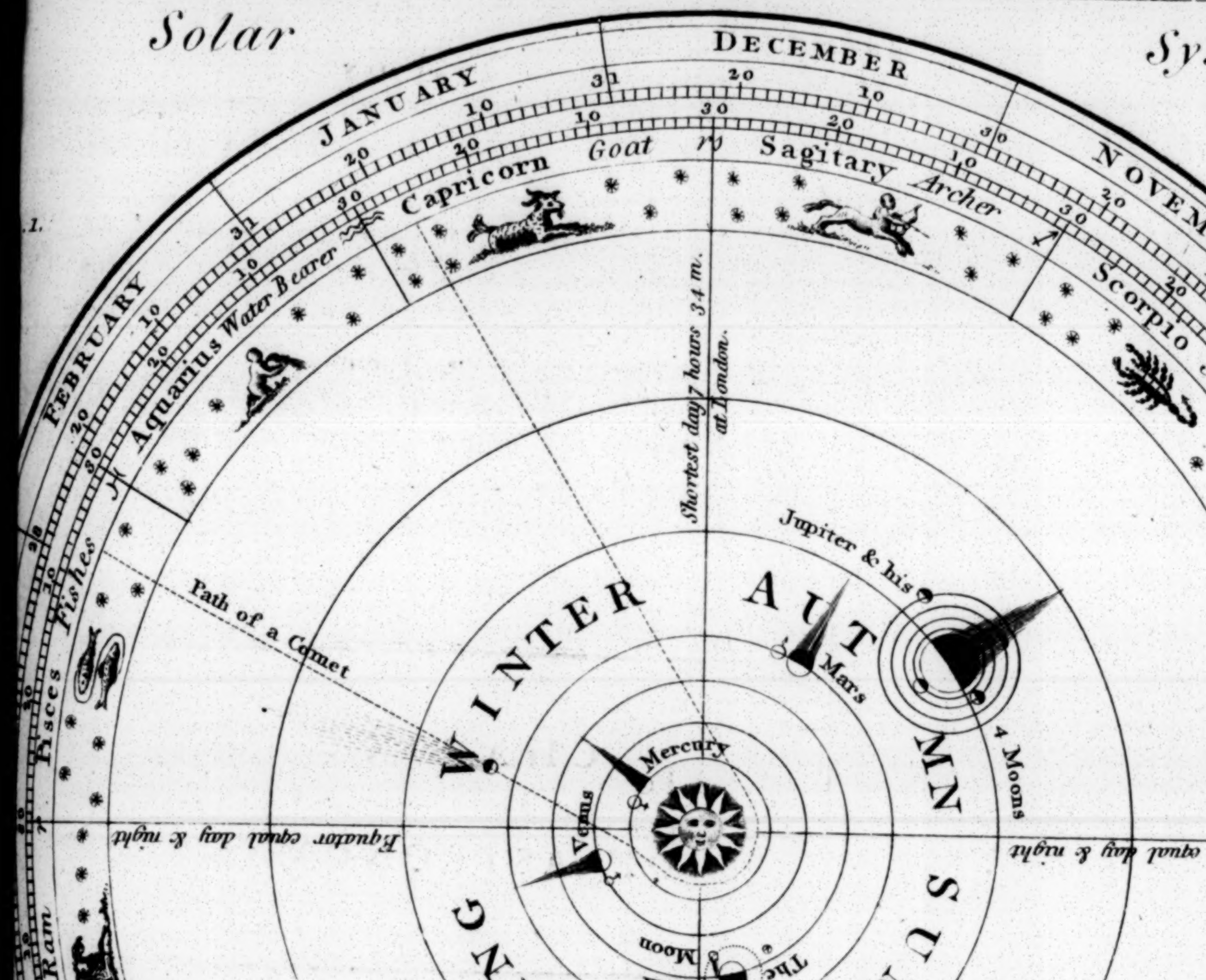
OF THE PRIMARY PLANETS.

BEFORE I proceed to the deſcription of the Primary Planets, it will be neceſſary to take a view of the ſolar ſyſtem, with the order and economy of the motions and courſes of thoſe planets.

The ſyſtems which have been moſt generally received in Aſtronomy, are the *Ptolomaic*, the *Copernican*, *Pythagorean*, and the *Tychonic*.

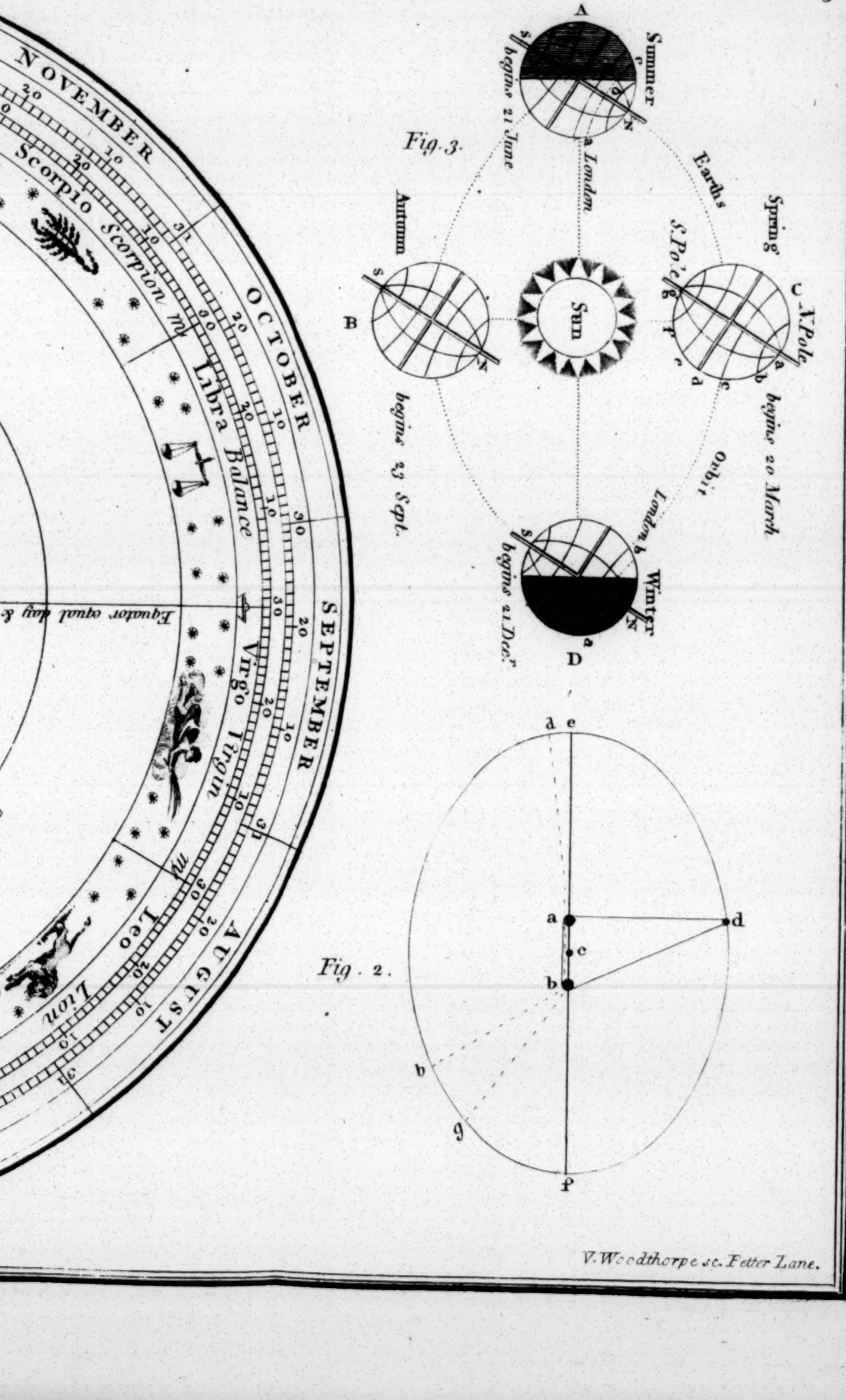
The

Solar Sy



DECEMBER
JANUARY
FEBRUARY
NOVEM
Capricorn Goat
Sagitary Archer
Scorpio
Aquarius Water Bearer
Fishes
Pisces
Ram
Aries
Taurus Bull
Gemini Twins
Cancer Crab
Lion
MAY
JUNE
JULY
Shortest day 7 hours 34 m. at London.
Longest day 16 hours 26 m at London.
Path of a Comet
Equator equal day & night
equal day & night
WINTER
AUTUMN
SUMMER
SPRING
Jupiter & his 4 Moons
Mars
Mercury
Venus
Earth
The Moon
Saturn & his 5 Moons
Georgium Sidus

System



V. Woodthorpe sc. Fetter Lane.

The *Ptolomaic* ſyſtem, ſo called from its inventor *Ptolomy*, ſuppoſes the earth to be placed at reſt in the centre of the univerſe, and the heavens revolving about it from eaſt to weſt, in the ſpace of 24 hours; and by this motion carrying the ſun, ſtars and planets, completely round the earth in that ſpace of time.

It was this ſyſtem which *Ariſtotle*, *Hiparchus*, and many of the philoſophers of antiquity defended ſo ſtrenuouſly; and was followed by the whole world for many ages, and longer retained in many learned univerſities. But latter improvements and more evident demonſtrations have now utterly exploded it.

In the *Tychonic* ſyſtem, invented by *Tycho*, the Dane; the earth is alſo ſuppoſed to be fixed in the centre of the univerſe, and all the heavenly bodies performing a revolution round it in the ſpace of twenty-four hours, as in the Ptolomaic ſyſtem; but with this difference; that it allows a monthly motion to the moon round the earth; and alſo the proper motions of the ſatellites, about Jupiter and Saturn. It alſo ſuppoſes the ſun to be the centre of all the primary planets: the primary planets being carried round the ſun in their reſpective periods; while the ſun, with all the aforeſaid planets, revolve round the earth every twenty-four hours. But this ſyſtem was ſo inconſiſtent with obſervations, that it had but few followers. It was therefore altered by Longomontanus and others, who allowed the diurnal motion of the earth on its own axis, but denied its annual motion round the ſun. This improved hypotheſis is called the ſemi-tychonic ſyſtem.

But theſe ſyſtems have now given place to that, called the *Copernican* ſyſtem, which undoubtedly is the moſt ancient in the world. It was firſt introduced into Greece and Italy, by *Pythagoras*; and from him called the Pythagorean ſyſtem. It was followed by *Philolaus*, *Plato*, *Archimedes*, and all the moſt ancient philoſophers, but was at length loſt under the Peripatetic philoſophy; and reſtored again about the year 1500, by *Nic Copernicus*.

This

This ſyſtem has been proved by the moſt eminent demonſtrations, to be the only true one. I ſhall therefore confine myſelf to the deſcription of this alone, and the Phenomena that ariſes from it,

Fig. 1. plate 15, is a repreſentation of this ſyſtem, where the ſeven concentric circles, marked Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and Georgian Sidus, repreſent theſe ſeven primary planets, performing each its annual rotation round the ſun, which is placed in the centre. The next two circles repreſent the twelve ſigns of the zodiac, with all its diviſions into thirty degrees in each ſign; and laſtly, the two outermoſt circles, ſhew the twelve calender months of the year, with their diviſions into days. Of each of which in their order.

A planet in the literal ſenſe of the word, ſignifies a wanderer or wandering ſtar; and is therefore uſed in contradiſtinction to the ſtars, which we call fixed. It is a celeſtial body, globular and opake, and revolving around the ſun, or ſome other planet, as a centre, at leaſt as a focus to its orbit, which always has a moderate degree of eccentricity; ſo that it is never much further from the ſun, or centre of motion, at one time than at another, in proportion to the diameter of its orbit.

The planets are either primary, or ſecondary.

Primary planets, ſometimes called planets by way of eminence, are thoſe ſeven above deſcribed, which move round the ſun, as their common centre, or focus of their orbits.

Secondary planets, or ſatellites, are ſuch as move round ſome primary planet, as their focus, in the ſame manner as the primary planets move round the ſun. Such are the moon which moves round the earth, and the four moons of Jupiter, the ſeven moons of Saturn, and the two of the Georgian ſidus.

The Primary planets are divided into ſuperior and inferior; the ſuperior planets, are thoſe that perform their revolutions round the ſun at a greater diſtance from the ſun, than the earth

earth is, as Mars, Jupiter, Saturn, and the Georgian. The inferior planets, are those included within the orbit of the earth; as Venus and Mercury; as may be seen in the figure.

The Planets were formerly represented by the same characters, which the Chymists made use of, to represent their metals by; on account of a supposed analogy, between the planets and those metals.

Thus, MERCURY, anciently called the messenger of the Gods, was signified by the character ☿ ; which stood for the metal Mercury, and also bore a rude resemblance to the heathen Deity of that name; being a man, with wings on his head and feet.

VENUS, the next Planet in order, so named from the Goddess of Love, was characterised by ♀, for the figure of a woman; and denoted the metal Copper.

TELLUS, or the Earth, is characterised by ⊖; and is the third planet distant from the Sun.

MARS, or the God of War, denoted by ♂, represented Iron; and supposed to signify a man, holding out a spear.

JUPITER, the chief of the heathen Gods, marked ♃, to represent thunderbolts; and signified the metal Tin.

SATURN, the Father of the Gods, is represented by ♄, to imitate an old Man, supporting himself with a staff; the same character being used for the metal Lead.

The GEORGIAN, or Herschel, is denoted by the initial of Dr. Herschel's name, with a cross for the christian planet.

The ORBIT of a PLANET, is the path it describes in going round the Sun, represented in the figure by concentric circles. The earth's orbit is called the ecliptic.

Kepler, was the first Astronomer, who discovered that the orbits of the planets, were not circular, but of an ecliptic form, in the form of the figure (fig. 2) having the Sun in one of the foci thereof: and, he further discovered these two primary laws, from hence called *Kepler's Laws*: viz.—That a

radius, drawn from the centre of the Sun, to the centre of the planet, always deſcribes equal areas, in equal times; or which is the ſame thing, in unequal times, it deſcribes areas proportional to thoſe times: and the ſquares of the periodical times of the planets, are as the cubes of the mean diſtance of the planets from their centre.

Thus, let *d e f* be the orbit of a planet, or an elipſe, in which *c* is the centre, and *a b*, the two foci of the elipſe, in one of which foci the Sun is placed, as ſuppoſe *b*: then if the area *b b g* be equal to the area *b g f* the planet will be as long in deſcribing that part of its orbit from *b* to *g*, as from *g* to *f*; and a ſtraight line *e f* drawn through the two foci of the orbit is called the line of the apſides, and the point *e*, being the fartheſt from the Sun, or centre of motion, is called the higher apſis or aphelion; and the point *f*, being that point of the line next the Sun, or centre of motion, is called the lower apſis or perihaelion; therefore, when the planet is in its aphelion, its motion is the ſloweſt of all, and when in the perihaelion, the quickeſt.

The axis of a Planet, is an imaginary line, ſuppoſed to be drawn through the centre, about which, the planet performs a diurnal rotation.

The method of deſcribing an ellipſe, is, having fixed two pins or points *a b*; with a piece of thread doubled, and a pen or pencil *d* in the double of the thread, deſcribe the ellipſe *d e f*, keeping the thread extended to the full length, by the pencil *d*. And note: the two fixed points *a* and *b*, which hold the thread, are always the two foci of the ellipſe: and the nigher theſe two foci are together, the nearer the ellipſe will approach to the form of a circle: and the farther they are diſtant from each other, the farther the ellipſe departs from a circular form. The diſtance *a c*, or *c b*, that is, the diſtance from the centre, to either foci, is called the eccentricity.

The

The orbits of the planets are not all in the ſame plane: neither is any two of them in the ſame plane: but the plane of the ecliptic, or Earth's orbit, interſects the plane of the orbit of every other planet in a right line, which paſſes through the Sun, called the line of the nodes; and the points of interſection of the orbits, are called the nodes.

The different inclinations of the axis of a planet, to the plane of its orbit, is the cauſe of the different ſeaſons of the year in that planet: and the diurnal rotation of the planet round its axis, produces the ſucceſſive changes of the day and night. Thus, in the earth's orbit, (fig. 3.) let A repreſent the earth in the ſummer ſeaſon, on the 21ſt of June. N S is the axis of the earth, N being the north, and S the ſouth pole, about which the earth performs her revolution every twenty-four hours. Let *c a* repreſent the path deſcribed by the city of London, in the rotation of the globe round its axis. Now, the reaſon why the days are longer than the nights at this ſeaſon of the year, in London, will be evident from a bare inſpection of the figure. For the axis of the earth being inclined to a perpendicular drawn to the ecliptic, in an angle of twenty-three degrees and a half, and the City of London being ſituated in the fifty-ſecond degree of north latitude, which is nigher the pole than to the equinoctial, it will not paſs through near ſo great a ſpace of the dark ſide of the Earth, or that ſide oppoſite to the Sun, as it will through the illuminated part of the earth, or part next to the Sun. The nights in this ſeaſon of the year, only continuing while London deſcribes that part of its tract, from *b* to *c*: but as ſoon as it arrives at *b*, it will be day break; and when it arrives at *a*, the Sun will be on the meridian, or it will be then noon-day. The length of the day, therefore, will be to that of the night, as the diſtance *c b*, to that of *b a*.

But when the Earth arrives at B, which is on September the 23d; the axis of the Earth always keeping in the ſame poſition, the days will be equal to the nights: for the illu-

minated parts of the Earth, will extend exactly from the north to the south poles: and therefore, in the passage of the Earth, from A to B, the length of the days will gradually decrease in the northern latitudes; but increase in the southern latitudes.

Fig. 1. MERCURY, is the smallest of all the primary Planets, and nearest the Sun, performing his revolution in a less space of time, than any of the rest round the Sun, with a very rapid motion. This occasioned the Greeks to give it its present name and character, calling it as before observed, the messenger of the Gods.

The mean distance of this planet from the Sun, compared with that of the Earth, from the Sun; is as 387 to 1000; therefore, his distance from the Sun is about thirty-six million of miles, or little more than one third of the Earth's distance from the Sun. Hence, the diameter of the Sun, seen from Mercury, will appear near three times as large as when seen from the earth; consequently, that planet receives about seven times as much heat and light from the sun, as this earth does. This is a degree of heat, sufficient to make water boil.

The diameter of Mercury, is not one third of that of the earth, or about 2600 miles; therefore, its surface is nearly one ninth; and his solidity, one twenty-seventh of that of the Earth.

The Orbit of Mercury, is inclined to the plane of the ecliptic, or Earth's orbit, in an angle of 6 degrees 54 minutes. He performs one entire revolution round the Sun, in the space of 87 days 23 hours; therefore, the summers and winters in that planet cannot be more than 44 days each. His greatest elongation from the Sun; that is, the greatest distance that he is seen by us to depart from the Sun, is 28 degrees. The eccentricity of his orbit, is one fifth of his mean distance, from the Sun; which is far greater than that of any of the other planets; and the pace with which he

moves

moves in his orbit, is at the amazing rate of about ninety-five thousand miles in an hour.

The place of his aphelion is 23 degrees 8 minutes of sagitary; the place of his ascending node is 14 degrees 43 minutes of taurus, and consequently, that of the descending node, 14 degrees 43 minutes of Scorpio.

The length of his days, or rotation on his own axis, inclination of his axis to his orbit, gravity on his surface, density and quantity of matter, are all unknown.

Mercury, is observed to appear with various phases, like the moon: varying according to his various positions, with regard to the earth and sun; therefore, he never appears with a full face towards us, except when he is too near the Sun to be distinctly seen; for his enlightened face is always towards the Sun. From these observations it is plain, that Mercury, like all the other planets, is a dark opake body, having no light of his own, but what he receives from the Sun; for if he had, he would always appear constantly round.

The best time to make an observation on this planet, is when it is seen on the Sun's disc, called its transit; at which time it passes before the Sun, and appears like a little round black spot, on the Sun's surface, eclipsing a small part thereof, and is only visible through a telescope. And as these transits can only happen when the Earth and Mercury are both in the same node of Mercury; that is on the sixth of November, and the fourth of May; and at which time Mercury must also be in an inferior conjunction, it will follow, that at these times, when Mercury is in the inferior conjunction with the Sun, he will appear to pass over the Sun's disc.

But in all the other parts of his orbit, he will never make a transit over the Sun, though he may be in an inferior conjunction; because he goes either above or below the face of the Sun. Gassendi, in November, 1631, first took an observation of this kind; and *Mr. Whiston* has given a list of

several

ſeveral tranſits of Mercury; viz. – November the 12th, at 3 hours, 44 minutes, afternoon, in 1782; May the 4th, at 6 hours, 57 min. forenoon, 1786; December the 6th, 3 hours, 55 min. afternoon 1789; and May 7th, 2 hours, 34 min. afternoon, 1799.

VENUS, the brighteſt, and to appearance the largeſt of all the planets, is the next inferior planet; and is diſtinguiſhed from all the reſt by her brightneſs, and white appearance; it is ſo conſiderable, that in a duſky place ſhe will often cauſe an object to project a ſenſible ſhadow; and her brightneſs is ſo great, as to often render her viſible in the day time.

As Venus moves round the Sun in an orbit within the Earth's orbit, like Mercury; ſhe can never be ſeen in oppoſition to the Sun; though ſhe departs farther from him than Mercury does; her greateſt diſtance from the Sun being about 48 degrees, as ſeen from the Earth.

When ſhe is on the weſt of the Sun, which happens from her inferior, to her ſuperior conjunction with him, ſhe riſes in the morning before the Sun, and is then called the *morning ſtar:* when ſhe is to the eaſt of the Sun, which is from her ſuperior to her inferior conjunction, ſhe ſets after the Sun in the evening, and is then called the *evening ſtar.*

The diameter of Venus, is nearly equal to that of the Earth; being about 7900 miles: her apparent mean diameter, ſeen from the Earth, is 59 ſeconds: but her apparent diameter ſeen from the Sun, or her horizontal parallax, is only 30 ſeconds: her diſtance from the Sun 70,000,000 of miles, her eccentricity $\frac{7}{1000}$ of her diſtance, or 490,000 miles. The inclination of her orbit to the plane of the ecliptic is 3 degrees, 23 minutes; the points of their interſections, or nodes, are in 14 degrees of Gemini and Sagittary. The place of her aphelion is 4 degrees 20 minutes of Aquarius. Her axis is inclined to her orbit in an angle of 75 degrees. Her periodical courſe round the Sun, is performed in 224 days, 17 hours.

The

The diurnal rotation round her axis is not certainly known; *Cassini*, makes it 23 hours, but others make it much more.

When Venus is observed through a good telescope, she is perceived to have various phases and changes, like those of the moon: her illuminated parts being constantly turned towards the Sun.

Dr. Herschel, made many observations on this planet, between the year 1777 and 1793. The results of his observations are, that this planet has a revolution about its own axis, but the periodical time of which he was not able to ascertain; that the position of its axis cannot certainly be known: that the planet's atmosphere is very considerable; the surface of the planet he also found to be diversified with hills and vallies, and other inequalities. But the atmosphere of the planet appeared so dense, or some other obstruction in the region of the planet, prevented him (as he says) from having a particular view of the mountains, though assisted by the best instruments.

The transits of Venus happen but seldom. One of these transits was seen in England, in the year 1639, by *Mr. Horrox, and Mr. Crabtree*; two more were seen in the present century: the one June the 6th, 1761; and the other in June 1769; there will not happen another, till the year 1874.

In all other respects Venus has the same appearance to us, regularly every eight years; that is, her conjunctions, elongations, times of rising and setting, being very nearly the same, and on the same days.

Some astronomers have discovered, or imagined they have discovered, a satellite belonging to Venus: of this number were *Cassini*, who with a telescope of thirty four feet, in the years 1672 and 1686, thought he saw a satellite move round this planet, at the distance of about $\frac{3}{5}$ of Venus's diameters. And *Mr. Short* in 1740, with a reflector of 16 $\frac{1}{2}$ inches, focus, perceived a small star near Venus, and with another telescope of the same focus, and magnifying to the sixtieth

ſixtieth power, he found its diſtance from Venus, about 10 minutes; and with a glaſs magnifying to the two hundred and fortieth power, he obſerved the different phaces of this ſatellite; and its diameter appeared to be near $\frac{1}{3}$ of that of Venus. And ſeveral other acute obſervers have imagined they diſcovered the ſame thing.

The EARTH, the next Primary Planet, in order from the Sun, is our habitation: and it performs its revolution round the Sun, at the diſtance of ninety-five million of miles. Like all the other primary planets, it has both a diurnal and annual motion. Its diurnal motion, is that by which it turns round its own axis, in the ſpace of twenty-four hours nearly, from weſt to eaſt, and thereby cauſing the continual ſucceſſion of day and night. Its annual motion, is that by which it is carried round the Sun in its own orbit, and between the orbits of Venus and Mars; having the orbits of Venus and Mercury within its own orbit, or between it and the Sun, in the centre; and thoſe of Mars, Jupiter, Saturn, the Georgian, &c. without or above it, which are therefore called the ſuperior planets, and Mercury and Venus the inferior ones. This annual motion is accompliſhed in the ſpace of a year, or 365 days, 6 hours nearly: or rather 365 days, 5 hours, 49 minutes; this is called the tropical year. But the time the Earth takes to perform its annual revolution, from any fixed ſtar to the ſame again, as ſeen from the Sun, is 365 days, 6 hours, 9 minutes, which is the ſidereal year. The figure of the Earth's orbit, as that of all the other planets, is elliptical; the eccentricity of the orbit, or diſtance of the Sun in the focus, from the centre of the orbit, is about one ſixtieth part of the mean diſtance of the Earth from the centre. The Earth, as well as all the other planets, performs his annual revolution according to the natural order of the ſigns.

By the diurnal rotation of the Earth, on its axis, the ſame appearance is produced as if it were fixed, and the ſun and ſtars

ſtars moved round them every twenty-four hours. For turning round from weſt to eaſt, cauſes the Sun, and all the heavenly bodies, to appear to move the contrary way, or from eaſt to weſt, as is imagined to be the caſe by the vulgar and illiterate. Thus, when by the rotation of the earth, the obſerver is brought to that part, where he ſees the Sun, or a ſtar, juſt riſing above the horizon, in the eaſt, they are then ſaid to be riſing; and as the Earth continues to move, other ſtars will appear to riſe, and advance weſtward; and when, by the motion of the Earth, the obſerver is brought directly under the Sun or ſtar, they are then ſaid to be on the meridian; after which, by a continuance of the ſame motion, the obſerver is brought to the eaſtern ſide of the Earth, when the Sun or ſtar will appear to ſet on the weſtern ſide thereof.

The circumference of the Earth is 25,000 miles; in diameter $7957\frac{3}{4}$ miles; the ſuperfices 198,944,206 ſquare miles; and the ſolidity 263,930,000,000 cubic miles.

The Earth throughout its annual orbit, always keeps its axis parallel to itſelf in every part of its orbit; thereby occaſioning all the varieties of different ſeaſons of the year; and the different length of the days and nights, as ſeen in fig. 3: the Sun enlightening more of the north polar parts, at one ſeaſon of the year, and more of the ſouthern parts, at the oppoſite ſeaſon of the year; thus, producing the different degrees of heat in the different ſeaſons. For in the ſummer ſeaſon, the heat of the Sun is increaſed by two cauſes: viz.—1. By the Sun's rays being more vertical to us in the north parts of the globe; and conſequently, the heat is not diminiſhed in paſſing through ſo great a portion of the atmoſphere, as when the rays come more oblique. And, ſecondly, Having the light, and conſequently, the heat of the Sun for a longer ſpace of time in the ſummer, than in the winter. All which will be evident from a peruſal of the figure.

What is here aſſerted concerning the Earth holds good in all the other planets; as each of them revolve about an axis, which is not perpendicular to the plane of their reſpective orbits, but inclined thereto in a greater or leſs angle: and which axis is always parallel to itſelf in every part of its orbit.

The figure of the Earth, like that of all the other planets, is that of an oblate ſpheroid, which *Sir Iſaac Newton* demonſtrated to ariſe from the rotation of the Earth about its axis: and by the obſervation and experiments of later Aſtronomers, the polar diameter is to the equatorial diameter, as 178 to 179.

The abſolute gravity, or denſity of the whole maſs of the Earth, is to that of water, as 9 is to 2, and to common ſtone, as 9 is to 5. Thus, we diſcover the very conſiderable mean denſity of the Earth, is almoſt double to that of common ſtone; from whence it may be preſumed that the internal parts of the Earth contain great quantities of metals.

Having the denſity of the Earth, its quantity of matter is eaſily found, being always equal to the product of its denſity, by its magnitude.

The Earth is moreover every where ſurrounded by an atmoſphere, which is that large quantity of fluid matter, extended over the whole ſurface of the Earth, conſiſting of air, aqueous, and other vapours, electric fluid, &c. which ſurround the Earth to a conſiderable height, and partake of all its motions, both annual and diurnal.

The atmoſphere, ſerves for innumerable purpoſes, and is even eſſential to both animal and vegetable life; it is this, inſinuating itſelf into all the vacuities of bodies, cauſes thoſe mutations of generation, corruption, diſſolution, &c.

This atmoſphere like all other matter, has the qualities of weight, and preſſure, the quantity of which, is now pretty well known, and is found by the barometer, to be equal to an equal column of quick ſilver, of 30 inches high: therefore,

fore, becaufe a cubical inch of quickfilver, is found to weigh near half a pound, avoirdupois, a column of quickfilver of 30 inches in height, whofe bafe is one fquare inch, will weigh near 15 pounds: therefore, the weight of the atmofphere on every fquare inch of the furface on the face of the Earth, is alfo 15 pounds. Thus, it appears that the preffure upon the human body, muft be very confiderable: for as every fquare inch of furface fuftains a preffure of 15 pounds, every fquare foot will fuftain 144 times as much, or 2160 pounds.

The atmofphere has been proved by late experiments, to become more rare, and of lefs denfity, the farther it is removed from the Earth, and in the following proportion: viz.—At the height of three miles and a half from the Earth, the denfity of the atmofphere is nearly two times rarer than at the furface of the Earth: at the height of 7 miles four times rarer: at the height of 14 miles, 16 times rarer: at the height of 21 miles, 64 times rarer: at the height of 28 miles, 256 times rarer: and at the height of 35 miles, 1024 times rarer, &c.

Mars, is the firft of the four fuperior planets, and placed immediately next above the Earth, including the orbit of the earth, within that of his.

The mean diftance of Mars from the Sun, is 1524 of thofe parts of which the diftance of the Earth from the Sun is 1000, or upwards of 145,000,000 miles: his eccentricity 141 of thofe parts, of which the Earth is diftant from the Sun 1000, his mean diameter is 4444 miles. The length of his year, or the period of his completing one revolution round the Sun, is 686 $\frac{23}{24}$ days. His revolution round his own axis, is performed in twenty-four hours and forty minutes. His mean diameter feen from the Sun, is 11 feconds. The inclination of his axis, to his orbit, is nothing, his axis being perpendicular. The inclination of his orbit to the ecliptic is one degree, fifty-two minutes. The place of his aphelion, the thirty-fecond minute of virgo.

virgo. His perihelion, or aſcending node, is 17 degrees, 17 minutes of Taurus.

Dr. Herſchel has made many obſervations on the rotation of this planet about its axis, from which he inferred that the mean diurnal rotation was between 24 hours 39 minutes 5 ſeconds, and 24 hours 39 minutes 22 ſeconds. He alſo obſerved ſeveral ſmall remarkable bright ſpots, near both the poles, which had a ſort of motion. He alſo concludes that the inclination of his axis to the ecliptic, is 59 degrees 22 minutes. And the node of the axis to be in 17 degrees 47 minutes of Piſces. The obliquity of the planet's ecliptic 28 degrees 42 minutes, and the point aries on Mars' ecliptic to anſwer to our 19 degrees 28 minutes, Sagittary.

The figure of Mars is that of an oblate ſpheroid, like the Earth, having his equatorial diameter, to the polar one, as 1355 to 1272, or nearly as 16 to 15.

This planet alſo has a conſiderable, but moderate atmoſphere, ſo that its inhabitants probably, are in their nature, ſimilar to the inhabitants of this Earth.

Mars always appears with a ruddy diſturbed light, occaſioned by the nature of his atmoſphere.

When he is in oppoſition to the Sun, that is, when the Earth comes between the Sun and him, he is five times nearer to us, than when in conjunction with the Sun, or when the Sun is between him and us, and conſequently, in the former caſe, he appears much more large and bright than at other times.

As he receives his light from the Sun, like the other planets, he muſt neceſſarily have an increaſe and decreaſe apparently like the Moon; he may alſo be obſerved ſometimes almoſt biſected when in the quadratures; but he is never ſeen cornicular as the inferior planets; which ſhews that his orbit includes that of the Earth, and that he ſhines with a borrowed light.

JUPITER,

Jupiter, is the next ſuperior planet, and the largeſt of all the planets in the Solar Syſtem; in brightneſs, he is next to the planet Venus: his orbit is ſituated between thoſe of Mars and Saturn. His diameter is above ten times that of the Earth; conſequently, his magnitude exceeds the magnitude of the Earth, above one thouſand times. His annual revolution round the Sun, is performed in 11 years 314 days 12 hours 20 miuutes and 9 ſeconds. His diurnal revolution about his own axis, he performs in the ſhort ſpace of 9 hours 56 minutes, by which motion, every part on his equator is carried round at the rate of 26,000 miles in an hour, being about 25 times faſter than the equatorial parts of the Earth moves.

The axis of Jupiter, is nearly perpendicular to the plane of his orbit; therefore, he has no ſenſible change of ſeaſons, except very near the poles. This is wiſely ordered by providence. For if his axis made any conſiderable angle, with the perpendicular of his orbit, juſt ſo many degrees as it was inclined thereto, would be near ſix years in darkneſs, round each of the poles, in their turn.

The orbit of Jupiter, is inclined to the ecliptic in an angle of 1 degree 20 minutes. The place of his aphelion, is 9 degrees 10 minutes of *Libra*. The place of his aſcending node, is 7 degrees 29 minutes of *Cancer*, and of his deſcending node 7 degrees 29 minutes of *Capricorn*. The eccentricity of his orbit, is one twentieth part of his mean diſtance from the Sun.

There are ſeveral faint ſhining ſubſtances, which ſurround Jupiter of different dimenſions, called his zones or belts, which are conſtantly changing in their ſize and ſituations, and are therefore, generally believed to be clouds. They have ſometimes appeared of different breadths, at other times of the ſame breadth. Large ſpots have alſo been obſerved in theſe belts; and whenever a belt vaniſhes, as is often the caſe, the ſpots contiguous to it, have alſo

vaniſhed.

vaniſhed. Theſe belts have ſometimes been interrupted and broken, and the broken ends of ſuch belts, have often been obſerved to revolve round the planet in the ſame time with the ſpots. The ſpots have alſo aſſumed different appearances, ſome of them changing their ſhape from a circular, to an oblong form; others uniting together in one; and ſometimes one large one dividing into two or three.

The difference between the equatorial and polar diameters of Jupiter, is upwards of 6000 miles, the former being to the latter as 13 to 12.

Jupiter, is attended by four moons, or ſatellites, ſome of them larger than our Earth, which perform their reſpective revolutions round Jupiter, in different periods of time; ſo that there is ſcarcely any part of this great planet, but what is enlightened by one or more of theſe moons during the whole night. The periods, diſtances in ſemidiameters of Jupiter, and the angles of the orbits of theſe moons, ſeen from the Earth, are as follow:

Moons.	Periods round Jupiter.	Diſtances.	Angles of the Orbits.
	D. H. M.		
1	1 18 36	$5\frac{2}{3}$	3′ 55″
2	3 13 15	9	6 14
3	7 3 59	$14\frac{1}{3}$	9 58
4	16 18 30	$25\frac{1}{3}$	17 30

The three neareſt moons of Jupiter, fall within his ſhadow, and are eclipſed once in every revolution; but the orbit of the fourth ſatellite, is ſo much inclined, that it paſſeth the ſhadow of Jupiter, without falling into it two years in every ſix.

An obſerver, placed in Jupiter, would ſee a very different appearance in the Solar Syſtem, from what is ſeen by us on the Earth:—1. He would never ſee any of the four primary

mary planets, included within the orbit of Jupiter: except as ſpots paſſing over the Sun's diſc, when they happen to come between him and the Sun. 2. The Sun would appear to him, under an angle of only ſix minutes; conſequently, the inhabitants of Jupiter, have but the forty-eighth part of the quantity of light and heat from the Sun, which the inhabitants of this earth enjoy.

SATURN, is the next primary planet, and the outermoſt from the Sun, except the Georgian. He ſhines with a feeble light, on account of his great diſtance from the Sun, which is alſo apparently increaſed by his great diſtance from us. This planet, has attracted the moſt attention of all the primary planets, on account of his wonderful ring. This ring, or rather a double ring, one within the other, ſurrounds the body of the planet, at a diſtance from the planet, equal to the diameter of the planet. Beyond the ring, ſeven moons perform their reſpective revolutions round the planet. The rings and the moons, are all in the ſame plane, and are all dark denſe bodies, and therefore, caſt their ſhadows upon each other.

The Phænomena of the rings, has engaged the attention of all the Aſtronomers, ſince their diſcovery; ſome contending it was one entire ring; others, dividing into two, or more; but the obſervations of *Dr. Herſchel*, have been more ſatiſfactory on this head, than thoſe of his predeceſſors. He divides them into two rings, one within the other. Their dimenſions and ſpaces, he ſtates in the following proportion to each other:—

	Miles.
Inſide diameter of ſmaller ring	146345
Outſide diameter of ditto	184393
Inner diameter of larger ring	190248
Outſide diameter of ditto	204883
Breadth of the inner ring	20000
Breadth of the outer ring	7200
Breadth of the vacant ſpace	2839

This

This ring revolves in its own plane, in ten hours thirty-two minutes, fifteen ſeconds.

From the above ſtatement, it appears that the outſide diameter of the larger ring, is almoſt twenty-ſix times the diameter of the Earth. This ring is inclined to the plane of the ecliptic in an angle of thirty degrees. When we ſee the ring moſt open, its ſhadow upon the planet is broadeſt; and from that time the ſhadow grows narrower, as the ring appears to do to us, until by the annual motion of Saturn, the Sun comes to the plane of the ring, or even with its edge; which being then directed towards us, becomes inviſible.

Saturn is found to have certain zones or belts, ſomewhat like thoſe of Jupiter. *Dr. Herſchel*, has diſcovered and demonſtrated that Saturn has a denſe atmoſphere; that he has a revolution about his axis; that his axis is perpendicular to the plane of his rings; that his figure like that of the other planets, is that of an oblate ſpheroid, the polar diameter being to the equatorial, as 10 to 11: that his rings have a revolution in their own plane, their axis being the ſame as that of Saturn.

The annual period of Saturn, about the Sun, is near thirty years, or 10759 days, ſeven hours; his diameter is about 67,000 miles, being near $8\frac{1}{2}$ times the diameter of the Earth; his diſtance from the Sun, is about $9\frac{1}{2}$ times, that of the Earth.

From his great diſtance from the Sun, ſome have imagined that the portion of light and heat, derived from the Sun are not ſufficient for animal life. But that they have a greater portion of light, and conſequently heat, than is at firſt imagined, is evident from the brightneſs of this planet, and its ſatellites in the night time. Alſo as the Sun's light to us, is 45,000 times as great as that of the full moon, the Sun will afford 500 times as much light to Saturn, as the

the full moon does to us; and 1600 times as much to Jupiter. Thus, theſe two planets without any moons to enlighten them, would receive more light from the Sun, than might be at firſt imagined, their number of ſatellites alſo, the rings of Saturn, and the nature of their atmoſpheres may alſo have a conſiderable effect, in increaſing their light and heat. For we find that in our Earth, the different degrees of heat, does not entirely depend on the rays of the Sun. The inhabitants alſo of thoſe planets are, no doubt, adapted to their ſituations.

Saturn has ſeven ſatellites, or moons, performing their revolutions round him, in their reſpective periods, as follows;—

Satellites.	Periods				Diſtances in Semidiam of Saturn.	Diſtances in Miles.	Diameters of the Orbits.
	d.	h.	m.	s.			
1	1	21	18	27	$4\frac{3}{8}$	170,000	1′ 27″
2	2	17	41	22	$5\frac{1}{2}$	217,000	1′ 52″
3	4	12	25	12	8	303,000	2′ 36″
4	15	22	41	13	18	704,000	6′ 18″
5	79	7	48	0	54	2,050,000	17′ 4″
6	1	8	53	9	$3\frac{5}{9}$	135,000	1′ 14″
7	0	22	40	46	$2\frac{5}{6}$	107,000	0′ 57″

The four firſt of theſe ſatellites, deſcribe ellipſes, like thoſe of the rings, and are alſo in the ſame plane. Their inclination to the ecliptic is from 30 to 31 degrees. The fifth ſatellite deſcribes an orbit which is inclined from 17 to 18 degrees to the orbit of Saturn; the orbit of this ſatellite lies between the ecliptic, and the orbits of the other ſatellites. *Dr. Herſchel* alſo diſcovered that this ſatellite turns

once round its own axis, in the time that it makes one revolution about the planet Saturn, in which respect it resembles our Moon.

The GEORGIAN SIDUS; or, HERSCHEL, is the most distant of all the primary planets from the Sun, its apparent diameter is about four seconds. It is but seldom he can be seen by the naked eye; but having his situation, he may be plainly seen with a good telescope, in a clear night.

This planet is twice the distance from the Sun that Saturn is, and is nearly eighty-three years performing his annual course. It is ninety times as large as the Earth. The degree of cold, in this planet, is supposed to be extreme, as it is computed that the light of the Sun is not above the three-hundredth part of what we enjoy on Earth.

This planet is attended by two satellites, which perform their revolutions round him as follows:

Satellites.	Periods.	Distances.
	h. m. s.	
1	8° 17 1 19	0′ 33″
2	13 11 5 $1\frac{1}{2}$	0′ $44'\frac{2}{9}$

The orbits of these satellites are nearly perpendicular to the plane of the ecliptic; and by the best observations that have been made, it is propable that their magnitudes are equal, if not greater, than those of Jupiter. The distance here set down is the angular distances of the satellites from their primary planets.

Those are the primary planets which constitute the Solar System, each of them moving, in his own proper orbit, round the Sun as the common centre. The Sun therefore can hardly be considered as a planet, though reckoned as such by the ancient Astronomers; but it may rather be considered

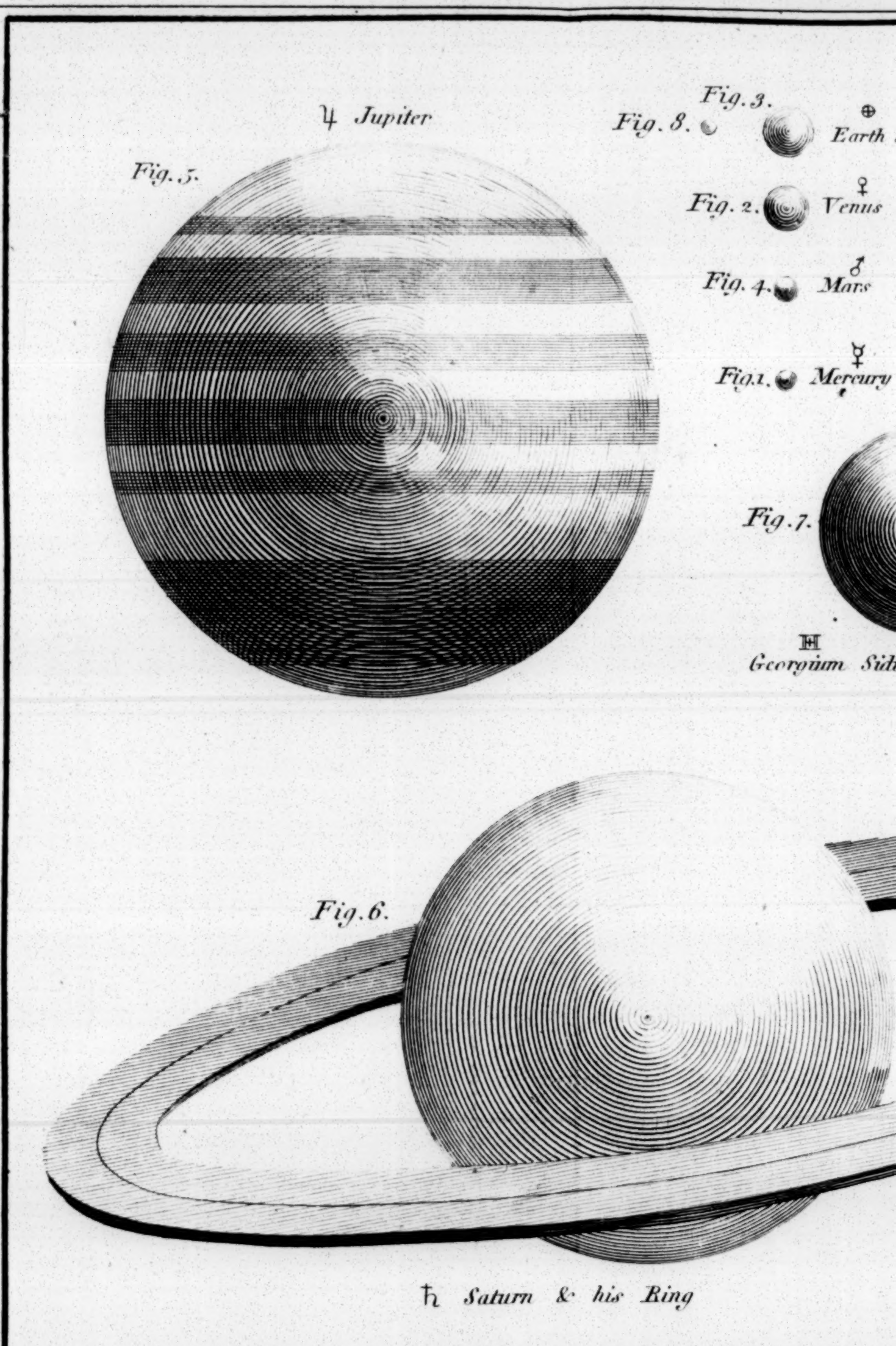
ſidered as a fixed ſtar. He has, in every reſpect, the ſame properties, being a fixed luminous body, imparting light and heat to all the planets, both primary and ſecondary, found in his ſyſtem. The reaſon he appears brighter and larger than any of the fixed ſtars, is on account of his nearneſs to the Earth, in compariſon with the great diſtance of the former. For a ſpectator, placed as near to any fixed ſtar as we are to the Earth, would ſee that ſtar, in every reſpect, as large and as bright as the Sun appears to us; and an obſerver, as far diſtant from the Sun as we are from the neareſt fixed ſtar, would ſee the Sun as ſmall as the ſtar appears to us, and would reckon it as one of the ſtars.

Though the Sun is ſaid to be a fixed body, yet he has a revolution round his own axis, which he performs in the courſe of 27 days, 12 hours, and 20 minutes, which is found, by an obſervation of the ſeveral ſpots, to be ſeen on the Sun's diſc, which paſs from the weſtern edge of his diſc, to the eaſtern edge thereof, in the ſpace of leſs than 14 days. And theſe ſpots are found to perform one entire revolution round the Sun in the ſpace of 27 days, 12 hours, and 20 minutes; therefore, we reaſonably ſuppoſe, that this is the Sun's proper motion from weſt to eaſt, like that of all the planets.

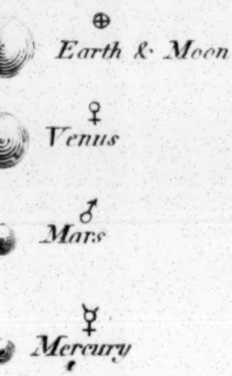
Philoſophers have been greatly divided concerning the matter of the Sun; ſome contending it was a ball of fire, from the property of the Sun's rays acting like fire, when collected by concave mirrours or convex Lenſes. Others, as Boerhaave, maintain the contrary, the particulars of which arguments I have not room to inſert, but the following properties of the Sun is demonſtrated by *Sir Iſaac Newton*;—1. That the denſity of the Sun's heat and light is ſeven times as great on the planet Mercury as it is with us.—2. That the quantity of matter in the Sun is to that in Jupiter nearly as 1100 is to 1; and that the diſtance of that planet from the Sun is in the ſame ratio to the Sun's ſemi-

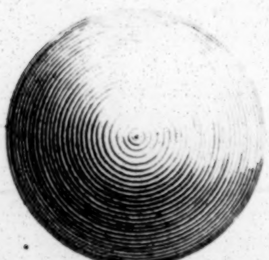
 diameter.—

diameter.—3. That the quantity of matter in the Sun is to that in Saturn as 2360 to 1; and the diſtance of Saturn from the Sun is in a ratio but little leſs than that of the Sun's ſemi-diameter. And hence the common centre of gravity of the Sun and Jupiter is nearly in the ſuperfices of the Sun; and that of the Sun and Saturn, a little within it. 4. Hence the common centre of gravity, of all the planets, cannot be more than the length of the ſolar diameter, diſtant from the centre of the Sun. This common centre of gravity is always at reſt; and though the Sun, by the various poſitions of the planets may be moved every way; yet it cannot recede far from this common centre.—5. The axis of the Sun is inclined to the ecliptic in an angle of 87 degrees, 30 minutes nearly. The Sun's apparent diameter being ſenſibly larger in December than in June, the Sun muſt be proportionably nearer to the Earth in winter than in ſummer. In winter, the Sun will be in the perihelion; in the Summer in the aphelion. This is demonſtrated by the Earth's motion being quicker in December than in June, by about the fifteenth part: for the Earth and every planet deſcribe equal areas in equal times; thus when it moves ſwifter it muſt be nearer the Sun. From this we find that we have about eight days more in real time from the Sun's Vernal equinox to the Autumnal, than from the Autumnal to the Vernal.—6. The Sun's diameter is equal to 100 diameters of the Earth; therefore, the body of the Sun is one million of times greater than that of the Earth.—7. The apparent mean diameter of the Sun is 32 minutes, 12 ſeconds. The Sun's horizontal parallax is now fixed at eight ſeconds, five twentieths of a ſecond.—8. If 360 degrees (the whole ecliptic) be divided by the quantity of time in the ſolar year, it will give 59 minutes, 8 ſeconds, &c. which is the mean quantity of the Sun's diurnal motion: And if this 59 minutes, 8 ſeconds, be divided by 24, the number of hours in a day, the quotient is 2 minutes, 28 ſeconds, which



The Diameter of the Sun according to this Proportion is 22 inches 3/

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...rgium Sidus or new Planet

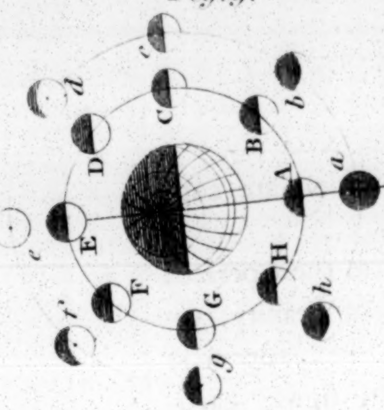
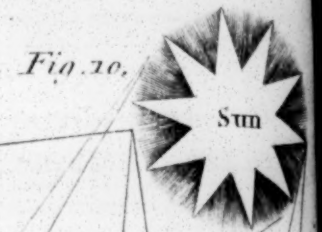


Fig.9.



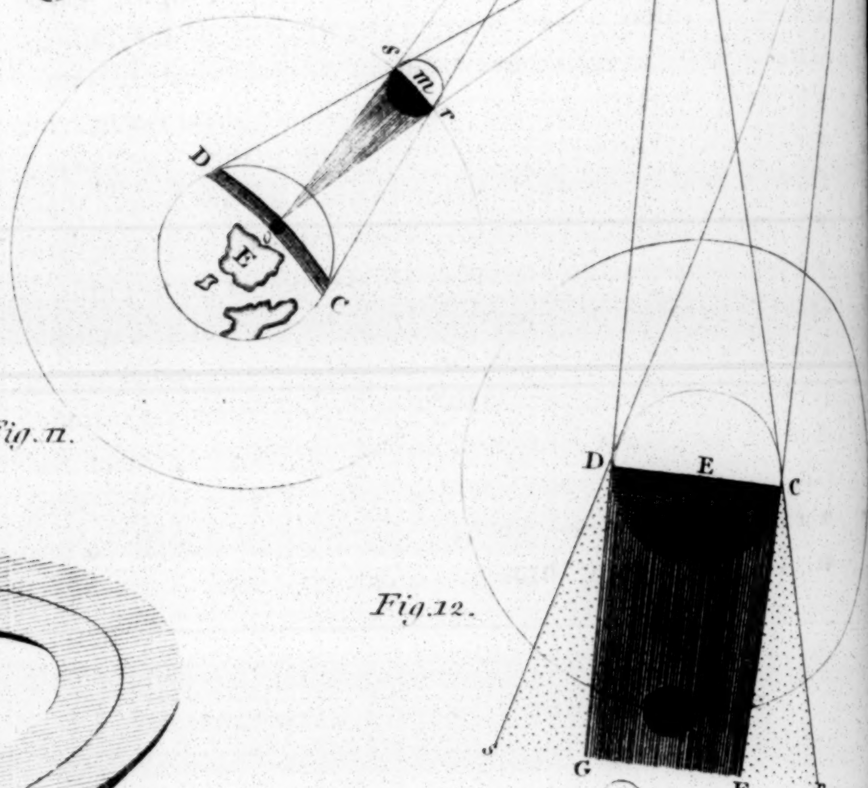
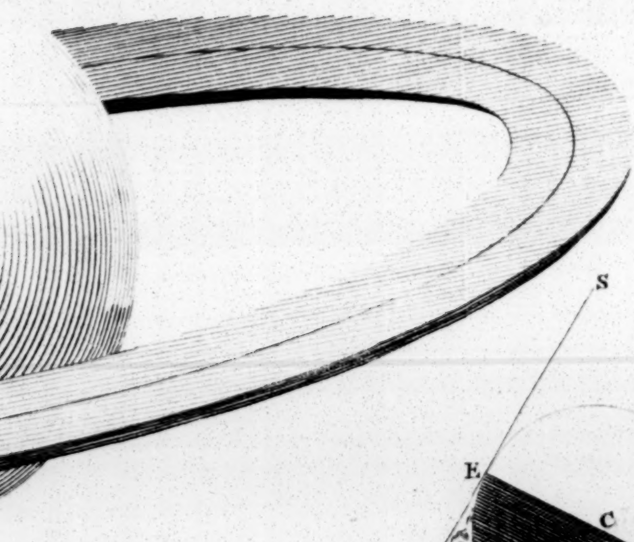


Fig.10.

Fig.11.

Fig.12.


2 inches 3/8

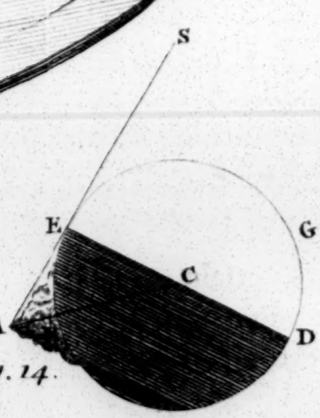


Fig.14.

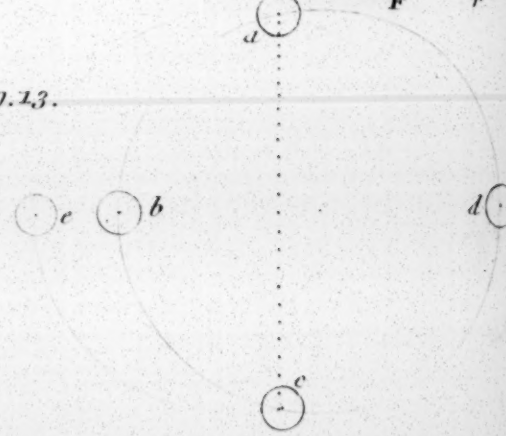


Fig.13.

which is the Sun's motion in one hour; and which, divided by 60, will give his motion in one minute, &c. By this method the Tables of the Sun's mean motion are conſtructed.

Though the planets above deſcribed perform their periods round the Sun, or rather round the centre of gravity, yet many of the planets, ſeen from the Earth, will appear to move in a contrary motion to the order of the ſigns; particularly the inferior planets; and ſometimes they may appear ſtationary, or not to move at all for ſeveral nights together. But theſe appearances are nothing but optical deceptions, ariſing partly from the motions of the planets, and partly from the motion of the Earth on which we are placed; for we always judge a planet to be in that part of the ecliptic which is on the oppoſite ſide of the planet to us; this is called its Geocentric Longitude; but the part of the ecliptic in which the planet is ſeen by an obſerver, ſuppoſed to be placed in the Sun, is called the Heliocentric Longitude. And the longitude of any planet or ſtar is an arch of the ecliptic, counted from the beginning of aries to the place where the ecliptic is cut by a circle perpendicular to the ecliptic, and paſſing through the ſtar or planet.

SECT. II.

OF THE SECONDARY PLANETS.

THE ſecondary planets, or ſatellites, are certain planets which perform a revolution round any other planet, as the Moon does round the Earth. They are called Satellites, becauſe

because they are always found attending their primary planet, and making the tour about the Sun together with it.

There are but four primary planets that are certainly known to have satellites, viz. the Earth, Jupiter, Saturn, and the Georgian Sidus; though some have imagined they have discovered satellites attending some of the other planets as hath been hinted in the last section: but these observations have not been sufficiently confirmed.

The Earth is attended by one satellite, called the Moon, and marked ☾. She performs her revolution round the Earth in an elliptic orbit. The mean eccentricity of which is one eighteenth part nearly of her mean distance from the Earth, or about thirteen thousand miles; her mean distance from the Earth being $60\frac{1}{2}$ semi-diameters of the Earth; or about 240,000 miles.

The mean time of one revolution of the Moon about the Earth, or from one New Moon to another, when she overtakes the Sun again, is 29 days, 12 hours, 44 minutes, 3 seconds, 11 thirds. But the mean time in which she moves once round her whole orbit is 27 days, 7 hours, 43 minutes, 8 seconds, which is at the rate of about two thousand two hundred and ninety miles in an hour. For the Moon has completed one revolution about the Earth before she comes again in conjunction with the Sun; because, while the Moon is performing her revolution the Earth has advanced about a thirteenth part of the ecliptic forward.

The Moon turns once round her own axis exactly in the time that she goes round the Earth. This is the reason the same side of the Moon is always turned towards the Earth: and day and night in the Moon, taken together, are just as long as a lunar month.

The diameter of the Moon is to that of the Earth, as 20 to 73; therefore, it is equal to 2180 miles. The surface of the Moon is to that of the Earth, as 3 is to 40, or as 1 to $13\frac{3}{4}$ nearly; therefore, the Earth reflects thirteen times

as

as much light upon the Moon, as ſhe does upon the Earth, when ſhe is at her full. The ſolid content of the Moon, is to that of the Earth, as 3 is to 146. The denſity of the Moon's body is to that of the Earth, as 5 is to 4; therefore, her quantity of matter is to that of the Earth, as 1 is to 39 nearly. The force of gravity on her ſurface, is to that on the Earth, as 100 is to 293. The axis of the Moon is almoſt perpendicular to the plane of the ecliptic; therefore, ſhe has little or no difference of ſeaſons. The mean apparent diameter of the Moon, is 31 minutes, 16 ſeconds and a $\frac{1}{2}$.

The various phaſes and appearances of the Moon, puzzled all the Aſtronomers of antiquity. Her wanings, and increaſings, her various poſitions, with regard to the Earth, and her frequent eclipſes, were matters of conſtant admiration. The Moon being a dark ſpherical body, and ſhining only with the borrowed light of the Sun, can only have one half of her body illuminated at the ſame time; the oppoſite half remaining in its native darkneſs. Therefore, as the Moon performs a revolution round the Earth, ſhe will ſometimes turn the whole of her illuminated face towards the Earth, at which time, ſhe appears perfectly round, and is a full moon. At other times, only a certain portion of her illuminated face, will be turned towards the Earth, ſhe will then appear either horned, half round, or gibbous, according to the quantity of her illuminated part, which is ſeen by us.

To illuſtrate this, let A B C D E F G H, repreſent the orbit of the Moon, (fig. 9, plate 16.) Now, when the Moon is at A, in conjunction with the Sun, her dark ſide will be turned towards the Earth, and therefore, ſhe will be inviſible, as at *a*, which is then called the new Moon. When ſhe arrives at B, or has run through one eighth part of her orbit, one quarter of her illuminated face will be turned towards the Earth, ſhe will then appear horned, as at

at *b*. When ſhe arrives at C, one half of her illuminated face is turned towards the Earth, as at *c*, when ſhe is ſaid to be in her quadrature. When ſhe arrives at D, which is called her ſecond octant, three parts of her illuminated face will be turned towards the Earth, and ſhe will appear gibbous, as at *d*. When ſhe arrives at E, the whole of her illuminated face is turned towards the Earth, and ſhe appears quite round, as at *e*, when ſhe is ſaid to be a full Moon. As ſhe proceeds through the other half of her orbit, ſhe decreaſes again from *e* to *a*, and nearly in the ſame ratio, as ſhe increaſed, in the former half of her orbit. And the Earth has all the ſame appearances to an obſerver in the Moon, as the Moon has to us, but in a contrary order, viz: the Earth being at the full to them, when the Moon changes to us, and *vice verſa*.

The motions of the Moon, are all very irregular; the only equable motion ſhe has, is the rotation on her own axis, in the ſpace of a month, being the time in which ſhe moves round the Earth; which is the reaſon that ſhe always expoſes the ſame face towards the Earth.

The orbit of the Moon is very changeable, and does not long preſerve the ſame figure; for though the orbit of the Moon, be an ellipſe, having the Earth in one of her focii thereof, yet the eccentricity is ſometimes greater than at other times.

The plane of the Moon's orbit, is inclined to that of the ecliptic, in an angle of five degrees.

The face of the Moon, has the appearance, when viewed through a teleſcope, of being diverſified with hills and vallies; this is alſo proved to be the caſe, from the edge or border of the Moon, appearing jagged, eſpecially about the line which ſeparates the illuminatied part of the Moon, from the dark ſide thereof. The ſpots alſo of the Moon, which are taken for mountains, are found to caſt a triangular ſhadow in the direction oppoſite to the Sun; and thoſe parts

parts, which are taken for vallies or cavaties, are always dark on that ſide next the Sun, and illuminated on the opposite ſide, which is agreeable to experience. Sometimes the tops of the mountains are ſeen illuminated by the Sun, while their baſes are in the dark ſide of the Moon, and by this means, we have a good method of taking the height of the lunar mountains.

Thus, let E D (fig. 14.) be the Moon's diameter, E C D the line dividing the dark from the illuminated part of the Moon; and A the top of a hill in the dark part, juſt beginning to be illuminated: with a teleſcope take the proportion of A E to the diameter E D, then there are given the two ſides A E, E C, of the right angled triangle A E C; the ſquares of the two ſides of the right angled triangle, being added together, gives the ſquare of the hypothenuſe A C, from the ſquare root of which, ſubtracting B C, the radius, there remains A B, the height of the mountain.

From late obſervations, *Dr. Herſchel* has diſcovered that very few of the lunar mountains, exceed half a mile in perpendicular height. The ſame gentleman has alſo obſerved three volcanos in the Moon, which he thus deſcribes. "I perceived, (April 19th, 10 hours, 36 minutes, ſideral time,) three volcanos, in different parts of the dark part of the new Moon. Two of them are either already nearly extinct, or otherwiſe, in a ſtate of going to break out; which, perhaps may be decided next lunation. The third, ſhews an actual eruption of fire or luminous matter. Its light is much brighter than the Nucleous of the comet, which Mr. Mechain diſcovered at Paris, the tenth of this month." The following night he diſcovered it burn more violently; and by meaſuring, he found the ſhining or burning matter, to be more than three miles in diameter. The actual fire, or eruption of a volcano exactly reſembled a ſmall piece of burning charcoal, when it is covered by a very thin coat of white aſhes; and it had a degree of bright-

neſs, about as ſtrong as that with which a coal would be ſeen to glow, in faint day light.

Theſe are the chief phenomena obſervable in the Moon. All the other ſatellites are of a ſimilar nature to this; but from their great diſtance from the Earth, we are unable to be ſo particular in our deſcription of them.

Though it is aſſerted that the Moon, and the other ſecondary planets, revolve round their primary planets, as their centre; and the primary planets revolve round the Sun for their centre; yet it muſt be remembered, that this aſſertion is not the real mathematical truth. For the primary planets do not regard the Sun, as their exact centre; but each primary planet, and the Sun, revolve round their common centre of gravity. Which common centre of gravity, is that point, where the two bodies, or the Sun and planet, will equiponderate each other. Thus, the centre of gravity in a common balance beam, or ſteel-yard, is the point of ſuſpenſion.

To diſcover the common centre of gravity of two bodies, is to find that point, whoſe diſtance from the greater body, is leſs than its diſtance from the leaſt body, in the ſame proportion as the gravity of the leſs body is leſs, than that of the greater. And in two equal bodies their common centre of gravity is equally diſtant from their two centres.

Thus, if B be a body, four times as great in magnitude as the body l, and both be ſuppoſed to be connected by an inflexible wire B l, the common centre of gravity of the two bodies, will be at the point g, which is four times nearer to B, than to l; or as B g is to g l, ſo is l to B.

Therefore, the common centre of gravity of the Earth and Sun, is nearer to the centre of the Sun, than to that

of

of the Earth, by as great a ratio as the quantity of matter in the Sun, exceeds that in the Earth, which centre of gravity is in the body of the Sun. The common centre of gravity of Jupiter and the Sun, is alſo within the body of the Sun, though very near its ſuperficies.

The Sun is not acted upon by our Eart only, ſo as to turn round the common centre of that and the Earth, without having regard to the other planets; but there is a common centre of gravity between the Sun and each of the primary planets; and each of theſe planets have its effect in cauſing the Sun to turn round their centres of gravity.

As the centre of gravity of Jupiter and the Sun, is the fartheſt diſtant from the Sun's centre, (owing to the great ſize of Jupiter, and its diſtance from the Sun,) and as this centre is within the body of the Sun, it follows that the Sun is never removed above one of its own diameters out of its place.

Each of the ſecondary planets, and its primary planet alſo, turn round their common centre of gravity.

The figures 1, 2, 3, 4, 5, 6, 7, and 8, ſhew the true proportions of the planets, Mercury, Venus, Mars, the Earth, Jupiter, Saturn, Georgian Sidus, and the Moon.

Jupiter has four ſatellites the times of whoſe periods and diſtances, have been noticed in the laſt Section. Their periods were found from their conjunction with Jupiter, after the ſame manner as the periods of the primary planets were diſcovered. Their diſtances from the body of Jupiter are meaſured by a micrometer, and computed in ſemi-diameters of Jupiter, and then reduced into miles.

Tht ſatellites of Jupiter, are of very great uſe in aſtronomy. For by an obſervation of the eclipſes of theſe ſatellites, we derive three great advantages. Firſt, In determining the diſtance of Jupiter from the Earth. Secondly, We find the progreſſive motion of light: for by theſe eclipſes,

 it

it is evident that light does not come to us from Jupiter in an inſtant. For, if the motion of light were inſtantaneous, it is evident, we ſhould ſee the commencement of the eclipſes of the ſatellites at the ſame moment they really happen, whatever diſtance they might be from us; but, on the contrary, if light have a progreſſive motion, it is evident, the farther we are from a planet, the later we ſhould be in ſeeing the beginning of its eclipſe, and ſo it is found to happen: the ſatellites of Jupiter appear to be eclipſed later than the true computed time, and always proportionably later, as the Earth is removed farther from the planet. When the Earth and Jupiter are neareſt to each other, that is, when they are both in conjunction, on the ſame ſide of the Sun, then the eclipſes of Jupiter's ſatellites are ſeen to happen ſooner than when the Sun is directly between Jupiter and the Earth; in which laſt caſe, the diſtance of Jupiter from the Earth, is greater than it is in the former caſe, by the whole diameter of the Earth's annual orbit, or by double the Earth's diſtance from the Sun: in this laſt caſe, we cannot obſerve an eclipſe of Jupiter's ſatellites, till near a quarter of an hour after the time, we could have diſcovered it in the former caſe, that is, when Jupiter was at his leaſt diſtance from the Earth. From hence it follows, that the light is near a quarter of an hour in paſſing through the ſpace equal to the diameter of the Earth's orbit, or near eight minutes in paſſing from the Sun to the Earth; which, is at the rate of about twelve million of miles in a minute.

But the third, and greateſt advantage, derived from the obſervation of theſe eclipſes, is the diſcovery of the longitude of different places on the Earth, for having the difference of time between two obſervations of the ſame eclipſe, taken in two different places, we have the difference of longitude between the two places. For example, ſuppoſe two obſervers of an eclipſe, the one at London, and the other, at Barbadoes; the eclipſe will appear at the ſame moment to each

each perſon; but being under different meridians, the hour of the day will be different at each place, if it be 12 o'clock at noon, at London. it will be 8 o'clock in the morning, at Barbadoes, by which the obſervers find the difference of longitude between the two places, to be ſixty degrees, or four hours in time.

The planet Saturn, has ſeven ſatellites, the ſixth and ſeventh of which were diſcovered by *Dr. Herſchel,* in the years 1787 and 1788. Their periods, diſtances, &c. have been deſcribed in the laſt Section.

The Georgian planet or Herſchel, is alſo found to have two ſatellites, revolving round him, like thoſe of Jupiter, and Saturn. Theſe ſatellites were diſcovered by *Dr. Herſchel* alſo, in the year 1787, for which, ſee the laſt Section.

Theſe are the only primary planets, which we are certain, are attended by ſatellites; ſome Aſtronomers, have imagined they diſcovered a ſatellite belonging to Venus, but the many repeated obſervations which have been made by others to obſerve it, and without effect, leave us room to ſuſpect they were deceived.

SECT. III.

OF THE FIXED STARS.

THE Fixed Stars, generally denominated *ſtars*, by way of eminence, are thoſe heavenly bodies, which uſually keep the ſame diſtance, with regard to each other. And all the heavenly bodies, except the primary and ſecondary planets, and comets, are of this claſs.

The

The diſtances of the fixed ſtars, is ſo great, that we have no diſtance in the planetary ſyſtem, with which we can compare them. For the diameter of the Earth's orbit, which is nearly one hundred and ninety million of miles, bears no ſenſible proportion to the diſtances of the neareſt fixed ſtars.

The diſtances of the fixed ſtars, have been the ſubject of inveſtigation, by ſeveral Aſtronomers: various methods have been purſued for this end, but without ſucceſs, on account of their almoſt infinite diſtance; the moſt accurate obſervations, only give us a diſtant approximation; but by the beſt obſervations, however, we can ſafely conclude that the neareſt fixed ſtar, is upwards of forty thouſand diameters of the Earth's orbit diſtant from us, or eighty thouſand times farther diſtant from us, than the Sun is.

The magnitudes of the fixed ſtars appear very different in different ſtars, owing, in ſome ſort, to their real magnitudes, which are different, but principally owing to their different diſtances from us.

The ſtars are generally divided according to their apparent magnitudes, into ſix claſſes. The firſt claſs called Stars of the Firſt Magnitude, are thoſe that appear largeſt. Stars of the Second Magnitude are ſomewhat leſs: and thus every following claſs comprehends thoſe ſtars next in ſize to the former claſs. The ſtars of the Sixth Magnitude, comprehending the ſmalleſt ſtars, viſible to the naked eye. All others that cannot be perceived but by the help of teleſcopes, are called *Teleſcopic Stars*. It is not to be inferred from hence that nature has divided the ſtars into thoſe claſſes; for there are almoſt as many claſſes as there are ſtars, ſo great is their variety of magnitude and brightneſs.

The number of the ſtars is alſo very great, and appears to be almoſt infinite; but Aſtronomers have deduced all that are viſible to the naked eye into catalogues. Mr. *Flamſteed* reduced 3000 ſtars into a catalogue, which contain all that are viſible to the naked eye, at any time of the year, and a great

great number that are only viſible through a teleſcope. The number of ſtars that are viſible, at one time, in the cleareſt heaven, ſeldom exceed one thouſand; their appearing ſo much more numerous, ariſes from their twinkling, and from viewing them confuſedly without reducing them into any order. But a good teleſcope will nevertheleſs bring great numbers to our view, and the more the magnifying power of the teleſcope is encreaſed, the greater will be the number of ſtars diſcovered, till the number becomes ſo great as to baffle our computation.

From the great diſtance of the ſtars, we are at a loſs to diſcover many of their properties. But from their phenomena we can with certainty deduce the following theorems concerning them.—1. That they are much greater than our Earth; for if that were not the caſe they could not be viſible at ſuch a diſtance. 2. They are farther diſtant than the moſt diſtant of the planets; for we often find them hid behind the body of the planets. And 3. They ſhine with their own natural light; for though they be much farther from the Sun than Saturn is, and appear much ſmaller to us, yet they ſhine much brighter than that planet. And it is known, that the more the teleſcope magnifies, the leſs is the angle under which the ſtar is ſeen; becauſe the teleſcope deſtroys all the adventitious rays. Thus a teleſcope magnifying two hundred times, will ſhew a ſtar leſs in magnitude than it appears to the naked eye, inſomuch that it will appear to be only an indiviſible point.

From hence we conclude, that the fixed ſtars are ſo many Suns; and that in all probability the ſtars are not much ſmaller than our Sun.

Therefore, it is generally believed that every ſtar is the centre of a ſyſtem, and has planets revolving round it in the ſame manner as the Earth and primary planets revolve round the Sun. For our Sun, together with the orbits of all the

the planets, would be almoſt inviſible from the neareſt fixed ſtar.

To imagine that the ſtars are only formed to afford us a faint light, would be abſurd, as we have incomparably more light from the Moon than from all the fixed ſtars taken together.

The fixed ſtars have two *apparent* motions, one called the firſt Common, or diurnal Motion; the other called the Second, or proper Motion. The former of theſe motions ariſes from the Earth's motion round its axis; by which the ſtars appear to be carried round the Earth, from eaſt to weſt, in the ſpace of twenty-four hours. The latter is that motion by which they appear to go backwards from weſt to eaſt round the poles of the ecliptic with a very ſlow motion, deſcribing only one degree of their circle in the ſpace of ſeventy-one and a half years. This apparent motion is owing to the preciſion of the equinoxes; or, in other words, the axis of the Earth is directed to the different part of the heavens, every year, deſcribing a circle, one degree of which it deſcribes in ſeventy-one years and a half.

The ZODIAC is an imaginary ring or zone in the heavens, in the ſpace of which all the primary planets revolve in their orbits: Its breadth is reckoned different by different Aſtronomers; but is from eight to ten degrees on each ſide the ecliptic; and is divided into twelve parts, called the Twelve Signs of the Zodiac, and each ſign into thirty degrees. But as the ſtars have a motion from weſt to eaſt, theſe conſtellations, or ſigns of the Zodiac do not now correſpond to their proper ſigns; for the Vernal Equinox formerly happened when the Sun was in the firſt degree of Aries, and the Earth in the oppoſite degree of the Zodiac, or firſt degree of Libra; whereas now, the Sun has advanced a whole ſign from that point at the Vernal Equinox.

The Twelve Signs of the Zodiac are diſtinguiſhed by the following names and characters, viz. ♈ *Aries*, ♉ *Taurus*, ♊ *Gemini*, ♋ *Cancer*, ♌ *Leo*, ♍ *Virgo*, ♎ *Libra*, ♏ *Scorpio*,

pio, ♐ *Sagittary*, ♑ *Capricorn*, ♒ *Aquarius*, ♓ *Pisces*: or, according to the English names, the Ram, the Bull, the Twins, the Crab, the Lion, the Virgin, the Ballance, the Scorpion, the Archer, the Goat, the Water Bearer, and the Fishes.

Besides these constellations in the Zodiac, the stars in every other part of the heavens are reduced into constellations of some figure to which it is supposed each set of stars bears some resemblance. In the Northern Hemisphere are twenty-one constellations, of which the following are the names:—the Little Bear, Great Bear, the Dragon, Cepheus, Bootes, the Northern Crown, Hercules, the Harp, the Swan, Cassiopeia, Perseus, Auriga, Serpentary, the Serpent, the Arrow, the Eagle, the Dolphin, the Horse, Pegasus, Dromeda, and the Triangle. In the Southern Hemisphere are fifteen constellations, viz. the Whale, Orion, the Eridanus, the Hare, the Great Dog, the Little Dog, the Ship, the Hydra, the Cup, the Raven, the Centaur, the Wolf, the Altar, the Southern Crown, and the Southern Fish.

This division was introduced by Ptolomy, and to these Bayer added twelve more, about the Southern Pole, viz. the Peacock, the Tucan, the Crane, the Phoenix, the Dorado, the Flying Fish, the Hydra, the Camelion, the Bee, the Bird of Paradise, the Triangle, and the Indian.

To these Mr. *Royer* has added eleven other constellations, viz. the Giraffe, the River Jordan, the River Tigrus, the Sceptre, and the Flower-de-Luce, being on the north. The following six are on the south part, viz. the Dove, the Unicorn, the Cross, the Great Cloud, the Little Cloud, and Rhomboid.

Hevelius also added the following new constellations, composed of some unformed stars, viz. the Unicorn, the Camelopardalis, the Sextant of Urania, the Dogs, the Little Lion, the Lynx, the Fox and Goose, Sobieski's Crown, the

 Lizard,

Lizard, the Little Triangle, and the Cerberus, to which Gregory has added the Ring, and the Armilla.

Besides the stars in the foregoing constellations, there are a great number of stars not included in any constellation, and therefore called Unformed Stars.

The GALLAXY, or Milky-Way, is that long white luminous tract which seems to encompass the heavens, and is easily seen in a clear night, when the Moon is not up. It is of a considerable breadth, and in some parts double. Its luminious appearance is owing to the great number of small stars with which it is every where bespangled; and with a good telescope may be plainly discovered.

Of Comets.

A Comet is a wandering body, appearing suddenly, and then disappearing; and moves, in its own proper orbit, like a planet.

It is usually attended with a long train of light, called its tail, which is always opposite to the Sun. Comets are divided into three kinds, viz. *bearded*, *tailed*, and *hairy* Comets, which division arises from the different situation of the Comet. Thus, when the comet is eastward of the Sun, and moves from him, it is said to be a bearded Comet, because the light precedes it in the manner of a beard: when the Comet is westward of the Sun, and sets after him, it is said to be a tailed Comet, because the train of light follows it in the manner of a tail; and when the Sun and comet are in opposition to each other, the Earth being between them, the train of the comet is hid behind its body; except the extremities of the train, which being broader than the body of the comet, appear, as it were, round the edges of it like a border of hair, from which it is called a hairy comet.

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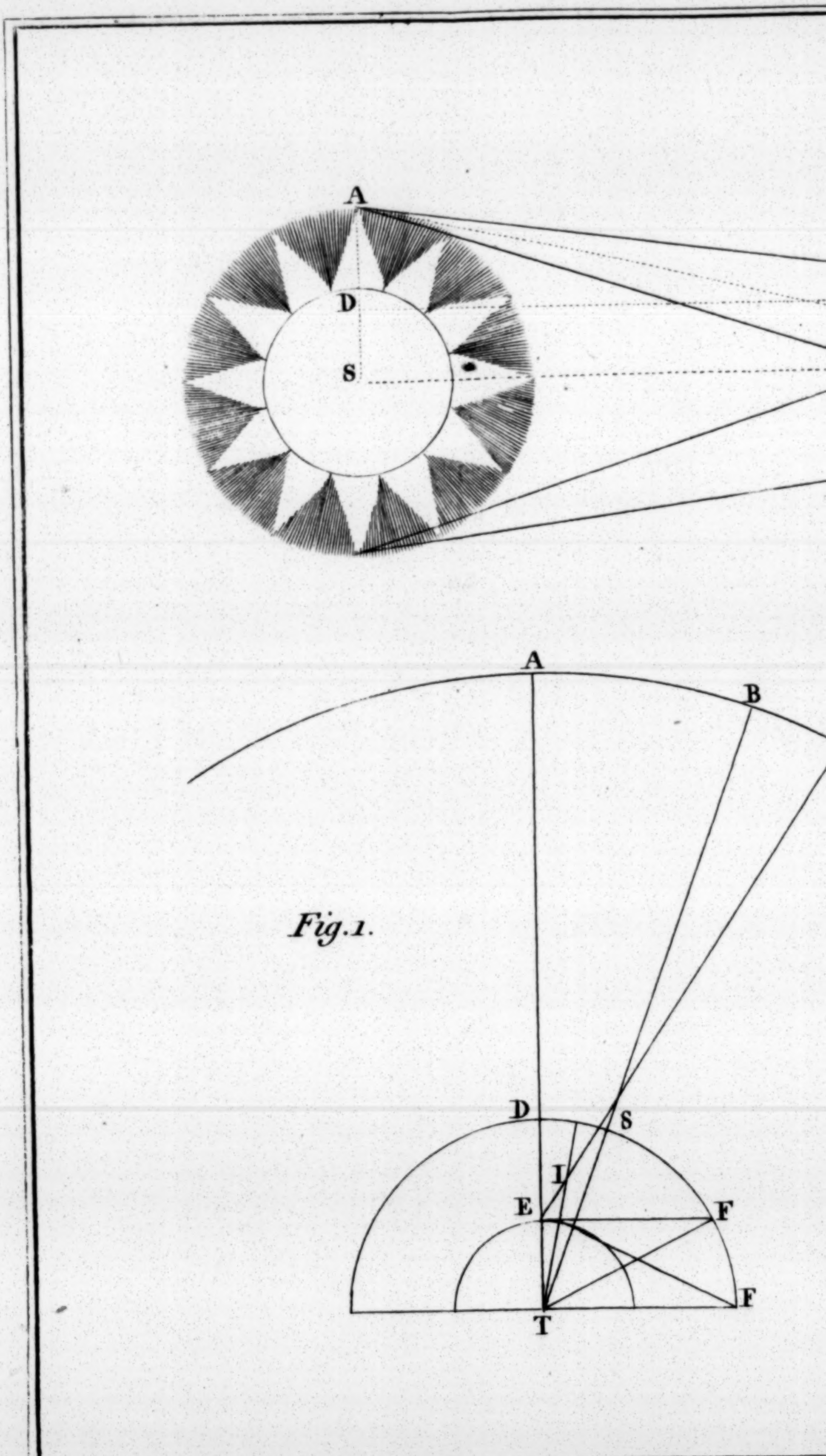

Fig.1.

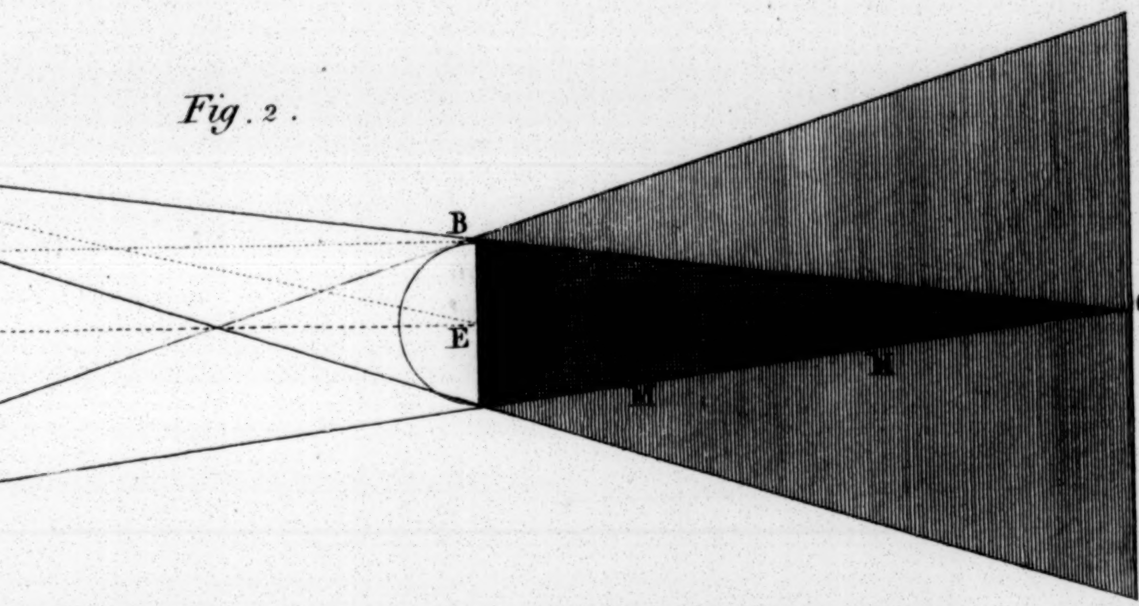

Fig. 2.

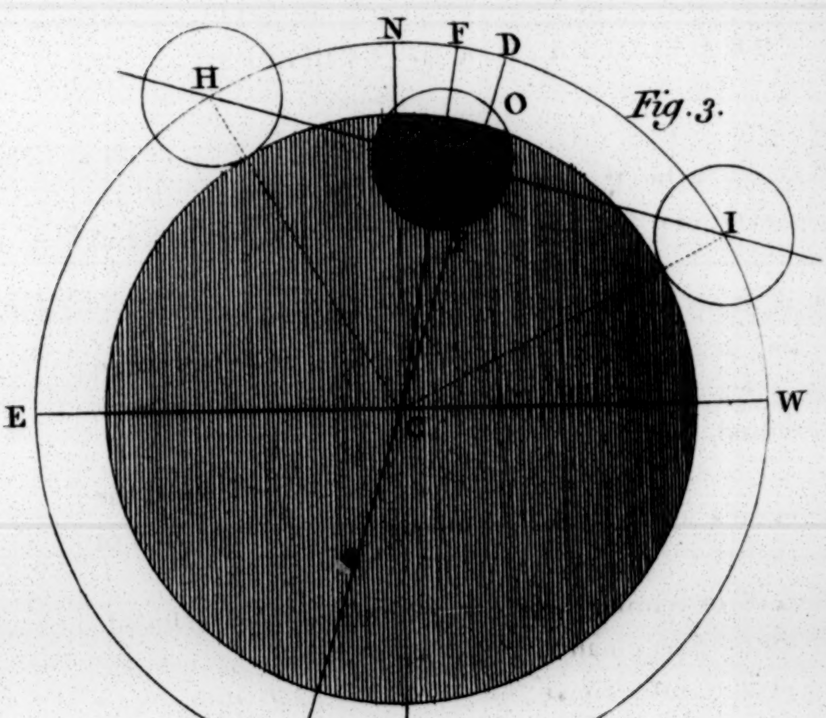

Fig. 3.

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The comets make a part of the ſolar ſyſtem, and move in eliptical orbits, having the Sun in one of their foci, and deſcribe Areas proportional to the times like the planets. The reaſon why they ſometimes appear viſible, and ſometimes not, is owing to the eccentricity of their orbits, which is very great; and when they are in that part of the orbit moſt remote from the Sun, they are much beyond the orbit of Jupiter; and in their perihelion they frequently deſcend within the orbit of Mars, and ſometimes within thoſe of the inferior planets.

SECT. IV.

THE CALCULATION OF ECLIPSES.

THE eclipſe is a privation of the light of one of the luminaries by the interpoſition of ſome opake body, either between the luminary and the eye, or between it and the Sun.

Duration of an eclipſe is the time of its continuance.

Immerſion, or *incidence* of an eclipſe, is the moment when the eclipſe begins; or when part of the luminary firſt begins to be obſcured.

Emerſion, or *Expurgation* of an eclipſe, is the time when the eclipſed luminary begins to re-appear, or emerge, out of the ſhadow.

Quantity of an eclipſe, is the part of the luminary eclipſed. To determine this quantity, the diameter of the eclipſed body, is divided into twelve equal parts, called digits; and the eclipſe is ſaid to be of ſo many digits, as are contained in that part of the diameter, which is eclipſed.

Eclipſes are either thoſe of the Sun, the Moon, or of the other ſatellites, and are either *total, partial, annular, central, &c.*

An *Annular* eclipſe, is when the whole body is eclipſed, except a ring or annulus, which appears round the border or edge.

A *Central* eclipſe, is when the centres of the two luminaries, and the Earth, come in a ſtraight line.

A *Partial* eclipſe, is when only a part of the luminary is eclipſed.

A *Total* eclipſe, is when the whole body of the luminary is darkened.

The eclipſe of the Moon, is a privation of the light of the Moon, and occaſioned by the body of the Earth, being directly between the Sun and the Moon, and ſo intercepting the Sun's rays, that they cannot arrive at the Moon; conſequently, the Moon paſſes through a part of the conical ſhadow of the Earth, as ſeen in fig. 12, where D E C repreſent the Earth, and D G F C the conical ſhadow thereof, in which is the Moon in an eclipſe. The dotted ſpace G D *s*, and F C *r*, ſhew that part of the ſhadow, called the penumbra, in which the Moon is only deprived of part of the Sun's light.

An eclipſe of the Sun, is an obſcuration of the Sun's body, occaſioned by the Moon's coming between the Earth and the Sun, and thus intercepting the light of the Sun from us, on which account, ſome have conſidered it an eclipſe of the Earth.

The *Solar* eclipſe, is repreſented, fig. 11, where *m* repreſents the Moon, C D the Earth, and *r m s o* the Moon's conical ſhadow, travelling over that part of the Earth C *o* D and cauſing a complete eclipſe of the Sun, to all the inhabitants, who reſide in the tract C D. The ſpace C D *r s* includes the penumbra, and all the inhabitants within that ſpace, will perceive a faint ſhadow of the eclipſe.

Hence,

Hence, an eclipſe of the Moon can only happen at the time of the full Moon, or when ſhe is oppoſite to the Sun; and an eclipſe of the Sun, will only take place, at the time of a new Moon, or when the Moon is between the Sun and Earth.

From hence, ſome may imagine that there may be two eclipſes, viz.—One of the Sun, and another of the Moon, in every lunation; which would really be the caſe, if the Moon moved in the ſame plane with the ecliptic; but the orbit of the Moon, not being in the plane of the ecliptic, but inclined thereto, in an angle of five degrees, thirty-one minutes, and paſſing through the plane of the ecliptic, it muſt neceſſarily follow that an eclipſe can only take place when the Moon is at that part of its orbit, which paſſes through the plane of the ecliptic. Theſe two oppoſite points where the Moon's orbit interſects the ecliptic, are called its Nodes. That point where the Moon aſcends from the ſouth to the north ſide of the ecliptic, is called the aſcending node, or dragon's head, marked ☊; and the oppoſite point where the Moon deſcends, from the north to the ſouth ſide of the ecliptic, is called the deſcending node, or dragon's tail, and marked ☋; and a line drawn from one node to the other, is called the line of the nodes. Thus, if (figure 13,) *a b c d*, be the orbit of the Moon, and *e*, the ecliptic, the points *a c*, where the orbit cuts the ecliptic are the two nodes, and the dotted line *a c* the line of the nodes. From a view of the figure, it is plain, when the full or new Moon happens, when the Moon is at the points *b* or *d*, there can be no eclipſe, the ſhadow of the Moon or Earth, falling either above or below the other luminary; but when the full or new Moon is at the points *a* or *c*, or within ſeventeen degrees of theſe points, there will be an eclipſe of one of the luminaries.

In

In order to calculate an eclipſe, it is neceſſary to know how to take the parallax of the Sun, or any heavenly body; as alſo to take the parallactic angle.

The parallactic angle, called alſo the parallax, is the angle E S T, (fig. 1, plate 17,) made at the centre of a ſtar, or other bodies, by two lines, one drawn from the centre of the Earth T, and the other from its ſurface E, or which is the ſame thing, it is the difference of the two angles C E A and B T A.

Parallax, is an arch of the heavens, intercepted between the true and apparent place of any ſtar, or heavenly body.

The true place of a ſtar, S, is that point of the heavens B, where it would be ſeen by an obſerver, placed in the centre of the Earth T. And the apparent place of the ſame ſtar, is the point C in the heavens, where it would appear to an obſerver on the ſurface of the Earth, at E. This difference of the two places of the ſame ſtar, is the parallax, ſometimes called for diſtinction ſake, the parallax of altitude; and is an angle formed by two viſual rays, the one drawn from the centre, and the other from the circumference of the Earth, and traverſing the body of the ſtar; the meaſure of it being an arch of a great circle, intercepted between the points of the true and apparent places, B and C.

The parallax B C is the difference between the true diſtance of the ſtar from the zenith A, and the apparent diſtance A C. Hence the parallax diminiſhes the altitude of a ſtar, or increaſes its diſtance from the zenith.

The parallax is greateſt in the horizon, which is therefore called the Horizontal Parallax, as E F T. From the horizon the parallax decreaſes all the way to the zenith A, where the true and apparent places of the ſtar coincide.

The parallax of the annual orbit of the Earth is the angle under which the ſemi-diameter of the Earth's orbit is ſeen.

To

To find the parallax of a celestial body, observe when the body is in the same vertical line with a fixed star which is near it; and while it is in that position measure its apparent distance from the star; then observe when the star and body are at equal altitudes from the horizon, and there measure their distances again; and the difference of these distances will be the parallax.

The Astronomy of Eclipses.

To calculate a lunar eclipse it is necessary first, to find the length of the Earth's conical shadow, which may be found by finding the distance between the earth and Sun, and the proportion of their diameters. Thus, suppose the semi-axis of the Earth's orbit to be ninety-five million miles, and the eccentricity of the orbit one million, three hundred and seventy-seven thousand miles, which, added together, make 96,377,000 miles, or 24,194 semi-diameters of the Earth; and the Sun's semi-diameter being to that of the Earth as 112 to 1, then, as A D is to B E, so is D B to E C (fig. 2.) that is, as 111 is to 1, so is 24194 to 218 semi-diameters of the Earth, equal to E C, the length of the Earth's shadow.

To find the apparent semi-diameter of the Earth's shadow, in the place where the Moon passes through it, add together the parallaxes of the Sun and Moon, and from the sum subtract the apparent semi-diameter of the Sun, and the remainder will be the apparent semi-diameter of the shadow at the place where the Moon passes through it.

NOTE. The Sun's parallax may very well be omitted in this calculation. And the apparent semi-diameter of the shadow increased by adding one whole minute.

It is also necessary to have the true distances of the Moon from the node at the mean opposition; also the true time of the opposition, with the true place of the Sun and Moon reduced

reduced to the ecliptic; and the Moon's true latitude at the time of the true oppoſition; likewiſe the angles of the Moon's way with the ecliptic, and the true horary motions of the Sun and Moon; from which every particular concerning the eclipſe may be computed by common Arithmetic and Trigonometry.

The method of conſtructing an eclipſe of the Moon is as follows: Let E W repreſent a part of the ecliptic, C the centre of the tranſverſe ſection of the Earth's ſhadow. Draw the line C N, (fig. 3.) perpendicular to the ecliptic, and towards the north, if the Moon have north latitude; but if ſhe have ſouth latitude, draw a line C S. Make the angle N C D equal to five degrees, thirty-five minutes, which is the angle the Moon's orbit makes with the ecliptic. Biſect this angle by the right line C F, and in this line the true equal time of oppoſition between the Sun and Moon, falls by the tables.

Take the Moon's latitude at the true time of full Moon from the ſcale of equal parts, which is ſuppoſed to repreſent minutes of a degree, and ſet this diſtance from C to G on the line C F. Through the point G draw a line H I at right angles to C D, which line repreſents a portion of the Moon's orbit. Then L is the point where the Moon's centre is at the middle of the eclipſe G; the place of her centre at the tabular time of her being full; and K, the point of her centre at the inſtant of her ecliptic oppoſition; I is the Moon's centre at the moment of her immerſion, and H her centre at the end of the eclipſe.

From the ſame ſcale take the Moon's ſemi-diameter and deſcribe three circles on the points I L H which repreſent the Moon in the beginning, middle, and end of the eclipſe.

Then, to find the length of time of the duration of the eclipſe, meaſure the line I H on the ſame ſcale, and ſay as Moon's horary motion from the Sun is to H I, ſo is one hour or ſixty minutes to the whole duration of the eclipſe.

From

From the above figure the eclipſes may be computed: For, firſt, in the right angled triangle C G L, there is given the hypothenuſe C G, which is the Moon's latitude at the time of the Full Moon: Alſo we have the angle of G C L equal to the half of 5 degrees, 35 minutes, wherefore the legs C L and G L may be found. Secondly, in the right angled triangle C H L, or C I L, are given the legs C L, and CH or C I, the ſum of the ſemi-diameter of the Moon and the Earth's ſhadow; therefore L H or L I may be found, which is equal to half the difference of the motions of the Sun and Moon during the eclipſe. Thirdly, as the difference of the horary motions of the luminaries is to one hour, or ſixty minutes, ſo is H L to the ſemi-duration of the eclipſe; and ſo is G L to the difference between the oppoſition and middle of the eclipſe. This laſt, therefore, taken from the time of Full Moon, gives the time of the middle of the eclipſe; from which, ſubtracting the time in L I, before found, gives the beginning of the eclipſe; and added to the ſame, gives the end of the eclipſe.

Laſtly, from C O, the ſemi-diameter of the Earth's ſhadow ſubtract C L, and the remainder L O, added to L P, gives O P the quantity eclipſed.

A Solar Eclipſe, or an eclipſe of the Sun, would be diſcovered in the ſame manner as a Lunar Eclipſe, if the Moon had no ſenſible parallax; but as the Moon has a parallax, the method is ſomewhat different.

1. Add the apparent ſemi-diameters of both Sun and Moon together both in the aphelion and perihelion, which gives thirty-three minutes ſix ſeconds for the greateſt ſum, and thirty minutes thirty-one ſeconds for the leaſt ſum.

2. As the parallax diminiſhes the northern latitude, and increaſes the ſouthern, let the greateſt parallax in the latitude be added to the former ſums, and alſo ſubtracted from them: thus may be had, in each caſe, the true latitude, beyond which there can be no eclipſe. Having this latitude, the

Moon's diſtance from the nodes are found in the ſame manner as for the lunar eclipſe.

To find the digits eclipſed, add the apparent ſemi-diameters of the luminaries together; from which ſubtract the Moon's apparent latitude, the remainder is the part of the diameter eclipſed. Then ſay, as the ſemi-diameter of the Sun is to the ſcruples eclipſed; ſo are 6 digits reduced into ſcruples, that is 360 ſcruples, or minutes, to the digits eclipſed.

To determine the duration of a ſolar eclipſe, find the horary motion of the Moon from the Sun for an hour before the conjunction, and one hour after it. Then ſay, as the former horary motion is to the ſeconds in an hour, ſo are the ſcruples of half the duration, to the time of immerſion; and as the latter horary motion is to the ſame number of ſeconds, ſo are the ſcruples of half duration to the time of emerſion. Then adding the times of immerſion and emerſion together, the ſum is the whole duration.

To find the beginning, middle, and end of a ſolar eclipſe, find the arch G L from the Moon's latitude for the time of conjunction. Then ſay, as the horary motion of the Moon from the ſun before the conjunction is to one hour; ſo is the diſtance of the greateſt darkneſs to the interval of time between the greateſt darkneſs and the conjunction. Subtract this interval in the firſt and third quarter of the anomaly from the time of the conjunction; but in the other quarters add it to the ſame; and the reſult is the time of the greateſt darkneſs. Laſtly, from the time of the greateſt darkneſs, ſubtract the time of incidence, to which is to be added, the time of emerſion; the difference in the firſt caſe will be the beginning; and the ſum, in the latter caſe, will be the end of the eclispe.

To calculate eclipſes of the Sun, it is neceſſary, firſt, to find the mean New Moon, and from thence the true one, with the place of the luminaries for the apparent time of the

the true one.—2. For the apparent time of the true New Moon, compute the apparent time of the New Moon obſerved.—3. For the apparent time of the New Moon ſeen, compute the latitude ſeen.—4. Thence determine the number of digits eclipſed.—5. Find the times of the greateſt darkneſs, emerſion, and immerſion.—6. And from thence determine the beginning and ending of the eclipſe.

From this it is evident, that the trouble in the calculation ariſes from the parallaxes of longitude and latitude, without which, the calculation of ſolar eclipſes, would be the ſame as thoſe of lunar ones.

SECT. V.

OF TIME.

TIME, is a mode of duration, marked by certain periods, meaſures and motions; and the chief method we have of meaſuring time, is by the revolution of the two luminaries, the Sun and Moon, and particularly by that of the Sun.

Mr. Locke obſerves, that the idea we have of time, is acquired by conſidering any part of infinite duration, as ſet out by periodical meaſures. The idea of any particular time, or length of time, as a day, an hour, &c. is acquired by obſerving certain appearances of ſome bodies, moving with a regular motion, and at regular and ſeemingly equidiſtant periods. Now, by being able to repeat theſe lengths or meaſures of time, as often as we pleaſe, we can imagine

duration where nothing really endures or exiſts, and thus, we imagine to-morrow, or next year, &c.

Time is alſo the duration of a thing, which has both a beginning and an end, and in this ſenſe, it is diſtinguiſhed from eternity.

Thus, time is the duration of ſome motion, for without ſome regular and uniform motion, we ſhould have no methods to compute time, or diſtinguiſh it from eternity.

Time, may be divided into *abſolute* and *relative*.

Abſolute time, is time conſidered in itſelf, and without any relation to motion.

Relative, or *apparent* time, is the ſenſible meaſure of any duration, by means of motion, as by the motions of the luminaries, the hand of a clock, watch, &c.

Relative time, is ſub-divided into *aſtronomical* and *civil*.

Aſtronomical time, is that, which is meaſured by the motions of the heavenly bodies.

Civil time, is formed for civil purpoſes, and diſtinguiſhed into years, months, days, hours, &c.

The *Year*, in the full extent of the word, is a ſyſtem of ſeveral months, or a ſpace of time, meaſured by the revolution of ſome celeſtial body in its orbit. Thus, the time in which the fixed ſtars make one revolution, is called the *great year;* and the times in which Jupiter, Saturn, &c. complete their revolutions, and return to the ſame point again, are reſpectively called the years of Jupiter and Saturn, &c. For a year, originally denoted a revolution, and is not limited to that of the Sun; therefore, we find ſome ancient nations at different times, called the revolutions of the Moon, or the ſpace of a month, a year, which occaſioned ſuch ſtrange accounts in the chronology of ſome very ancient nations, as the Egyptians, Babylonians, &c.

The *Solar Year*, called alſo year, by way of eminence, is the ſpace of time, in which the Sun moves through the twelve ſigns of the ecliptic. This year, by the beſt obſervations,

vations, is found to contain 365 days, 5 hours, 48 minutes, 48 feconds; but the quantity according to the authors of the Gregorian calender, is 365 days, 5 hours, 49 minutes. In the civil account, this year is faid to contain only 365 days; and one day is added to every fourth year, to make up for the odd hours, which is therefore, called leap year.

The Solar Year, is either Aftronomical or civil. The Aftronomical Solar Year, is that which is determined precifely by aftronomical obfervations, and is of two kinds, viz. *tropical*, and *fideral* or *aftral*.

The tropical or natural year, is the time the Sun takes to pafs through the twelve figns of the zodiac, and is the only proper natural folar year, becaufe the feafons always fall in the fame months. The *fideral* or *aftral* year, is the fpace of time, the Sun takes in paffing from any fixed ftar, till his return to the fame again; and is 20 minutes, 29 feconds, longer than the true folar year.

The *Lunar* year, is the fpace of twelve lunar months, and is either aftronomical or civil.

The *Lunar Aftronomical* year, confifts of twelve lunar fynodical months; and is therefore, 354 days, 8 hours, 48 minutes, 38 feconds; being 10 days, 21 hours, 10 feconds, fhorter than the folar year.

The *Lunar Civil* year, is either common or embolifmic. The common lunar year, confifts of twelve lunar civil months, and contains 354 days. The embolifmic lunar year, confifts of thirteen lunar civil months, and contains 384 days.

The *Civil*, or *Legal* year, in England, formerly began on the twenty-fifth of March, or the day of the annunciation of the Virgin Mary; but the hiftorical year, begun on the firft of January. The part of the year between thefe two terms, was ufually expreffed thus, 1735—6, or $173\frac{5}{6}$. But according to the new ftile, the civil year now begins, on the firft of January.

The

The ancient Roman year, as first settled by the Romans, contain only ten months, and in all 304 days.

The *Julian* year, so named from Julius Cæsar, who established it, consists of 365 days, 6 hours, which exceeds the true solar year by upwards of 11 minutes, which excess amounts to a whole day in near 131 years. And one day is added to the end of February every fourth year, which is composed of the odd six hours every year. This year is, therefore, called Bissextile, or Leap-Year.

The *Gregorian Year*, introduced by Pope Gregory the thirteenth, in 1582, is the Julian year, corrected by this rule, viz. That instead of every hundredth year being a leap year, as it would be in the Julian Calendar, in this way, only one hundredth year out of four is a leap year, the other three being common years. By this omission of three days in four hundred years, the civil year would nearly keep pace with the solar year for time to come.

In the year 1752, this stile was adopted in England, and the eleven days were thrown out after the second of September, by accounting the third the fourteenth of that month, This was called the New Stile in distinction from the former, which was called the Old.

Yet this year is not quite perfect, for, as in four centuries the Julian year gains three days, two hours, forty minutes; and as there are only three days omitted in the Gregorian account, there is still an excess of two hours, forty minutes in four hundred years, which amounts to a whole day in three thousand six hundred years.

The *Egyptian* year, called also the Year of Nabonassar, from the epoch of that name, contains only 365 days divided into twelve months of 30 days each, with 5 intercalary days added at the end. Thus the Egyptian year loses a whole day of the Julian year every four years, and after the space of 1460 years, it begins with the Julian year, which length of time is called the Sothic Period.

The

The *ancient Greek Year*, conſiſted of twelve months, which at firſt was divided into 30 days each, but afterwards each month contained 29 and 30 days alternately; and is computed from the firſt appearance of the New Moon, with the addition of an emboliſmic month of 30 days, every 3d, 5th, 8th, 11th, 14th, 16th, and 19th year, in order to keep the New and Full Moons to the ſame ſeaſons of the year.

The *Ancient Jewiſh* year conſiſted of 12 months, containing 29 and 30 days alternately. To which was added 11 or 12 days to make it agree with the ſolar year.

The *Syrian* year is the ſame in quantity as the Julian year, but beginning in the beginning of October, according to the Julian year.

The *Perſian* year contained twelve months, of thirty days each, with five intercalary days added.

The *Arabic*, *Mahometan*, or *Turkiſh* year; called alſo the Year of the Hegira, conſiſts of 354 days, 8 hours, 48 minutes, divided into 12 months, containing 29 and 30 days alternately; though ſometimes it contains 13 months; and intercalary days alſo added every 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21ſt, 24th, 26th, and 29th year. The months commence with the firſt appearance of the New Moon.

The year is divided into 12 parts, called months, from the Moon, by whoſe motions it was regulated; and is properly the time in which the Moon paſſes through the Zodiac, and is of ſeveral kinds, as—

1. The illuminative month, which is the interval between the appearance of one New Moon and that of the next; and always varies in quantity. This month is uſed by the Turks and Arabs.

2. The Lunar periodical month, or the exact time in which the Moon runs through the Zodiac, and conſiſts of 27 days, 7 hours, 43 minutes, and 8 ſeconds.

3. *Lunar-ſynodical* month, called the Lunation, is the time

time between two New Moons, as ſeen from the Earth, and conſiſts of 29 days, 12 hours, 44 minutes, 3 ſeconds, and 11 thirds.

4. The *Solar* month, is the time the Sun runs through one ſign of the ecliptic, and conſiſts at a mean rate of 30 days, 10 hours, 29 minutes, 5 ſeconds.

5. *Civil*, or *Common* month, is an interval of a certain number of whole days—ſuch are the calendar months.

6. *Civil Lunar* months conſiſt, alternately, of 29 and 30 days. Thus two of theſe months are equal to two aſtronomical months, and the New Moon will be kept to the firſt day of ſuch civil months for a long time together. This month was in moſt common uſe till the time of Julius Cæſar.

7. The *Civil Solar* month, which conſiſted, alternately, of 30 and 31 days, excepting one month, which had 29 days, introduced by Julius Cæſar. But, under Auguſtus, the ſixth month, till then called *Sextilis*, received the name of Auguſtus, from thence called Auguſt; and one day more added to it, which was taken from February. This is the regular civil month in uſe in England.

A week is a ſpace of time containing ſeven days, and originated from the diviſion of the lunar month into four parts.

This diviſion into weeks was uſed by the Syrians, Egyptians, and moſt of the Oriental nations. The Romans week conſiſted of nine days; and the ancient Greeks uſed decades, or a ſyſtem of ten days.

But the Jews uſed the week of ſeven days. The days of the week they denominated, the firſt, ſecond, third, fourth, fifth, and the ſixth day they called the preparation of the Sabbath, the Sabbath being the ſeventh day: and this method is ſtill obſerved by the Chriſtians, Arabs, Perſians, Ethiopians, &c.

The

The ancient heathens, denominated the days of the week from the ſeven planets, calling each day after that planet which they ſuppoſed governed the firſt hour of the day. Thus, the firſt day was called *Dies Solis;* or *Sunday;*—the ſecond, *Dies Lunæ;* or *Monday*, &c.

But our Saxon anceſtors, before their converſion to Chriſtianity, named the days of the week from the Sun and Moon, and ſome of their deified heroes, to whom they were peculiarly conſecrated, and repreſented the ancient Gods or planets; which names we ſtill retain. Thus, Sunday was devoted to the Sun; Monday to the Moon; Tueſday to Tuiſco, or Mars; Wedneſday to Woden, or Mercury; Thurſday to Thor, the Thunderer, or Jupiter: Friday to Friga, or Friya, the wife of Thor, or Venus; and Saturday to Seater, or Saturn. And the days of the week are often expreſſed by modern Aſtronomers by the characters of the planet, as, ☉ for Sunday; and ☽ for Monday, &c.

The day is that ſpace of time which ariſes from the appearances or diſappearances of the Sun, and is either natural or artificial.

The natural day is the portion of time in which the Sun performs one revolution round the Earth apparently; that is, the time in which the Earth makes a rotation on its own axis.

The artificial day is the time from Sun riſing to Sun ſetting.

The natural day is either aſtronomical or civil.

The *Aſtronomical* day begins at noon, or when the Sun's centre is on the meridian, and is counted at 24 hours to the following noon.

The *Civil* day is the time allotted for the ſpace of a day in civil purpoſes, and includes one entire rotation of the Earth on its axis. This day begins at different times in different nations. It begins at Sun riſing among the ancient Babylonians, Perſians, Syrians, and moſt other Eaſtern nations, and the preſent inhabitants of the Balearic Iſlands,

 the

the Greeks, &c. It begins at Sun ſetting among the ancient Athenians, and Jews; alſo with the Auſtrians, Bohemians, Marcommanni, Sileſians, Modern Italians, and Chineſe. At noon with all modern Aſtronomers and the ancient Umbri, and Arabians; and at midnight among the ancient Egyptians, Romans, and with the modern Engliſh, French, Dutch, Germans, Spaniards, and Portugueſe.

The natural day is divided into twenty-four parts, called Hours; but ſometimes it is divided into twelve parts only. With us it is the twenty-fourth part of the Earth's diurnal rotation on its axis, and anſwers to fifteen degrees of longitude on the equator.

There are various kinds of hours, as, 1. Equal hours, which are the twenty-fourth part of a natural day. They are ca[illegible] equinoctial hours, becauſe they are meaſured on the equinoctial; and aſtronomical, becauſe uſed by Aſtronomers.—2. Babyloniſh hours, which are twenty-four equal hours in the day, and reckoned from Sun riſing.—3. European hours, uſed in civil computations, and are reckoned from midnight; 12 hours from thence till noon, and 12 more from noon to midnight.—4. Jewiſh, or planetary, or ancient hours, which are the twelfth parts of the artificial day, and the ſame parts of the artificial night. They are called ancient or Jewiſh hours, becauſe uſed by the ancients, and ſtill uſed by the Jews. They are called Planetary hours, becauſe ancient Aſtrologers pretended that a new planet predominated every hour.—5. Italian hours, which are twenty-four equal hours to a day, reckoning from Sun ſet.

The hour is divided into ſixty minutes; and each minute ſub-divided into ſixty ſeconds, and each ſecond into ſixty-thirds, &c.

As time for the purpoſes of chronology is calculated by years, it is neceſſary to have ſome certain fixed point of time from which calculations can be made with certainty, which fixed point of time is called an *Epocha*, or *Epoch*.

Different

Different nations uſe different epochs or areas; the Chriſtians chiefly uſe that of the nativity of Jeſus Chriſt.—The Mahometans, that of the Hegira, or flight of Mahomet—The Jews, that of the creation of the World, or that of the Deluge.—The ancient Greeks, that of the Olympiads.—The Romans, that of the building of Rome.—The ancient Perſians and Aſſyrians, that of Nabonaſſar, &c.

The doctrine and uſe of epochs, is of great importance in chronology. And to find what year of one epoch, correſponds with that of another; a period of years has been invented, which commenced before all the epochs, and is a common ſtandard of them all, and called the Julian period. To this period, all the epochs are reduced; that is, the year of this period, when each epoch commences is determined. Thus, adding the given year of one epoch, to the year of the period, correſponding with its beginning, and from the ſum, ſubtracting the year of the ſame period, correſponding to the other epoch, the remainder is the year of that other epoch.

A TABLE

A TABLE,

OF THE

Years of the most remarkable Epochs.

N. B. The years before Christ, are those before the reputed year of his birth, and not reckoned back from the first year of his age, as is usually done.	Julian Period.	Years of the World.	Years before Christ.
Creation of the World - - - -	706	0	4007
The Deluge, or Noah's Flood - - - -	2362	1656	2351
Assyrian Monarchy, founded by Nimrod -	2537	1831	2176
Kingdom of Athens, founded by Cecrops	3157	2451	1556
Entrance of the Israelites into Canaan -	3262	2556	1451
The Destruction of Troy - - -	3529	2823	1184
Solomon's Temple founded - - -	3701	2995	101[illegible]
The Argonautic Expedition - - -	3776	3070	937
Lycurgus formed his Laws - - -	3829	3103	884
Arbaces, first King of the Medes - -	3838	3132	875
Olympiads of the Greeks began - -	3938	3232	775
The Building of Rome - - - - -	3961	3255	752
Area of Nabonasser - - - - -	3967	3261	746
First Babylonish captivity, by Nebuchadnezzar	4107	3401	606
The Second Baba. captivity and birth of Cyrus	4114	3408	599
Solomon's Temple destroyed - - -	4125	3419	58[illegible]
Cyrus began to reign in Babylon - -	4177	3471	536
Peloponnessian War began - - -	4282	3576	431
Alexander the Great, died - - -	4390	3684	323
Captivity of 100,000 Jews, by Ptolemy - -	4393	3687	320
Archimedes killed at Syracuse - - -	4506	3800	207
Julias Cæsar invaded Britain - - -	4659	3953	54
He corrected the Calendar - - -	4667	3961	46
The true year of Christ's birth - - -	4709	4003	4

The

The Christian Area.	Julian Period.	Years of the World.	Years since Christ
Dionysian, or vulgar Area of Christ's birth	4717	4007	0
Christ Crucified, Friday, April 3d. -	4746	4040	33
Jerusalem destroyed - - - - - -	4783	4077	70
Adrian's Wall built in Britain - -	4833	4127	120
Dioclesian Epoch of Martyrs - -	4997	4091	284
The Counsel of Nice - - - - -	5038	4332	325
Constantine the Great, died - - -	5050	4344	337
The Saxons invited into Britain - - -	5158	4452	445
Hegira, or flight of Mahomet - - -	5335	4629	622
The Death of Mahomet - - -	5343	4637	630
The Persian Yesdegird - - - - -	5344	4638	631
The Moon, and all the primary planets seen in the sign Libra, from the Earth -	5899	5193	1186
Art of Printing discovered - - -	6153	5447	1440
The Reformation began, by Martin Luther -	6230	5524	1517
The Calendar corrected by Pope Gregory -	6295	5589	1582
Oliver Cromwell died - - - - -	6371	5665	1658
Sir Isaac Newton born, December 25th -	6355	5649	1642
Made President of the Royal Society -	6416	5710	1703
Died March 20th - - - - - -	6440	5734	1727
The New Planet discovered by Dr. Herschel -	6494	5788	1781

SECT. VI.

OF ASTRONOMICAL PROBLEMS, AND THE USE OF THE GLOBES.

THE Celestial Globe differs from the Terrestial one in having the images of the constellations and figures of the stars upon it, instead of the several parts of the Earth. The

The meridian circle drawn through the poles and the point Cancer repreſents the ſolſtitial colure; and that meridian drawn through the point Aries, repreſents the equinoctial colure.

PROBLEM I.

To exhibit the true representation of the face of the heavens at any given time and place.

Rectify the globe for the latitude of the place (as taught in Geography) placing the north pole of the globe towards the north pole of the world. Having found the Sun's place in the ecliptic, and brought it to the meridian, there ſet the index to 12 o'clock at noon; then turn the globe on its axis till the index points to the given hour of the preſent time. In this poſition the globe exactly repreſents the face of the heavens as it appears at that time, every conſtellation and ſtar in the heavens correſponding in ſituation to thoſe on the globe.

PROBLEM II.

To find the declination and right ascension of any star.

Bring the ſtar to the brazen meridian; and the number of degrees on the meridian between the equator and the ſtar is its declination, and the degree of the equator, cut by the meridian, is the right aſcenſion of the ſtar. Thus the right aſcenſion of any ſtar is an arch of the equator, intercepted between the firſt degree of Aries, and that point where the meridian or circle paſſing through the ſtar cuts the equator.

PROBLEM III.

To find the latitude and longitude of any star.

Bring the ſolſtitial colure to the meridian, and there fix the quadrant of altitude over the pole of the ecliptic in the ſame

ſame hemiſphere with the ſtar, and bring its graduated edge to the ſtar; then the degree on the quadrant, cut by the ſtar, is its latitude, counted from the ecliptic; and the degree of the ecliptic, cut by the quadrant, is its longitude.

PROBLEM IV.

To find the place of any star or heavenly body, having its declination and right ascension.

Find the point of right aſcenſion on the equinoctial, and bring it to the meridian; then count the degrees of declination upon the meridian from the equinoctial, and there make a mark on the globe, which will be the place of the ſtar, &c.

PROBLEM V.

To find the place of a star, planet, comet, &c. having the latitude and longitude.

Bring the pole of the ecliptic to the meridian, and there fix the quadrant of altitude, which turn round till its edge cut the given longitude on the ecliptic; then count the given latitude from the ecliptic upon the quadrant of altitude, and there make a mark upon the globe, which will be the place of the ſtar, planet, &c. The place of any ſtar, planet, &c. being found by this or the foregoing problem, its riſing, ſetting, or any other circumſtance concerning it, may be found by the proper problems, as thoſe of the ſun are found.

PROBLEM

PROBLEM VI.

To find the rising, setting, and culminating of a ſtar or any celestial body, with its continuance above the horizon for any place and day; also its oblique ascension and descension, with its eastern and western amplitude and azimuth.

Adjuſt the globe to the ſtate of the heavens, at 12 o'clock at noon, on the given day. Bring the ſtar, &c. to the eaſtern ſide of the horizon, which will give its eaſtern amplitude and azimuth, and the time of riſing, as for the Sun. Again, turn the globe till the ſame ſtar come to the weſtern ſide of the horizon; ſo will the weſtern amplitude and azimuth with the time of ſetting be found. Then the time of riſing ſubtracted from that of ſetting, leaves the continuance of the ſtar above the horizon; which ſubtracted from twenty-four hours, leaves the time it is below the horizon. Laſtly, bring the ſtar to the meridian, and the hours to which the index then points, is the time of its culminating, or ſouthing.

PROBLEM VII.

To find the altitude of a star, &c. for any given hour.

Adjuſt the globe to the poſitions of the heavens, and turn it till the index point to the given hour; then fix the quadrant of altitude at ninety degrees from the horizon, and turn it to the place of the ſtar; then the degrees of the quadrant intercepted between the horizon and the ſtar will be the altitude ſought.

PROBLEM

PROBLEM VIII.

Having the altitude of a Star by night, or the altitude of the Sun by day to find the hour of the day or night.

Rectify the globe as in the foregoing problem; and turn the globe and quadrant till that degree of the ecliptic where the Sun is, or the ſtar itſelf, cut the quadrant in the given degree of altitude; then the index will point to the hour required.

PROBLEM IX.

Having the azimuth of a star, or the Sun, to find the time of the night or day.

Rectify the globe as before, and bring the quadrant to the given azimuth in the horizon; then turn the globe till the ſtar or Sun come to the quadrant, and the index will then ſhew the hour of the night or day.

Motions, Distances, &c.	Mercury.	Venus.	Earth.
Greatest elongation of inferior, and parallax of superior planets.	28° 20′	47° 48′	* *
Periodical revolutions round the Sun.	d. h. m. 87 23 15½	d. h. m. 224 16 49¼	d. h. m. 365 6 9¼
Diurnal rotation upon their axis.	* *	23h 22m	23h 56m 4s
Inclinations of their orbits to the ecliptic.	7° 0′	3° 23⅓′	* *
Place of the ascending node	1s 15° 46¾	2s 14° 44′	* *
Place of the aphelion, or point farthest from the Sun.	8s 14° 13′	10s 9° 38′	9s 9° 15′¼
Greatest apparent diameters seen from the Earth.	11″	58	*
Diameters in English miles—that of the Sun being 883217.	3222	7687	7964
Proportional mean distances from the Sun.	38710	72333	100000
Mean distances from the Sun in semidiameters of the Earth,	9210	17210	23799
Mean distances from the Sun in English miles.	37 millions.	68 millions.	95 millions.
Eccentricities; or, distances of the focus from the centre.	7960	510	1680
Proportion of light and heat, that of the Earth being 100.	668	191	100
Proportion of bulk, that of the Sun being 1380000.	$\frac{1}{15}$	$\frac{9}{8}$	1
Proportion of density; that of the Sun being ¼	2	1¼	1

Mars.	Jupiter.	Saturn.	Herſchell, or Georgian.
47° 24′	11° 51′	6° 29′	3° 4¼′
d. *h.* *m.* 686 23 30¾	*d.* *h.* *m.* 4332 8 51½	10761*d.* 14*h.* 36*m*¾	*d.* *h.* 30,445 18
24h 39m 22s	9h 56m	* *	* *
1° 51′	1° 19¼′	2° 30⅓′	48°
1s 17° 59′	3s 8° 50′	3s 21° 48¼′	3s 13° 1′
5s 2° 6¼′	6s 10° 57′½	9s 0° 45½′	11s 23° 23′
25″	46″	20″	4″
4189	89170	79042	35109
152369	520098	953937	1903421
36262	123778	227028	453000
144 millions.	490 millions,	900 millions.	1800 millions
14218	25277	53163	4759
43	3.7	1.1	0,276
7/24	1 2/5	1000	90
.7	23	.02	*

An Explanation of the principal Terms used in Astronomy.

Æras, certain periods of time, from whence Chronologers and Aſtronomers, begin their computation.

Altitude, the height of the Sun, Moon, or ſtars above the horizon, and reckoned upon a vertical circle.

Amplitude, an arc of the horizon, contained between the eaſt or weſt point of the heavens, and the centre of the Sun or a ſtar, at the time of its riſing or ſetting.

Anomaly (true) the diſtance of a planet, in ſigns, degrees, &c. from that point of its orbit, which is the fartheſt from the Sun.

Anomaly (mean) is that which would take place, if the planet moved uniformly in the circumference of a circle.

Antecedentia, the motion of any heavenly body, when it is contrary to the order of the ſigns, as from aries, towards piſces, aquaries, &c.

Aphelion, that point in the orbit of a planet, which is the fartheſt diſtant from the Sun.

Apogeon, that point in the orbit of a planet, in which it is at its greateſt diſtance from the Earth.

Apsides, the two points in the orbit of the planet, which are its greateſt and leaſt diſtance from the Sun; the line joining theſe points, is called the line of the Apſides.

Armillary Sphere, an inſtrument having the principal circles, which are uſually drawn upon the artificial globe.

Ascensional Difference, an arc of the equinoctial, contained between that point of it which riſes with the Sun, Moon or Star, and that point which comes to the meridian

ridian with them, or it is the time the Sun rifes or fets before or after fix o'clock.

Atmosphere, that collection of vapours or body of air, that furrounds the Earth.

Axis of the Earth, or any planet, is an imaginary line, paffing through the centre, from one pole to the other.

Azimuths, great circles paffing through the zenith and nadir, and are perpendicular to the horizon. The Azimuth of any celeftial object, is an arc of the horizon, contained between the eaft or weft point of the heavens, and a vertical circle paffing through the centre of that object.

Bissextile, or Leap Year, every fourth year, fo called, becaufe the Romans reckoned the fixth day of the calends of March, twice over.

Cardinal Points, the eaft, weft, north, and fouth points of the compafs.

Cardinal Points of the Ecliptic; the firft points of the figns, aries, cancer, libra, and capricorn.

Centrifugal Force, that force by which any body, revolving in a circular orbit, endeavours to fly off from the centre of motion in a right line, or tangent to the circle.

Centripetal Force, that force which attracts any heavenly body towards the centre of its orbit; and which, together with the centrifugal force preferves the body in the proper path of its orbit.

Colures, two great circles or meridians, one of which, paffes through the folftitial points, cancer, and capricorn, and is called the folftitial colure; the other paffes through the equinoctial points, aries and libra, and called the equinoctial colure.

Conjunction, is when two ftars or planets, feen from the Sun or Earth, appear in the fame point of the heavens.

Constellation, feveral ftars lying near each other, which Aftronomers, for the fake of remembering with more eafe,

eafe, fuppofed to be circumfcribed by the outlines of fome animal, or other figure.

Cosmical, rifing or fetting of a ftar, is when they rife with the Sun in the morning, or fet with him, in the evening.

Consequentia, the motion of the planets, according to the order of the figns, as from Aries, towards Taurus, &c.

Culminating, of the Sun or a Star, is when they come to the meridian of any place.

Cycle of the Moon, a period of nineteen years, in which time, the changes and eclipfes of the Moon, are nearly the fame, and happen at the fame time.

Day, that portion of time, in which the Earth performs an entire revolution upon its axis, and is either natural, artificial or aftronomical.

Declination of any celeftial body, is its diftance north or fouth from the equator, reckoned in degrees, minutes, &c. upon a circle, perpendicular to the equator.

Degree, the three hundred and fixtieth part of a circle.

Direct, a planet is faid to be direct, when it moves according to the order of the figns.

Disc of the Sun or Moon, is its round face, which, on account of its diftance from us, appears flat, like a plane furface.

Digit, is the twelfth part of the Sun, or Moon's diameter.

Eccentricity, the diftance between the centre of an ellipfes, and either of its foci.

Ecliptic, that great circle, in which the Sun appears to move.

Elevation of the pole, is an arc of the meridian, contained between the pole and the horizon, and is always equal to

to the distance of the zenith from the equator, that is, the latitude of the place.

ELONGATION, the angular distance of a planet from the Sun, as it appears to a spectator upon the Earth.

ELIPSIS, a figure formed by cutting a cone obliquely. The orbits of all the planets are of this form.

EMERSION, the time when any planet that is eclipsed, begins to recover its light again.

EPACT, the Moon's age at the end of the year, or the difference between the solar and lunar year.

EQUATIONS, certain quantities by which are estimated the inequalities in the motion of a planet: the Moon being subject to many irregularities, has a great number of equations.

EQUATION OF TIME, the difference between equal and apparent time, or that shewn by a clock and a sun-dial.

EQUINOXES, the two points where the ecliptic cuts the equator.

GALAXY, or the milky-way, a large irregular zone in the heavens, illuminated with a great number of stars.

GEOCENTRIC place of a planet, is that part where it is seen from the Earth.

HELIACAL, rising of a star, is when it appears above the horizon, before the Sun in the morning: and heliacal setting of a star, is when it is not seen after the Sun in the evening.

HELIOCENTRIC place of a planet, is that part in which the planet is seen from the Sun.

HEMISPHERE, the half of a globe or sphere, and is either celestial or terrestrial.

HORIZON, is the circle, which separates the visible from the invisible hemisphere, and is either sensible or rational. The former passing over the surface of the Earth, and the latter through the centre.

HOUR CIRCLES, are great circles passing through the poles of the World.

IMMERSION,

IMMERSION, the moment when an eclipse begins on a planet.

INCLINATION, the angle which the orbit of one planet makes with that of another.

LATITUDE of a ſtar, or planet, is its diſtance from the ecliptic, reckoned in degrees, minutes, &c. upon the arc of a great circle.

LONGITUDE of a ſtar or planet, is its diſtance from the firſt point of aries, in degrees, minutes, &c. upon the ecliptic.

LUMINARIES, the Sun and Moon, ſo called, by way of eminence.

LUNATION, the time between one new Moon, and the next.

MAGNITUDES, the different claſſes of the ſtars, of which there are uſually reckoned ſix.

MEAN MOTION of a planet, is that which would take place if it moved in a perfect circle, and an equal ſpace every day.

MERIDIAN, that great circle which paſſeth through the poles, and the zenith of any place.

MINUTE, the ſixtieth part of an hour in time, or the ſame part of a degree of ſpace.

NADIR, that point in the heavens, directly under our feet.

NODES, the two points, where the orbit of a planet interſects the ecliptic.

NORTHERN SIGNS of the ecliptic, are thoſe ſix on the north of the equinoctial, viz.—aries, taurus, gemini, cancer, leo, and virgo.

NUCLEUS, the head of a comet, or the central part of a planet.

OBLIQUE ASCENSION, is an arc of the equinoctial, contained between the firſt degree of aries, and that point of it which riſes with the Sun or ſtar.

OBLIQUE

OBLIQUE SPHERE, is a poſition of the globe, when either pole is above the horizon, leſs than ninety degrees.

OPPOSITION, when two ſtars or planets are one hundred and eighty degrees diſtant from each other.

ORBIT, the path a planet deſcribes in its courſe round the Sun.

ORBIS MAGNUS, the orbit of the Earth.

PARALLAX, the difference between the places of any celeſtial body, as ſeen from the centre, and from the ſurface of the Earth.

PARALLAX of the Earth's annual orbit, is the angle at any planet, which is ſubtended by the diſtance between the Sun and Earth.

PARALLELS of latitude, are ſmall circles of the ſphere, parallel to the equator.

PERIGEON, that point of a planet's orbit, in which it is neareſt the Earth.

PERIHELION, that point of a planet's orbit, neareſt the Sun.

POLE STAR, a ſtar of the ſecond magnitude in the tail of the great bear, ſo called from being ſituated near the north pole of the World.

POLES OF THE WORLD, the two points at the extremities of the Earth's axis.

PRECESSION of the equinoxes, a ſlow motion of theſe two points, whereby they are found to go backwards, about fifty ſeconds in a year.

QUADRANT, the fourth part of a circle, alſo an inſtrument for meaſuring angles.

RETROGADE, is that motion, by which ſome of the planets ſeem to go backwards, or contrary to the order of the ſigns.

RIGHT ASCENSION, is that degree of the equator, which comes to the meridian with any celeſtial body, reckoning from the firſt point of aries.

SATELLITES, the ſecondary planets.

SECOND, the ſixtieth part of a minute, either of time, or ſpace.

SOLSTITIAL POINTS, are the two points in the ecliptic, through which the ſolſtitial colure paſſes.

STATIONARY, a planet is ſaid to be ſtationary, when it has no apparent motion.

SYSTEM, a number of bodies revolving round a common centre, as the ſolar ſyſtem,

SYZIGIES, thoſe points of the Moon's orbit, where ſhe is at the new and full.

TELESCOPIC STARS, are ſtars only diſcoverable, by means of a teleſcope.

TRANSIT, is the paſſing of celeſtial bodies, before another.

TWILIGHT, that faint light, we perceive before the riſing, and after the ſetting of the Sun, occaſioned by the Earth's atmoſphere.

VECTOR RADIUS, a line ſuppoſed to be drawn from any planet to the Sun, which moving with the planet, deſcribes equal areas, in equal times.

ZENITH, that point of the heavens, directly over our heads.

ZODIAC, that zone ſurrounding the heavens on each ſide of the ecliptic, in which all the planets perform their motions.

CHAP.

CHAP. XV.

OF MECHANICS.

Definitions.

1. THE mechanical powers are certain, ſimple machines, uſed for raiſing greater weights, or overcoming greater reſiſtances than the natural ſtrength of man can perform without them.

2. Theſe ſimple machines are reckoned ſix in number, viz. 1. The lever.—2. The wheel.—3. The pulley.—4. The ſcrew.—5. The wedge.—6. The inclined plane.

3. Force is a power exerted on a body to move it; if it act inſtantaneouſly it is called Percuſſion, or Impulſe; if conſtantly, it is an accelerative force.

4. Gravity is that force wherewith the body endeavours to fall downwards; it is called Abſolute Gravity, when, in an empty ſpace, and relative gravity, when immerſed in a fluid.

5. Specific gravity is the proportion which the weight of one body bears to that of another.

6. The centre of gravity is a certain point in a body, upon which the body, when ſuſpended, will reſt in any poſition,

7. Centre of motion is a fixed point round-about which a body moves; and the axis of motion is that fixed line about which it moves.

8. Power and weight, when oppoſed to each other, ſignify the body that moves another, and the body that is moved;

the body which communicates the motion is the power, and that which receives the motion is the weight.

9. Friction is the resistance which any machine suffers by the parts rubbing against each other.

In the practice of mechanics, though all bodies are rough in some degree, and all engines imperfect; yet it is necessary to consider all planes as perfectly even; all bodies perfectly smooth; and all bodies and machines to move without friction or resistance; all lines straight and inflexible; all cords very pliable, &c.

SECT. I.

OF THE SIX MECHANICAL POWERS.

THE whole principle of relative motion in mechanics depend upon this one single rule:—That the whole force of a moving body is the result of its quantity of matter multiplied by the velocity of motion. Thus, when the product arising from the multiplication of the particular quantities of matter in any two bodies by their respective velocities are equal—the entire forces are so too. For example:—Suppose a body A, which weighs forty pounds, move at the rate of two miles in a minute, and another body B, which weighs only four pounds, to move twenty miles in a minute: the entire forces with which these two bodies will strike against any other, would be equal to each other; and therefore, it would require equal powers to stop them; for, 40 multiplied by 2, gives 80, the force of the body A, and 80 is also the product of 4, multiplied by 20, the force of the body B: Thus, the heavier any body is, the

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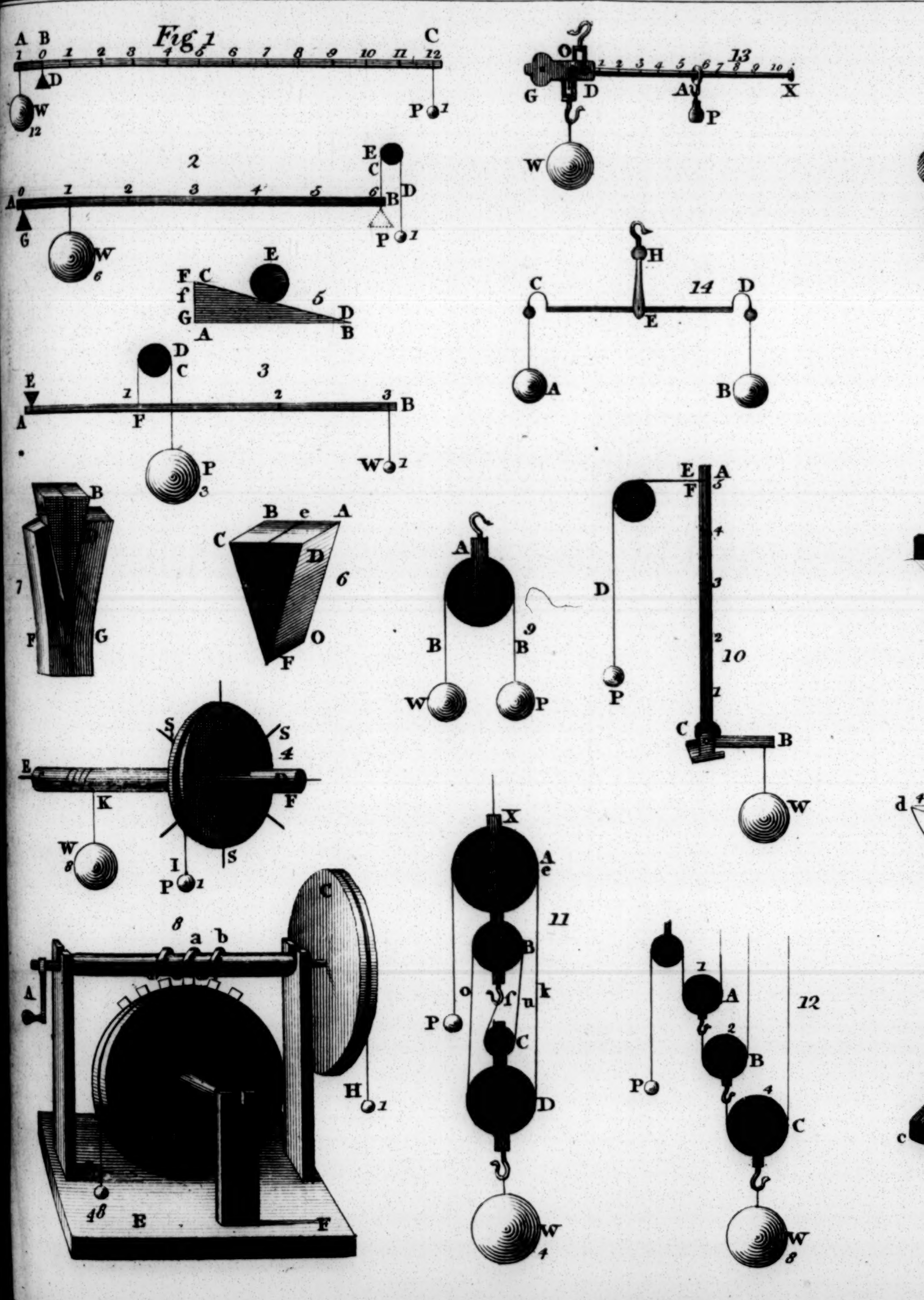

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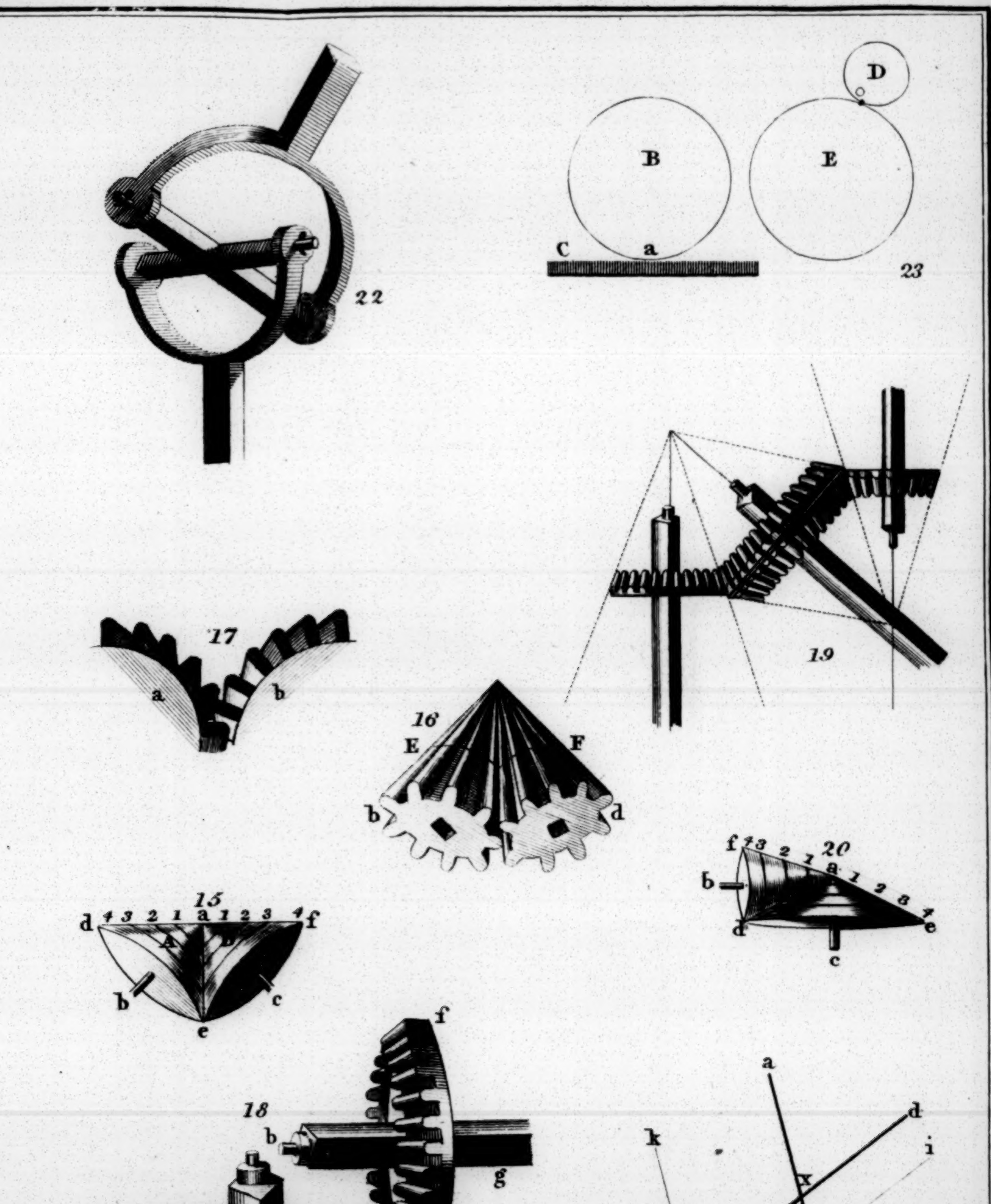

22
D
B
E
C
a
23
19
17
a
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16
E
F
b
d
20
15
18
21

the greater is the power required, either to move or ſtop it. And again, the ſwifter it moves, the greater is its force; therefore, when two bodies are ſuſpended on any machine, ſo as to act contrary to each other; if the machine be put in motion, and the perpendicular aſcent of one body, multiplied into its weight, be equal to the perpendicular deſcent of the other body, multiplied into its weight; thoſe bodies, how unequal ſoever in their weights, will ballance one another in all ſituations; for as the whole aſcent of one is performed in the ſame time with the whole deſcent of the other, their reſpective velocities muſt be directly as the ſpaces through which they move; and the exceſs of weight in one body, is compenſated by the exceſs of velocity in the other: Upon this principle the power of any machine may be eaſily computed; for it is only finding how much ſwifter the power moves than the weight does; (that is, how much farther in the ſame time) and juſt ſo much power is gained by the engine.

A Lever is a bar, either of iron or of wood, one part of which is ſupported by a prop, as its centre of motion; and the velocity of every part or point in the lever is directly as its diſtance from the prop.

There are four kinds of levers: 1. The common lever, where the prop is placed between the weight and power; but much nearer the weight than the power.—2. Where the prop is at one end of the lever, the power at the other end, and the weight between them.—3. Where the prop is at one end, the weight at the other end, and the power applied between them.—4. The bended lever, which only differs from the lever of the firſt ſort in being bent. Levers of the firſt and ſecond kind are often uſed in mechanical engines; but the third kind are ſeldom uſed, as no power can be gained by it.

When the power is at the ſame diſtance from the prop as the weight is, and the power and weight be both alike, the machine will remain in equilibrium, and no power can be gained.

gained. This is the principle upon which the common balance is formed. Let C D (fig. 14.) be a beam or lever, E the middle point, or centre of motion, which may be considered as the prop; A B two weights hanging at the end, C and D; then, when the machine is suspended at the point E, the points C and D being equidistant from E, will describe equal arches, therefore their velocities will be equal; and if the bodies A and B be also equal, then the motion of A will be equal to that of B, as the velocities and quantities of matter are equal, and consequently, if the machine be at rest, neither of the weights can move the other, but they will remain in equilibrium.

The use of the balance, or common pair of scales, is to compare the weights of different bodies: for any body whose weight is required, put into one scale, will be balanced by a body of the same weight, put into the other scale.

In order to have a pair of scales perfect, they should have the following properties:—1. The points of suspension of the scales, and the centre of motion in the beam C, E, D, must be in a right line. 2. The arms C E and D E, must be of equal length. 3. The centre of gravity, must be in the centre of motion E. 4. There should be as little friction as possible. 5. The scales must be in equilibrium when empty.

If the centre of gravity of the beam, be above the centre of motion, and one end of the balance be put lower than the other, that end will continually descend, till it be stopped at the handle H; but if the centre of gravity of the beam be below the centre of motion, the balance will preserve an equilibrium.

Hence, to examine a pair of scales, let the weights in the two scales be in equilibrium, then change the weights to the contrary scales, and if they remain in equilibrium, the balance is true, otherwise it is false.

Let

Let A B C, (fig. 1, plate 18,) reprefent a lever of the firft kind, fupported by the prop D; the parts A B and B C, on each fide of the prop D, are called arms of the lever; the end A of the fhorter arm A B, is applied to the weight to be raifed, and the power is applied to the end C, of the other arm B C. The principal ufe of this lever, is to loofen large ftones, which are fixed in the ground, or to raife great weights to a fmall height, in order to place rollers under them, or ropes for raifing them higher by other Machines.

In this lever, the fhorter arm A B, fhould be as much thicker than the longer arm B C, as will be fufficient to balance it, on the prop D. Thus, if P reprefent a power whofe weight is equal to one ounce, and W a weight of twelve ounces, and if the power be twelve times farther from the prop than the weight is, they will exactly counterpoife each other; and a fmall addition applied to the power P, will raife the weight W: and the velocity with which the power defcends, will be to the velocity with which the weight rifes, as 12 is to 1; that is, directly as their diftances from the prop; and confequently, as the fpaces through which they move. Thus, it is evident, that if a man, by his natural ftrength, could lift an hundred weight, he will by a lever of this fort, be able to raife twelve hundred weight. If the weight be lefs, or the power greater, than in the foregoing cafe, the prop may be placed fo much farther from the weight, and then it can be raifed to a proportionable greater height, by the fame addition of force: but if the weight be greater, or the power lefs, the prop fhould be placed fo much nearer the weight. For, univerfally, if the gravity of the weight, multiplied by its diftance from the prop, be equal to the gravity of the power, multiplied by its diftance from the prop, the power and weight will exactly balance each other. Thus, if the weight W, be twelve ounces, and its diftance

from

from the prop 1 inch; the product of 12 multiplied by 1, is 12; and if the power P, be 1 ounce, and its distance from the prop 12 inches, the product of these two, is also 12; therefore, they counterpoise each other. And if a power, equal to two ounces, be applied at 6 inches distance from the prop, it will also balance the weight W, for 6 multiplied by 2 is 12. And if the power be 3 ounces, and placed at 4 inches distant from the prop, it would also, balance the weight W, for 3 times 4 is 12. And the like in any other proportion.—A poker stirring a fire, is a lever of this kind, the bar, upon which it rests is the prop, the hand applied to the end of it, is the power, and the incumbent coals, on the other end is the weight. Several sorts of instruments are formed of two levers of this kind; as, scissars, snuffers, pincers, &c. the prop, or centre of motion, is the pin which holds them together.

The *Statera*, or *Roman Steelyard*, is a lever of this kind, and is used to find the weight of any body, by one single weight, placed at different distances from the prop. G X, (fig. 13,) is a steelyard, suspended by the hook O, from the centre of motion D; the shorter arm D G, is of such a weight, as exactly to counterpoise the longer arm D X; if this longer arm be divided into as many equal parts, as it will contain, and each part equal to O D, the single weight P, will weigh any body as heavy as itself, or as many times heavier as there are divisions in the arm D X. Thus, if the weight P, be one pound, and placed at the first division 1, in the arm D X, it will balance one pound in the scale W; if it be removed to the second division, at 2, it will balance two pounds in the same scale; if to the third three pounds, &c. And if each of these integral divisions, could be divided into as many equal parts, as a pound contains ounces, then the weight P, placed at any of these sub-divisions, will shew the odd ounces, over and above the number of pounds of the body in the scale.

The

The ſecond kind of levers have the weight between the prop and the power, (fig. 2.) In this as well as the former, the advantage gained is as the diſtance of the power from the prop, to the diſtance of the weight from the prop; and the rules for computing the force of this lever are the ſame with thoſe of the former. Thus, if W be a weight of ſix ounces hanging at the diſtance of one inch from the prop G, and P a power or weight of one ounce hanging at the end B, 6 inches diſtant from the prop, by the cord C D running over the fixed pulley E, the power will juſt ſupport the weight: and a ſmall addition to the power will raiſe the weight one inch for every 6 inches that the power deſcends; thus the power acts with the ſame force upon the weight as it would do if the weight were at the ſame diſtance from the prop, and on the other ſide thereof, in which caſe it would be a lever of the firſt ſort.

Two men carrying a burthen upon a ſtick, exhibit a ſpecimen of a lever of this kind; and the portion of weight, borne by each man, is in proportion to his diſtance from the weight. Alſo, in yoking two horſes of an unequal ſtrength to draw any load, the point of taction is placed as much nearer to the ſtronger horſe than to the weaker, as the ſtrength of the former exceeds that of the latter.

Of this kind of levers are, oars, rudders of ſhips, doors turning upon hinges, cutting knives fixed at the point, &c.

The third kind of lever has the power applied between the weight and the prop, in which, in order that the power may counterpoiſe the weight, the gravity of the power muſt exceed that of the weight as much as the diſtance of the weight from the prop exceeds the diſtance of the power from the prop. Thus, if E (fig. 3,) be the prop of the lever A B, and W a weight of one pound, which is placed three times as far from the prop as the power P, which acts at F by the cord C going over the pulley D; the power P muſt be three pounds to counterpoiſe the weight of W of one pound;

and for every inch, the power P defcends, the weight W will afcend three inches, &c.

Levers of this kind are very little ufed, becaufe they give no advantage in point of force, though they give an advantage in point of motion; but in fome cafes they become neceffary, as in raifing a ladder againft a wall, in which cafe, the foot of the ladder, which is fixed againft the ground, is the prop, the man's hand who raifes it the power, and the whole length of the ladder, from the hand to the upper end, is the weight.

The bones of a man's arm are likewife levers of this kind, for the mufcle which raifes the arm, is fixed to the bone about a tenth part as far below the elbow as the hand is. Therefore, the elbow may be confidered as the prop upon which the lower part of the arm turns, and the mufcle muft, confequently, exert a force ten times as great as the weight, which is raifed in the hand.

The fourth kind of lever has all the properties of the firft kind, and differs from it only in being bent, which is done for the fake of convenience. A C B (fig. 10.) is a lever of this fort, bent at C, its prop or centre of motion. W the weight, and P the power acting at A, over a pulley by means of the cord D. As the the mechanical power of this lever is the fame as that of the firft fort, it need not be repeated. A hammer drawing a nail is a lever of this fort.

The fecond mechanical power is the *wheel* and *axle* (fig. 4.) in which the power is generally applied to the circumference of the wheel, and the weight W to that of the axle; the weight being raifed by a rope winding round the axle as the wheel turns round. In this inftrument it is evident that the velocity of the power muft be, to that of the weight, as the circumference of the wheel, is to the circumference of the axle: and the power gained is in proportion to the circumference of the wheel to that of the axle. Therefore, when the gravity of the power is to that of the weight, as the circumference of the axle is to the circumference of the wheel, the power

power and weight will balance each other. Again, let A B be a wheel. (fig. 4.) E D the axle, and the circumference of the wheel eight times as great as that of the axle; then a power P of one pound weight, added by the cord I, which goes round the wheel, will balance the weight W of eight pounds hanging by the rope K, which goes round the axle; and a ſmall addition to the power will cauſe it to deſcend and raiſe the weight; but the weight will riſe with only an eighth part of the velocity wherewith the power deſcends; and conſequently will move through only an eighth part of an equal ſpace in the ſame time. If the wheel be pulled round by the handles S S, the power will be increaſed in proportion to their length.

In this mechanical power the radius of the wheel, and the oppoſite radius of the axle, may be conſidered as the longer and ſhorter arms of a lever of the firſt kind, the centre of the axle being the prop.

Sometimes the wheel or the axle is indented or cut into teeth, which have another wheel working in them, as in jacks, clocks, mill-work, &c. by which means they give a much greater mechanical force. To compute the power of a combination of wheels, multiply the radii of all the axles continually together; as alſo the radii of all the wheels; then, as the former product is to the latter, ſo is a given power applied to the circumference of a wheel, to the weight it can ſuſtain. For example, in a combination of five wheels and axles, to find the weight a man can ſuſtain or raiſe, whoſe force is equal to 150 pound: the radii of the wheels being 30 inches, and the radii of the axles 3 inches. Here 3 multiplied 5 times into itſelf, produces 243; and 30 multiplied 5 times into itſelf, produces 243,00000; therefore, as 243 is to 24300000, ſo is 150 to 15000000 pounds, the weight he can ſuſtain, which is more than 6696 tons; or above 100000 times as great a weight as he could ſuſtain by his own natural force.

But here it muſt be obſerved, that though there is a prodigious gain of power in theſe combinations of wheels, yet there is a great loſs of time; that is, the weight, in this caſe, will move 100,000 times ſlower than the power; and this is true in all mechanical caſes whatever.

The third mechanical power is the *pulley*, or ſometimes a ſyſtem of pullies; ſometimes they are fixed in a block or caſe, which is alſo fixed; at other times they are in a block, which is moveable, and riſes with the weight. The ſingle pulley A, (fig. 9.) gives no mechanical advantage, though it may ſerve to change the direction of the power: but is only as a beam of a balance, whoſe arms are of equal length and weight, and is, properly, but another form of the balance.

The ſyſtem of pullies is repreſented (fig. 12.) where the four pullies are faſtened to an immoveable block above; three of them, A, B, C, by the three diſtinct cords running under them. The power of this ſyſtem of pullies is diſcovered by ſuppoſing W a weight of 16 pounds, ſuſpended from the pulley C, which is alſo ſuſpended by the cord C, one end of which is faſtened to the block above, and the other end ſupported by the pulley B; therefore the pulley B, ſuſtains only half the weight of the weight W, or eight pounds; the other half being ſuſtained by the cord C, fixed to the block. Then the cord B, which goes under the ſecond pulley, ſuſtains the weight of eight pounds, which is alſo divided, four pounds being ſuſtained by the cord B, fixed in the block above, and the other four pound by the next pulley A. This next pulley A, alſo has its weight divided, one half being ſupported by the cord A fixed to the block, and the other half ſupported by the ſmall pulley, which ſmall pulley again divides the weight it ſupports, ſo that the power P is equal to only one pound, which will counterpoiſe the weight W of 16 pounds.

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The velocity of the weight to that of the power, is, as the gravity of the power is to that of the weight. Thus if P deſcend 8 inches, A will aſcend 4 inches; B 2 inches, and C or H 1 inch.

A, B, C, D, (fig. 11.) are four pullies, two of which A and B are in a fixed block X; the two others, C and D in a moveable block. Here the weight W is raiſed by pulling the cord at P, which goes ſucceſſively over the four pullies, and is faſtened at the end to the fixed block at ſ. The purchaſe of this machine is ſeen to be as 4 to 1, for P is ſuſtained by the ſingle cord; but W, by four folds of the cord, viz. *o*, *s*, *u*, *k*, ſo that if P be 1 pound, W will be 4.

The velocity of the power to that of the weight is alſo as in the former caſe, as the gravity of the weight to that of the power; or as 4 to 1; for when P deſcends 4 inches, the parts of the cord at k will aſcend 4 inches towards e, and all the other parts of the cord will equally follow each other; and as there are four folds in the cords, viz. *o*, *s*, *u*, *k*, they will each of them be ſhortened one inch, and C or W will be ſo much raiſed.

In the ſame manner the purchaſe of any combination of pullies may be determined; for the *momenta* of the weight and power will always be equal, as in the other mechanical powers.

The fourth mechanical power is the *Inclined Plane:* In this machine the advantage gained is as great as its length exceed its perpendicular height. Let A B (fig. 5) be a plane parallel to the horizon, and C D a plane inclined to it: if the length C D be three times as great as the perpendicular height, G F, the cylinder E will be ſupported on the plane C D, by a power equal to a third part of the weight of the cylinder; or, it may be rolled up the plane with a third part of the power, which would be ſufficient to draw it up the ſide of an upright wall. If the plane were four times as long as the perpendicular height, it would only require the fourth

fourth part of the power, and ſo on in proportion. The uſe of this power is to raiſe a great weight to any eminence, which is uſually done by puſhing it up a ſtout plank, which is ſet ſloping to the place deſigned; and ſuch plank, or other contrivance ſimilar thereto, is called an inclined plane.— Now, it is evident, the ſteeper the aſcent is, the more difficult it is to puſh any weight up it; and the more the aſcent inclines to the horizon, the eaſier the weight may be puſhed up. This is evident, from the eaſe with which a rolling weight is forced up a hill that riſes gently, while it is ſo difficult to roll the ſame weight up a hill which is very ſteep.

The force wherewith a rolling body deſcends upon an inclined plane is to the force of its abſolute gravity, as the height the plane is to its length. Thus, if the perpendicular height, G F of the plane be equal to half its length A B, the cylinder E, will roll down the plane with a force equal to half its weight; and it would require a power equal to half its weight to ſuſtain it. If the plane be ſo much elevated as to be perpendicular to the horizon, the cylinder E would deſcend with its whole weight, becauſe the plane contributes nothing to its ſupport or hindrance.

In an inclined plane a power acts to the greateſt advantage, when its direction is parallel to the ſurface of the plane.

The *wedge* conſtitutes the fifth mechanical power, and which may be conſidered as two equally inclined planes, A D F (fig. 6.) and C B F joined together at their baſes F O; then D C is the whole thickneſs of the wedge A B C D, the back of the wedge where the power is applied: E F the height or depth; D F the length of one of its ſides, equal to C F, the length of the other ſide, and O F its ſharp edge, which is driven into the wood, intended to be ſplit by the force of a hammer or mallet ſtriking on its back. Thus, A B is a wedge (fig. 7.) driven into the cleft C E D of the wood F G.

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The power gained by the wedge, is in proportion of the ſlant ſide to half the thickneſs of the back. Thus, if the back of the wedge, be two inches thick, and the ſide 20 inches long, any weight performing on the back, will balance twenty times as much, acting againſt the ſides. To uſe a wedge to the greateſt advantage, it ſhould be forced, not by preſſure, but by percuſſion, as by the blow of a hammer or mallet; by which means, a wedge may be driven in below any weight, and ſo made to lift it up, as the largeſt ſhips, &c.

The wedge has a very great mechanical force, and effects what would be impoſſible by the lever, wheel and axle, or pulley; for the force of the blow ſhakes all the adjacent parts, and thereby makes them ſeparate more eaſily; ſo that not only wood, but even rocks can be ſplit by it.

To the wedge may be referred the axe or hatchet, the chiſſel, the ſpade and ſhovel, knives of all kinds; as alſo the bodkin and needle, and all ſorts of inſtruments which beginning from an edge or point, become gradually thicker.

The ſixth and laſt mechanical power is the *ſcrew*, which is not properly a ſimple machine, becauſe it cannot be uſed without a lever to turn it, called the winch or handle. It is a compound engine of very great force, and is a kind of perpendicular or endleſs inclined plane, ſtill farther aſſiſted by the power of the handle or lever: and the gain of power is in proportion of the circumference, deſcribed by the power to the diſtance between one thread, and the next in the ſcrew.

Thus, let C be a wheel, (fig. 8,) having a ſcrew *a b*, on its axis, working in the teeth of the wheel D, which ſuppoſe to be 48 in number. Then it is evident that for one revolution of the wheel C, ſcrew *a b*, and winch A, the wheel D, will be moved, one tooth by the ſcrew; and therefore in 48 revolutions of the winch A, the wheel D will be turned once round.

round. Then if the circumference of the circle deſcribed by the handle of the winch A, be equal to the circumference of a groove round the wheel D, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove; conſequently, if a line G, goes round the groove D, and has a weight of 48 pounds unto it, below the pedeſtal E F, a power equal to one pound at the handle, will ſupport this weight; or, if a groove be made in the wheel C, equal in radius to the circle deſcribed by the handle, the weight H of one pound, ſuſpended therefrom, by a line in the groove, will balance the 48 pounds, as before. If the line G, inſtead of going round the groove of the wheel D, go round its axle I, the power of the machine, will be as much increaſed, as the circumference of the groove exceeds that of the axle, as ſhewn under the wheel and axle. And if a ſyſtem of pullies were applied to the cord H, the power could be encreaſed to an amazing exceſs.

The uſes to which the ſcrew is applied, are various: it is chiefly uſed for preſſing bodies cloſe together, as the preſſes for book-binders, packers, hot preſſers, &c.

The friction in the ſcrew, is very conſiderable, as it is alſo in the wedge, which generally requires a third part more of the power to work them when loaded, than what is ſufficient to conſtitute a balance, between the weight and power.

If machines or engines, could be made without friction, the leaſt degree of power, above what is ſufficient to balance the weight, would be ſufficient to raiſe it. In the lever the friction is little or nothing. In the wheel and axle, it is but ſmall. In pullies it is conſiderable; and in the inclined plane, wedge and ſcrew, it is very great.

Wood greaſed, or metal oiled, have nearly the ſame friction, and the ſmoother they are, the leſs is their friction, provided they be not too highly poliſhed. In poliſhed ſteel, moving upon poliſhed ſteel or pewter, the friction is about a fourth

fourth part of the weight; on copper a fifth part; and on brass a sixth part of the weight. Iron or steel, running in brass, has the least friction of any. And metals of the same sort have more friction, than different sorts; and in general, the friction increases in the same proportion with the weight; but is greater, with a greater velocity.

The friction in pullies, is now almost reduced to nothing, by the contrivance of *Mr. Garnett*, in his patent friction rollers, which produce a great saving of labour and expence, as well as wear of the materials, both when applied to pullies, and the axles of wheel carriages. By this contrivance, there is a hollow space left between the nave and axle, or centre and pin-box, which is filled up by solid equal rollers, nearly touching each other, and furnished with axles, each of which, is inserted into a circular ring at each end, by which their relative distances are preserved; and they are kept parallel by means of wires fastened to the rings between the rollers, and which are rivetted to them.

It is a general property in all the mechanic powers, that when the weight and power balance each other, if they be put in motion, the power and weight will be to each other, reciprocally, as the velocities of their motion; or the power is to the weight, as the velocity of the weight is to the velocity of the power; so that their two *momentas* are equal, viz.—The product of the power, multiplied by its velocity, equal to the product of the weight, multiplied by its velocity. And hence, the general rule, viz.—That what is gained in power, is lost in time; for the weight moves as much slower as the power is less.

SECT. II.

OF THE APPLICATION OF THE POWERS TO MILLS AND MACHINES.

IN order to discover the properties of any machine, consisting of the mechanical powers, it is necessary to consider the weight that is to be raised, or the resistance to be overcome; and also the power required to raise the weight, or overcome the resistance. For this purpose, there are two principal problems, the resolution of which, is requisite, to shew the phenomena of any engine--The first Problem is, *to determine the proportion, that the power and weight ought to have to each other, that they may be in the just equilibrium.*—The second is, *to determine what the proportion should be between the power and weight, that the machine may produce the greatest effect in a given time.*

The first problem is solved by this general rule, viz.—That the power and weight sustain each other, or are in equilibrium, when the power and weight are reciprocally proportional to the distances of the directions, in which they act from the centre of motion; or when the product of the power, by the distance of its direction, is equal to the product of the weight, by the distance of its direction. This is the proportion of the weight and power, when they are in equilibrium, so that the one would not prevail over the other if the engine were at rest; and if it be set in motion, it would continue to proceed uniformly, if there were no friction of its parts, and other resistances. And, in general, the effect of any power or force is, as the product of that force, multiplied by the distance of its direction from

from the centre of motion; or the product of the power, and its velocity when in motion, since this velocity is proportional to the distance from that centre.

The second general problem, in Mechanics, is of the greatest importance, though it has been little attended to by Mechanical writers, viz.—To determine the proportion, between the power and weight, so that when the power prevails, and the machine is in motion, the greatest effect possible, may be produced by it, in a given time. When the power is only a little greater than what is sufficient to sustain the weight, the motion is usually too slow; and though a greater weight be raised, in this case, it is not sufficient to compensate for the loss of time. And when the power is much greater than what is sufficient to sustain the weight, the weight is raised in less time; but it often happens, that this is not sufficient to compensate for the loss which arises from the load being reduced; therefore, the only general rule, that can be given, is to find when the product of the weight, multiplied by its velocity, is the greatest; for this product measures the effect of the machine in a given time, which is always greater in proportion, as both the weight and velocity is greater.

In the construction of compound machines, where it is necessary to alter the direction of the motion, recourse must be had, to which is called *Bevel Geer*, the principle of which is as follows:—

Let A and B, (fig. 15,) be two cones, revolving on their centres *a c* and *a b*; if their bases be equal, they will each perform their revolution in the same time; and any two points in each cone equally distant from the centre, as *d* 1, *d* 2, *d* 3, &c. will revolve in the same time as *f* 1, *f* 2, *f* 3, &c. respectively. But if one cone be twice the diameter of the other, as the cone *a d e*, (fig. 20,) which is twice the diameter of the cone *f a d*, then, as they turn upon their centres, when the cone *a f d* has made one revolution, the

cone *a d e* will have made but half a revolution; and every part in each cone, equally distant from the centre *a*, will have the same proportion in their revolutions to each other; as *f* 1, *f* 2, *f* 3, &c. will have made two revolutions to the points *e* 1, *e* 2, *e* 3, &c. for one revolution of the other cone respectively, &c. Now, if the cones are fluted, or have teeth cut in them, diverging from the centre *a*, to the bases *d e*, *d f*, (fig. 16,) they would then become Bevel Geer. The teeth at the point of the cone, being small, and of little use, may be cut off; or instead of the two cones, may be used two shafts, with bevel wheels fixed to them, as the shaft *a b*, (fig. 18,) with the bevel wheel *c d*, which turns the bevel wheel *e f*, with its shaft *b g*, and the teeth work freely into each other, as in figure 16. The teeth may be made of any dimensions, according to the strength required, and by this means, a motion may be communicated in any direction, or to any part of a building, with very little trouble and friction.

The method of constructing the wheels, for any proportion, is as follows:—Draw the line *a b*, (fig. 21,) to represent a shaft of a wheel; draw the line *e d* to intersect the line *a b*, in the direction that the motion is to be conveyed, and the line *e d* will represent the other shaft of the motion.

Then suppose, the shaft *e d* is to revolve three times in the time that the shaft *a b* revolves once; draw the parallel line *i i* at any distance, from a scale, suppose one foot, then draw the other parallel line *k k*, at three feet distance; after which, draw the line *w x*, through the intersections of the two shafts *a b* and *e d*, and likewise, through the intersections of the two parallel lines *i i* and *k k*, in the points *x y*, which will be the pitch line of the two bevel wheels, or the lines where the teeth of the two wheels act on each other, as may be seen in figure 19, where there are three wheels.

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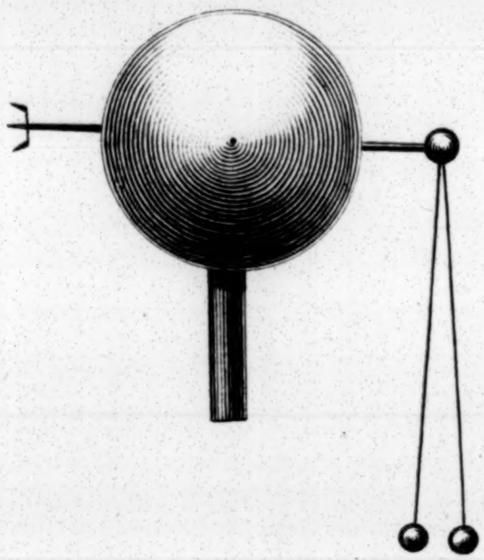

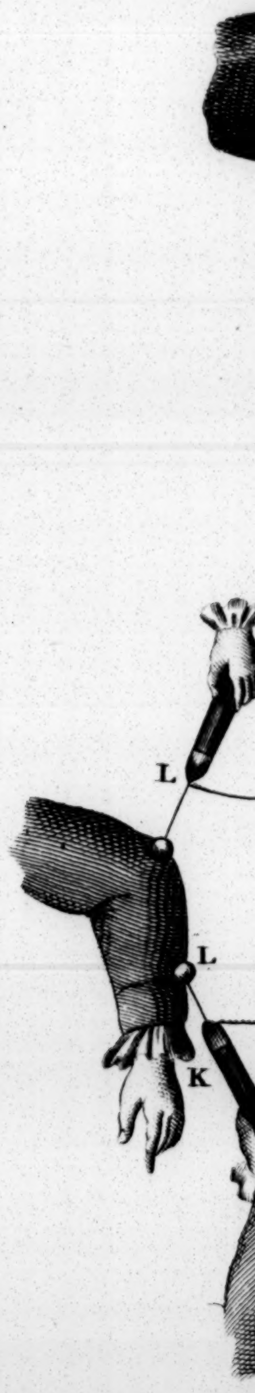

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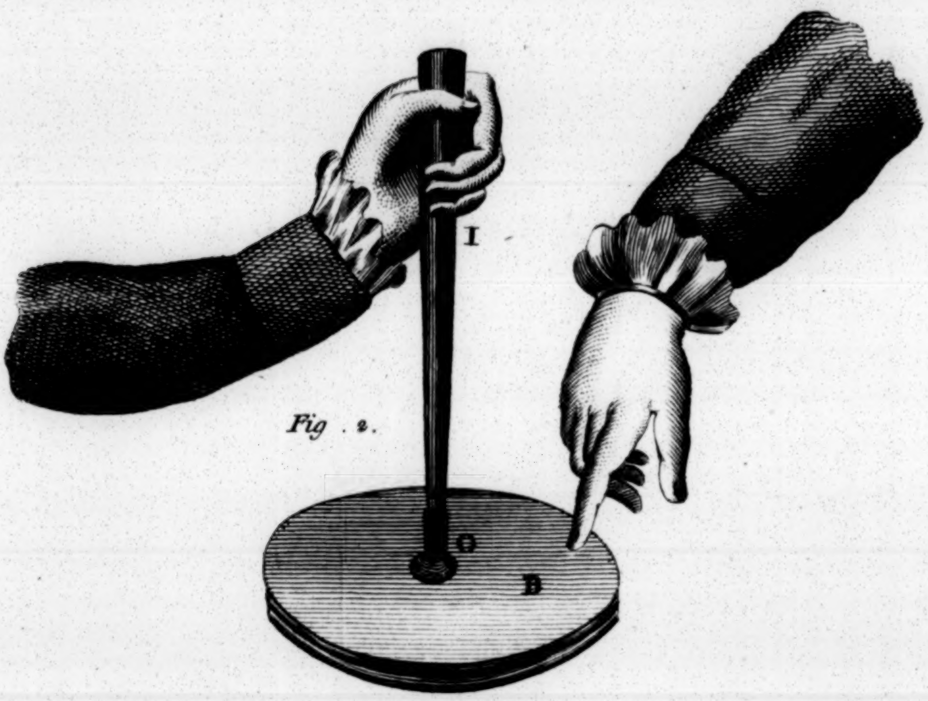

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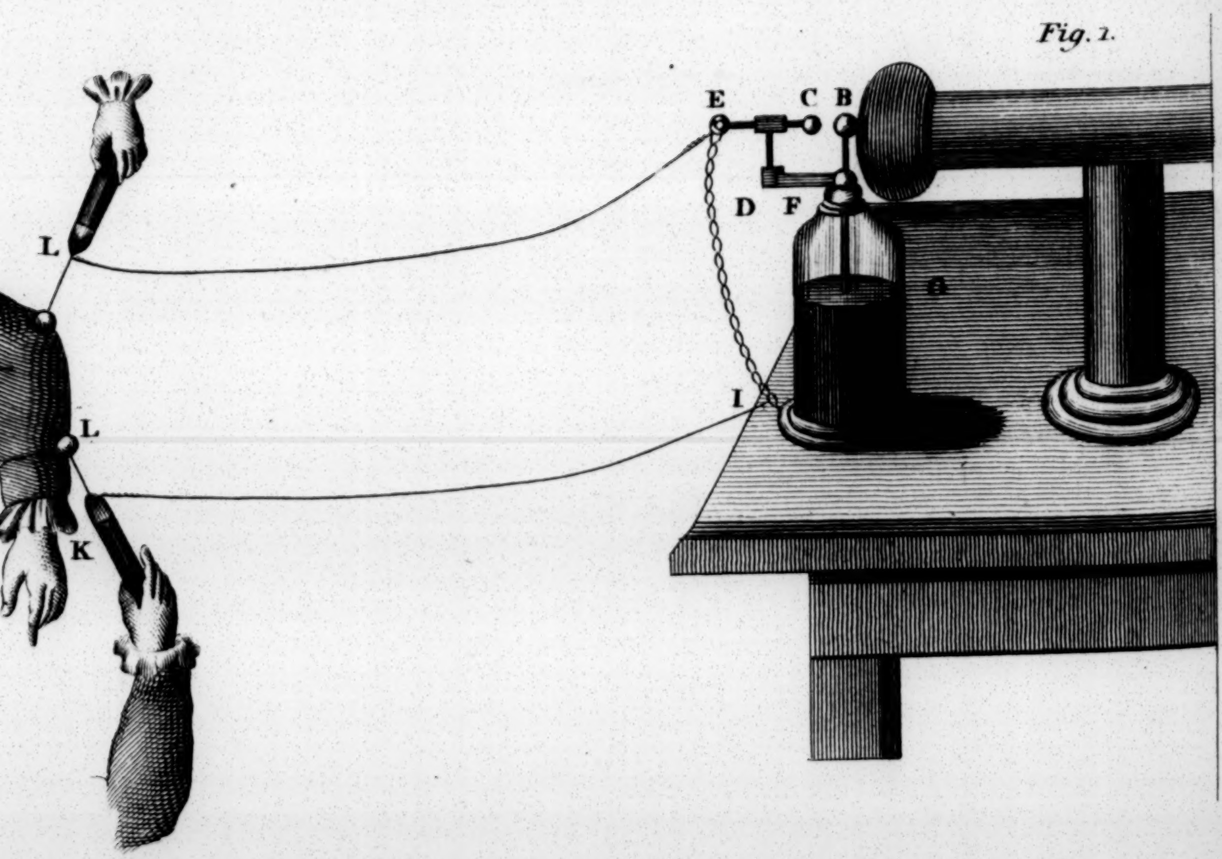

V. Woodthorpe sc. 27 Fetter Lane.

and alſo where the equality of the motion is not regarded, the univerſal joint may be uſed, (fig. 22,) inſtead of the *Bevel Geer*. This joint may be conſtructed by a croſs, as ſhewn in the figure; or with four pins faſtened at right angles, upon the circumference of a hoop, or ſolid ball. This is of great uſe in ſome machines, where the tumbling ſhafts are continued to a great diſtance from the moving power, as it is in cotton mills. The ſhafts, by applying this joint, may alſo be cut to any length, which is a great advantage, where there is much reſiſtance.

CHAP. XVI.

OF ELECTRICITY.

SECT. I.

THE PRACTIC PART OF ELECTRICITY.

THE Earth, Air, and all Terreſtrial bodies, are ſuppoſed to contain a certain quantity of an elaſtic ſubtle fluid, called by philoſophers, the *electric fluid*; and when any body poſſeſſes more or leſs of this fluid than what naturally belongs to it, ſeveral effects are viſible in it, and the body is ſaid to be electrified.

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This certain quantity of electric fluid, found in all bodies, could never be increased or diminished, if all bodies admitted the passage of this electric fluid through their pores, or along their surfaces; but there are many bodies which will not suffer this fluid to pass through them, while others freely permit it. Those bodies through which the electric fluid can pass, are called *conductors* of electricity, of which the most perfect are metals of all kinds. And those bodies through which the electric fluid cannot pass, are called *non-conductors* of electricity, of which the most perfect are glass, resin, sealing-wax, sulphur, bees-wax, and baked wood among solids; and oils and air among fluids. But all substances become conductors, when they are made very hot. Conducting substances, are also called *non-electrics*, and non-conducting substances, are called *electrics*. Into these two classes, all bodies are divided by electricians.

When any body has acquired an additional quantity of electric matter, and is surrounded with other bodies through which it cannot pass, or non-conductors, it must remain overloaded; or if it have lost part of its natural share of electric matter, it must remain exhausted; because, the bodies which surround it, prevent any of the electric fluid, from entering or coming out of it, and the body is then said to be *insulated*.

There are two principle theories of electricity, each of which has had its advocates. The one is, that of two distinct electric fluids, repulsive with respect to themselves, and attractive of one another, adopted by *M. Du Fay*, on discovering the two opposite species of electricity, viz.—The vitreous and resinous, which is since new modelled by *Mr. Symmer*. Upon this hypothesis these two fluids are equally attracted by all bodies, and exist in intimate union in their pores; and in this state they shew no mark of their existance. But the friction of an electric body, against a rubber, separates these fluids, and causes the vitreous electricity

city of the rubber to paſs to the electric, then to the prime conductor of the machine; while the reſinous electricity of the conductor and electric is communicated to the rubber; thus the quality of the electric fluid poſſeſſed by the conductor and the rubber is changed, while the quantity remains the ſame in each. In this ſeparated ſtate the two electric fluids will exert their reſpective powers; and any number of bodies charged with either of theſe, may repel each other, attract thoſe bodies that have leſs of each particular fluid than themſelves, and be ſtill more attracted by bodies that are either only deſtitute of it, or loaded with the contrary. In this theory the electric ſpark makes a double current; one fluid paſſing to the electrified conductor from any ſubſtance preſented to it, while the ſame quantity of the other fluid paſſes from it; and when each body receives its natural quantity of both fluids, the balance of the two powers is reſtored, and both bodies are unelectrified.

The other theory, and that which is commonly received, is that diſtinguiſhed by the name of *poſitive* and *negative electricity*, ſuggeſted by Dr. Watſon, and demonſtrated by Dr. Franklin; in which it is ſuppoſed, that all bodies poſſeſs a certain ſhare of one and the ſame fluid, which is extremely ſubtle and elaſtic, by which the particles of it are ſtrongly attracted, as they are repelled by one another. When bodies poſſeſs their natural ſhare of this fluid, they are ſaid to be in an unelectrified ſtate; but when the equilibrium is deſtroyed, and they have an additional quantity *from* other bodies, or when they loſe part of their natural ſhare by the communication *to* other bodies, they then become electrified, and exhibit electrical appearances which are generally the ſame in both caſes. In the former caſe, they are ſaid to be electrified poſitively, or *plus*; and in the latter caſe negatively, or *minus*. It is alſo ſuppoſed, that electrics always contain an equal quantity of this fluid; ſo that there can be no increaſe on one ſide, without a proportional decreaſe or loſs

loſs on the other, and *vice-verſa;* and as the electric will not ſuffer the fluid to paſs through its pores, there will be an accumulation on one ſide, and a correſponding deficiency on the other; then, connecting both ſides together by proper conductors, the equilibrium will be reſtored, by the ruſhing in of the redundant fluid from the overcharged ſurface to the exhauſted one. Thus, if an electric be rubbed by a conducting ſubſtance, the electricity is only conveyed from one to the other, the one giving what the other receives: and if one be electrified poſitively, the other will be electrified negatively, unleſs the loſs be ſupplied by other bodies connected with it, as in the caſe of the electric and inſulated rubber of a machine. Thus, bodies differently electrified, will naturally attract each other, till they mutually give and receive an equal quantity of the electric fluid, and then the equilibrium between them will be reſtored.

From what has been ſaid, it is evident that the method of diſturbing the equilibrium of the electric fluid, in bodies, or of making it paſs from one to another, is chiefly friction, or a ſlight rubbing of them one againſt the other; when the electric fluid will generally leave the rougher ſurface and paſs upon the ſmoother; or it leaves the leaſt perfect electric and paſſes to the more perfect one of the two. Thus, if a ſmooth glaſs tube (fig. 1, plate 19,) be drawn through the hand, the effect of the friction makes the electric fluid leave the hand and paſs to the glaſs tube, which is the more perfect electric of the two, where it will remain in addition to its natural quantity. For the electric fluid cannot poſſibly leave the glaſs, becauſe neither the glaſs nor the ſurrounding air are conductors of electricity; but if a conducting ſubſtance, as the finger or a piece of metal be preſented to any part of the glaſs, the electric fluid will leave the glaſs, and paſs into them; and if the finger or metal be preſented to every part of the tube ſucceſſively, the whole of the redundant fluid will leave the tube, and it will retain only its natural

natural ſhare. Here the glaſs is ſaid to be *excited*, becauſe the friction ſeems to excite the electric power which was in the glaſs.

In the ſame manner the friction of the glaſs globe, againſt the rubber in the electrical machine, makes the electrical fluid which was in the rubber, paſs on the glaſs, from whence it is conveyed to the prime conductor, the points of which are preſented to every part of the globe in ſucceſſion, as it is turned into the machine; and as the friction is continued, there will be a conſtant ſupply of electric fluid to the prime conductor, though other bodies be preſented to it, and keep diſcharging all the while in viſible ſparks. The hand in the former of theſe caſes, and the rubber in the latter, part with their natural ſhare of electric fluid to the glaſs againſt which they were rubbed, but receive an immediate ſupply from the conducting ſubſtances to which they are connected; and theſe are again ſupplied by the general maſs of fluid that is in the Earth.

Again, if a ſtick of ſealing wax, a piece of ſulphur, or a tube of rough glaſs, be drawn through the hand, the electric fluid belonging to them will paſs from them to the hand, and being ſurrounded by the air, which is a non-conductor, they remain exhauſted, and are ready to take ſparks of electric fire from any bodies preſented to them. The ſulphur, ſealing wax, &c. in this caſe, are ſaid to be excited, as well as the glaſs, which was overloaded with fluid; though the ſtate they are in be the reverſe of one another. It is impoſſible to diſtinguiſh, by the eye, the courſe of the electric matter, its velocity is ſo great.

There are a variety of inventions for the conſtruction of the electrical machine; but the moſt ſimple is that repreſented (fig. 2.) which, by reaſon of its ſimplicity, is not liable to be put out of order, as it has neither wheel nor ſtring, though both might be attached thereto, if required. It may alſo be fixed firm on a table, and eaſily taken off:—

The globe may alſo be taken out with the greateſt eaſe, in order to be packed up. This machine is the ſame as that uſed by Mr. *Prieſtly*, and when the inſide of the globe is lined with his compoſition following, it will produce more fire than any of thoſe in common uſe.

A is the baſe, which is a piece of mahogany about nine inches ſquare, and 1 ¼ thick, in which is fixed the pedeſtal B to ſupport the globe G, which is fixed in an iron axle C, to which is fixed a braſs cap. The globe is turned by the handle H running in the braſs ſocket E; R is the rubber, made of wood, cut to the curve of the globe, and covered with a leather covering, which is at a little diſtance from the wood in the middle of the curve, that it may the better yeild to the preſſure of the globe.

Over this leather is another leather, made to take off by moving a pin. On this leather the amalgam is rubbed, and as it is eaſily taken off it is more readily brought into order than thoſe which are fixed to the rubber. To this leather is fixed a piece of black ſilk, which extends half round the globe, and greatly increaſes the fire; ſo that this machine will give fire well if the rubber ſcarcely touch the globe. This machine will alſo ſuit any kind of conductor.

For thoſe who do not chuſe to have the rubber inſulated, there is a ſpring S; but the more curious may have them made with the rubber well inſulated by a glaſs pillar, that will hold the rubber to the back of the globe, as in fig. 3.

Mr. Prieſtly's compoſition for lining the inſide of Globes, or Cylinders.

This compoſition conſiſts of an equal quantity of linſeed oil and reſin, which is boiled over a gentle fire for two hours. When the globe or cylinder is to be lined, it muſt be put into an oven, with a ſufficient quantity of the compoſition broken, and put into the inſide, and when it is melted,

melted, the globe is turned round every way, in order to ſpread it all over equally.

Inſtead of the above compoſition, ſome uſe a compoſition made of four parts of Venice turpentine, one part of reſin, and one of bees-wax; which is prepared and uſed in the ſame manner as the former.

There are alſo other methods for making amalgam, as,—1. By four parts of ſpelter and ſix parts of mercury.—2. Alſo by adding 6 ounces of quickſilver to 1 pound of molten tin, which, when cold, and reduced into powder, is to be mixed with 7 ounces of ſulphur, and 6 ounces of ſal ammoniac; the whole is ſublimated in a mattreſs.

The parts of the machine which are inſulated ſhould be varniſhed over with a varniſh made of highly rectified ſpirits of wine and ſealing-wax; as alſo the glaſs pillars, in or er to keep off the moiſture they would imbibe from the damp air.

It is neceſſary for the young practitioner to attend to the following rules, in the performing of his experiment; as it will often happen, that though he be in poſſeſſion of very good inſtruments, yet, through ſome inadvertencies, his experiments will not ſucceed according to his expectation, for want of a ſufficient practice in the art.

1. The electrical machine, coated jars, and every part of the electric apparatus, ſhould be kept clean, and free from duſt and moiſture.

2. In clear weather, when the air is dry, and particularly in froſty wheather, the machine will always work well; but in hot weather, and damp weather, except it be brought in a warm room, and the apparatus made thoroughly dry, it will not work ſo well.

3. The cylinder ſhould always be wiped clean with a ſoft dry linen cloth that is warm, and then with a clean hot flannel before the machine be uſed; then applying a little amalgam, turn the winch of the machine, and the electric fluid will

 from

come like a wind from the cylinder to the knuckle, and some sparks and cracking will soon follow. This indicates that the machine is in good order. But if these appearances be not produced, there is a fault, which is generally in the rubber; to remedy which, remove the rubber from the glass pillar, and dry the silk part before the fire, then grease the leather with a bit of tallow or mutton suet.

4. When the table on which the machine stands, and to which the chain of the rubber is connected, is very dry, it is a bad conductor, and hinders the operation of the machine. The floor, and walls of the room also, in very dry weather, have the same effect on the machine. In this case the chain of the rubber should be connected by a long wire, with some moist ground, or with the iron-work of a water-pump; by which means the rubber will be supplied with a sufficient quantity of electric fluid.

5. If there be too much amalgam upon the leather of the rubber, the machine will not work well until a little be scraped off.

6. If the globe or cylinder contract any black spots, as is often the case, they should always be wiped off.

7. In charging electric jars, they should be made a little warm before they are used, and they will produce a greater effect.

8. When a large battery is to be discharged, never discharge it through a good conductor, except the circuit be at least five foot long, otherwise some of the jars would be found broken in it.

To shew the effects of electrical attraction and repulsion.

Suspend a plate of metal (fig. 2.) F from the conductor, which is supported by two glass pillars, and supplied with electricity from the globe; and at the distance of three or four inches below this put another plate P, of the same size; upon the bottom

bottom plate lay a feather, or ſmall ſlips of paper; and when the machine is ſet in motion, the feather or the papers will be attracted to the upper plate F, from which they will be immediately repelled, and will fly to diſcharge themſelves upon the lower plate P, which is ſupported on the pedeſtal G H; after which, they will be attracted and repelled again as before, and fly from one plate to the other with great rapidity, if the electrification be ſtrong. It is uſual to cut the pieces of paper into the figures of men and women, when they exhibit a kind of dance, which affords ſome entertainment to the beholders.

The electrical bells are often uſed in concert with the above experiment, and depend on the ſame principle. Theſe are four bells *a*, *b*, *c*, *d*, which hang from the ends of two braſs rods, (fig. 5.) communicating with the prime conductor, and with another bell *e*, fixed on a pedeſtal A, reaching to the ground. Between the four bells hang four braſs balls, ſuſpended by ſilken ſtrings; each of theſe balls hang between the centre bell *e*, and one of the outermoſt bells. The outermoſt bells being connected with the prime conductor by braſs chains, are electrified and attract the braſs balls, which hang between them, and the centre bell; and the attraction being ſtrong, each ball ſtrikes its outer bell with ſome violence, and makes it ring: being then loaded with electricity, it is immediately repelled, and flies to unload itſelf by ſtriking upon the centre bell, which is inſulated by the glaſs pillar B upon the pedeſtal A, and from which pillar the electric matter paſſes to the floor, by means of the braſs pedeſtal A. The balls are then again attracted by the outermoſt bells as before; and thus the ringing may be continued as long as neceſſary.

When a perſon is to be electrified, and ſtands upon a ſtool with glaſs feet, or baked wood, (fig. 7.) having the chain in his hand, faſtened to the prime conductor, he is then ſaid to be inſulated, and may be conſidered as part of the prime conductor;

conductor; for every part of his body will exhibit the ſame appearance as the prime conductor. For, if the finger of any perſon ſtanding upon the floor be preſented to him, a ſpark of fire will be ſeen to iſſue from him, and both he and the perſon that receives it will feel a painful ſenſation, and a ſnapping noiſe will be heard. Every part of his body will then attract all light ſubſtances, as feathers, bits of paper, &c. The hairs of his head alſo, or of his wig, if they happen to be looſe, will repel each other, and many of them ſtand upright.

Pointed bodies have a remarkable property in electricity: for the more acutely pointed any body is, the more eaſily does it take, or part with, electric matter. Thus, if a needle, or ſharp-pointed wire, be faſtened to the prime conductor, it will retain but a ſmall degree of electricity, and conſequently will give but a ſmall ſpark when the finger or a piece of metal is preſented to it. Again, if the needle or wire be held in a perſon's hand, ſtanding upon the floor, and preſented to the conductor, it will be found to retain but a ſmall degree of electricity. In the former caſe, the needle being in contact with the prime conductor, the electric fluid went off at the point, and was diſperſed in the air. In the latter caſe, the needle being preſented towards the conductor, received the electric fluid from it at a conſiderable diſtance.

If the ſharp-pointed wire be given out the electric fluid, the flame will be larger, (for a flame will be ſeen at the point of the needle or wire, if the experiment be made in the dark) the parts of which it conſiſts will be fewer, and a ſnapping noiſe will be heard if the point be not very acute; whereas, if the pointed wire be receiving the electric fluid, the flame will be much ſmaller and more globular; the parts of which it conſiſts will be more in number; and inſtead of the ſnapping noiſe there will be a kind of hiſſing. The flame iſſuing from a body, is called a pencil,

on account of its oblong ſhape: and when the rays come to a point, they project more equally from the centre; and it is then called a ſtar.

As pointed bodies tranſmit the electric fluid with ſo much eaſe, it affords an opportunity of proving the identity of lightning, and the electric fluid; for, if a long rod or pole, having a ſharp pointed wire at the end of it, be ſupported by electric ſubſtances, the point projecting towards the clouds, will draw the electric matter from them, and become ſenſibly charged with electricity, as if it had been connected with the prime conductor of an electrical machine. It will attract all light bodies, and ſparks of electric matter may be drawn from it; and, in ſhort, it will exhibit every appearance of common electricity; and on the other hand, by common electricity, may be produced in miniature, all the known effects of lightning.

This diſcovery was effected by Dr. *Franklin*, by raiſing a kite, called the Electrical Kite; and formed of a large thin ſilk handkerchief, extended and faſtened at the four corners to two ſlender ſtrips of cedar, and accommodated with a tail, loop, and ſtring, ſo as to riſe in the air like a common paper kite. To the top of the upright ſtick of the croſs was fixed a ſharp-pointed wire, extending a foot upwards, above the wood; and to that end of the twine which is next to the hand, was tied a ſilk ribband. At the junction of the twine and ſilk was ſuſpended a key, from which, when the kite is raiſed during a thunder ſtorm, a phial may be charged, and electric fire collected, as by means of an electrical machine. From which it appears, that points have a remarkable property, both of throwing off, and receiving the electric fluid; from whence has riſen that uſeful invention of applying metallic conductors to houſes or buildings, in order to preſerve them from the dreadful effects of lightning, as will be hereafter ſhewn.

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SECT. II.

ELECTRICAL EXPERIMENTS.

1. CUT a piece of tin in the form of a ſtar, and let it be ſupported on its centre by a wire projecting from the prime conductor; then, as ſoon as the machine is ſet in motion, and the ſtar electrified, a flame will appear at the extremity of every point of the ſtar, which will have a beautiful appearance, if the experiment be made in the dark. And if the ſtar be made to turn ſwiftly on its centre, an entire circle of fire will be ſeen. This experiment may be rendered more diverting, if the operator now and then touch the prime conductor with his finger, or a piece of metal; for by theſe means he will make it diſappear, and appear again at pleaſure; for, in every experiment, if the prime conductor be touched, the effect of the experiment will be ſtopped. And, if inſtead of the ſtar, two ſharp-pointed wires be uſed, with the four ends at right angles, and in the ſame plane, but pointed different ways, and turning upon a centre; when it is electrified, a flame will be ſeen at each of the points, and what is more ſurpriſing is, that the wires will begin to turn round of themſelves, and in the direction oppoſite to that to which the points be turned; and if the electrification be continued, the motion will become more rapid. It is by this experiment that what is called the Electric Horſe Race is performed; which is done by cutting the figures of horſes in paper, and faſtening them ſo, that the points of the wires may be in *their tails*; by which they will ſeem to purſue one another, though without a poſſibility of any one overtaking the reſt.

2. Preſent

2. Preſent the point of a wire, which is fixed on the prime conductor, to the inſide ſurface of a glaſs tumbler, graſping it on the outſide with the hands. The glaſs then will ſoon become charged with electric matter: its inſide ſurface acquiring the electricity from the point of the wire, and its outſide ſurface looſing its natural quantity of electric fluid through the hands, and which, in this caſe, ſerves as a coating to the glaſs. Then put a few pith-balls upon the table, and cover them with this glaſs tumbler, and they will immediately jump up by the ſides of the glaſs, (fig. 9,) and continue in motion for ſome time, being attracted and repelled by the electric fluid, upon the ſurface of the glaſs, which they gradually conduct to the table; the outſide of the glaſs acquiring the electric fluid, from the ſurrounding air.

3. Tie the ſilk ſtring G, (fig. 10,) to a crooked glaſs, tube A E F B, by which it may be held: fill the middle part of this tube E F with reſin, ſealing-wax, or any other electric ſubſtance: then fix the two wires A E, B F, in the ſealing-wax, &c. Hold the tube over a clear fire to melt the reſin or ſealing-wax within it; at the ſame time connecting one of the wires A or B, with the outſide of a charged Jar, and touching the other wire with the knob of the jar; then endeavour to make the diſcharge through the reſin, wax, &c. and it will be obſerved, that while the reſin is cold, no ſhock can be tranſmitted through it, but as it melts it becomes a conductor; and when perfectly melted, the ſhocks paſs through it freely: by which, it may be ſeen, that glaſs and other electrics, become conductors when they are made very hot.

4. A, (fig. 11,) repreſents the ſide of a houſe, being a board, about three quarters of an inch thick. This is fixed perpendicularly upon the bottom board B, upon which is alſo fixed in a hole in the ſame board, the glaſs pillar C D, about eight inches diſtant from the board A. In the board A, is a ſmall ſquare hole I L M K, about a quarter of an

inch deep, and an inch wide, which is filled by another ſmall board nearly of the ſame dimenſions; and made to fit eaſy in the hole; ſo that it may drop out by any ſudden ſhock. This ſmall piece of wood, repreſents a ſhutter, or door, in the ſide of the houſe A. L K is a wire faſtened diagonally to this piece of wood. I H is another wire of the ſame thickneſs, having a braſs ball H, ſcrewed on its upper point. M N, is another wire, turned into a ring at O: theſe three wires are all fixed to the board A. From the upper extremity of the glaſs pillar C D, proceeds a crooked wire, having a ſpring ſocket F, through which, is a double knobbed wire; the lower nob G, falls juſt above the nob H of the conductor. The glaſs pillar D C muſt be fixed in the board looſe, ſo that it may be eaſily moved round; by which the braſs ball G may be brought nearer, or further from the ball H, without touching the part E F G. Now, when the ſquare piece of wood M L I K, is fixed in the hole, ſo that the wire L K may ſtand in the dotted line I M, then the metallic communication from H to O is complete; and the inſtrument exhibits a houſe furniſhed with a proper metallic conductor: but if the ſquare piece of wood is ſo fixed, that the wire L K ſtands as repreſented in the figure; then the metallic conductor H O, which goes from the top of the houſe to the bottom, is interrupted at I M; in which caſe, the houſe is not properly ſecured. Then let the ball G, be about half an inch in perpendicular diſtance from the ball H; and by turning the glaſs pillar D C, the former ball will be removed from the latter; then by a wire or chain, connect the wire E F, with the wire Q, of the jar P, and let another wire or chain, faſtened to the hook O, touch the outſide coating of the jar P. Let the wire Q be connected with the prime conductor of the machine, and charge the jar: then by turning the glaſs pillar D C, bring the ball G, gradually near the ball H, and when they approach ſufficiently near one another, the jar will explode, and the piece of wood L M I K will be puſhed

out

out of the hole, to a conſiderable diſtance. In this experiment, the ball G, repreſents a thunder cloud, which being arrived ſufficiently near the top of the houſe A, the electricity ſtrikes it; and as the houſe is not ſecured with a proper conductor, the exploſion will break part of it, by knocking out the piece of wood I M.

Again, let the piece of wood I M, be ſo ſituated, that the wire L K may ſtand in the direction I M; in this caſe, the conductor H O, is not diſcontinued: and repeating the experiment as before, it will be ſeen that the exploſion will have no effect upon the houſe; as the piece of wood L M will remain in the hole unmoved: which ſhews the uſefulneſs of a metallic conductor. The inſtrument uſed in this experiment, is called the Thunder Houſe, as it ſhews the effect it has upon an houſe, both ſecured, and unſecured.

5. The *Electrical Battery*, is the moſt formidable and entertaining part of electricity; and is formed of a number of glaſs coated jars, connected together, ſo that their whole force may be united. And if a battery of no great power is required, as containing about eight or nine ſquare feet of coated glaſs, common pint, or half pint phials, will anſwer the purpoſe very well: but when a large battery is required, it is neceſſary to have cylindrical glaſs jars, of about fifteen inches high, and four or five inches in diameter.

The beſt method of coating theſe jars, is to coat them with tin-foil on both ſides, which may be fixed upon the glaſs, with paſte made of wheat flour: but in coating the inſide of phials or jars, whoſe mouths are not large enough to admit the tin-foil, braſs-filings are uſed, mixed with gum-water, or bees-wax, &c. And the coatings ſhould not come within two inches of the mouth of the jar; otherwiſe, the jar may diſcharge itſelf. Some kinds of glaſs, is not capable of holding any charge; the jars or phials ſhould therefore be examined before any experiment be performed.

A very good battery may be formed of twelve jars, coated

on both ſides with tin-foil, containing in the whole, about twelve ſquare feet of coated glaſs. In the middle of each jar, is a cork, that ſuſtains a wire, which, at the top, is faſtened to the wire E, knobbed at each end, (fig. 12,) and which connects the inſide coatings of three jars; and by four ſuch wires, the inſide coatings of all the other jars are connected together. Each of the wires F, has a ring at one end, through which, one of the wires E paſſes; and the other end has a braſs knob, reſting on the next wire E. If the whole force of the battery is not required, one, two, or three rows of jars may be uſed at pleaſure. The wooden box, that contains theſe jars, is lined at the bottom with tin-foil. It has a hole on one ſide, through which an iron hook paſſes, communicating with the metallic lining, and conſequently, with the outſide coating of the jars; to this hook, is faſtened a wire, the other end of which is connected with the diſcharging rod.

The diſcharging rod, conſiſts of two curved wires, B B, (fig. 4,) which move by a joint C, fixed to the braſs cap of the glaſs handle A. The wires are pointed at the ends, on which points are ſcrewed the two knobs D D, ſo that it may be uſed either with the points or knobs. When a large battery is required, it is better to uſe two, three, or more ſmall ones, and their force may be united by a wire or chain; but the beſt method of uniting their force, is to have a wire from every jar, connected at the top with a ball in the form of a wire cage.

The force of electricity, thus accumulated by ſeveral jars or batteries, is aſtoniſhing. Metals which reſiſt the greateſt effect of chemical fire, are inſtantly made red hot, and melted. But in performing experiments of this kind, the operator ſhould be careful that no perſon touches, or even come too near any part of the apparatus; otherwiſe, it may produce ſerious conſequences. And it is to be obſerved, in charging a battery,

a battery, a ſmall conductor is more proper, than a large one, as the diſſipation of the electricity is not ſo great.

6. The experiment, called animating the ſpider by electricity, is performed by ſuſpending a piece of cork B, (fig. 14,) by a ſilk thread; in the cork, a few ſhort threads are drawn, to repreſent its legs. It is to be hung in the midway, between the knob E of the wire D E, which is connected with the jar A D and the knob A: then the jar being charged by connecting its knob A, with the prime conductor, the ſpider will be attracted by the knob A, and then repelled by it to the knob E, where it diſcharges its electricity; and is then again attracted with the knob A, and again repelled to the knob E; and will continue this motion, till it has completely diſcharged the jar.

7. The ſpiral tube, is compoſed of two glaſs tubes C D, (fig. 18,) one within the other, the ends being cloſed with two braſs caps A and B. On the outſide of the innermoſt tube, is ſtuck a ſpiral row of ſmall round pieces of tin-foil, about a twelfth of an inch diſtant from each other; then holding it by one end, and preſenting the other end to the prime conductor, ſmall ſparks will appear between all the pieces of tin-foil, and in the dark, it will have the appearance of a ſpiral line of fire. If, inſtead of the ſpiral tube, the tin-foil be ſtuck upon a flat plate of glaſs A B C D, (fig. 19,) it may be ſo formed to repreſent any other figures, letters, flowers, &c.

8. A B D, (fig. 16,) is a wire about ten feet long, at the ends of which, is fixed a piece of glaſs G, to keep the ends A B at a proper diſtance, and to let them ſlide within half an inch of each other, if required; then connect the chains belonging to the ſliding wires, with the hook of the battery, and the diſcharging rod, and ſend the charge of the battery through them. On making the exploſion, a ſpark will be ſeen between A and B; which proves that the electric fluid chuſes a ſhort paſſage through the air, rather than a long one,

one, through good conductors; for very little of the electric fluid, will pass through the bent wire A D B.

9 Roll a piece of tobacco-pipe clay in the form of a small cylinder, and in the two ends insert two wires A B, (fig. 22.) so that their ends, within the clay, may be within a fifth part of an inch of each other. Then, if a shock be sent through this clay, by connecting one of the wires with the outside of a charged jar, and the other wire with the inside, it will be inflated by the spark that passes between the two wires, as represented in fig. 23. If the shock be too strong, and the clay not very moist, it will be broken by the explosion, and its fragments scattered in every direction; as may be proved by using a piece of the tube of a tobacco-pipe, instead of the clay.

10. Immerse two knobbed wires, A B, (fig. 17.) in a glass of water, so that the knobs of the wires may be within a little distance of each other. Then, if one of these wires be connected with the outside coating of a jar, and the other wire be touched with the knob of it, the explosion in passing through the water, from the knob of one wire to that of the other, will break the glass with a surprising violence; and great caution is necessary in performing this experiment, as it is sometimes attended with danger. If, instead of a drinking glass, a glass tube be used, stopped with a cork at each end, through which the wires are inserted, and the charge be very weak, the electric spark will appear in the water passing between the wires.

11. A B (fig. 24.) is the electrical thermometer, and consists of a glass tube, about ten inches long, and nearly two inches in diameter, and closed air-tight at both ends by two brass caps. H A is a small tube, open at both ends, passing through a hole in the upper cap, and immersed at the bottom in some water at B, in the bottom of the large tube. F G, and E I are two wires, inserted through the middle of each of the brass caps, and having a brass knob at the head of

of each, within the braſs tube. This inſtrument is faſtened to the pillar C D, by a braſs ring C. When the air within the tube A B is rarified, it will preſs upon the water at the bottom of the tube, and ſo cauſe it to riſe in the ſmall tube; and the riſe and fall of the water ſhew the rarification of the air in the glaſs tube A B, which has no communication with the external air.

If the knobs G I of the two wires be brought into contact with each other, and the ring E or F be connected with one ſide of the charged jar, and the other ring with the other ſide, and a ſhock be made to paſs through the wires, the water in the ſmall tube will not be at all moved; which ſhews that the paſſage of the electric fluid, through conductors, ſufficiently large, occaſions no rarification of the air. But if the knobs G I be placed a little diſtant from each other, and the ſhock ſent through the wires as before, the ſpark between the two knobs will conſiderably rarify the air, and the water will be ſuddenly forced up the ſmall tube quite to the top.

12. To ſhew the courſe of the electric fluid in the diſcharge of a jar, and to make it viſible by the ſtar and pencil For this purpoſe, the jar muſt be charged; then taking the diſcharging rod, (fig. 4.) without its knobs, preſent one point within an inch of the knob A, (fig. 15.) and the other point at an equal diſtance from the outſide coating of the jar. By theſe means the jar will be diſcharged ſilently; and if its inſide be electrified poſitively, the point C of the diſcharging rod will be illuminated with a ſtar, becauſe it receives the electric fluid; and the point B, with a pencil, becauſe it gives out the electric fluid to the outſide of the jar; and if the jar be electrified negatively on the inſide, the pencil will appear upon the point C, and the ſtar upon the point B.

13. The univerſal diſcharger is an inſtrument of very extenſive uſe, and is compoſed of the following parts:—A is a flat board, about fifteen inches long, four broad, and one thick.

thick. (fig. 8.) B B are two glafs pillars, cemented in two holes in the board. At the top of each is a brafs cap, having a turning-joint and a fpring tube, through which flide the wire C D. Thus, each of thefe wires have two other motions, viz. an horizontal and vertical one; each wire is alfo furnifhed with an open ring at one end C, and a brafs ball at the other end D, which ball may be taken off at pleafure. E is a ftrong circular piece of wood, 5 inches in diameter, on the furface of which is a flip of ivory; and furnifhed with a ftrong cylindrical foot which fits the focket, and which, by means of the fcrew G, may be made faft, and alfo raifed higher, or brought down lower.

The *Leyden Phial* is an inftrument to prove the hypothefis of a fingle electric fluid, and is formed by coating a fmall phial about three inches up the outfide, with tin foil, (fig. 20.) To the top of the neck a brafs cap is cemented, having a hole with a valve; from this cap proceeds a wire, being blunted at the point, and terminating a few inches within the phial. When the phial is exhaufted of air, a glafs ball is fcrewed on the brafs cap to prevent any air from getting into the phial. This phial fhews the direction of the electric fluid, both in charging and difcharging; for, if it be held by its bottom with the brafs knob prefented to the prime conductor, which is pofitively charged, the electric fluid will caufe the pencil of rays to proceed from the wire within the phial, as in fig. 21; but if it be difcharged, a ftar will appear inftead of the pencil, as in fig. 20. But if the wire be held by the brafs cap, and its bottom be touched by the prime conductor, the point of the wire on its fide will appear illuminated with a ftar when charging, and with a pencil when difcharging. If it be prefented to the prime conductor, electrified negatively, all thefe appearances both in charging and difcharging, will be reverfed.

Inflammable air, that will take fire by the electric fpark, is thus made: A D, (fig. 25.) reprefent two bottles; in the bottle

bottle D is put two or three ounces of filings of iron, and ſome oil of vitriol, mixed with four times its quantity of water; the bottle A is filled with water, and the bent glaſs pipe C. is fixed with one end, air-tight, into the neck of the bottle B, and the other end a little way up the neck of the bottle A; in a ſhort time the mixture will boil and emit a fluid which will paſs through the tube C, into the bottle A, and at length fill it, expelling the water into the baſon B. The bottle A, is then to be quickly corked up for uſe.

The *Electrical Piſtol* is repreſented, (fig. 26.) where *c a* is of thin braſs; to the mouth *a b* is fitted a cork, and a perforated piece of braſs, *e* ſcrews on the bottom of the piſtol at *e*, having a glaſs tube with a wire cemented into it, bent over the glaſs tube ſo as to reach within one-eighth of an inch of the braſs; when the piſtol is to be charged uncork the inflammable air bottle before-mentioned, likewiſe the piſtol, and place the mouth of the piſtol upon the top of the bottle, and the common air, which is within the piſtol will deſcend, while the other aſcends. Having held the piſtol in this ſituation a few ſeconds, in order to fill it with inflammable air, cork both it and the bottle expeditiouſly, and it is then charged. When it is to be diſcharged, fill a ſmall jar, or a hollow handle, and apply it to the knob of the wire *e*: it will then explode and draw out the cork to a conſiderable diſtance, with a report as loud as that of a piſtol filled with gunpowder.

Fig. 28, repreſents an inſtrument to cure the tooth-ach, in which, A is a flat piece of box wood, about an inch broad, and a quarter of an inch thick. Near its oppoſite edges are made two longitudinal holes, through which are put two braſs wires, *a b c*, and *d e f*, and fixed in with ſealing-wax, and then bent at *c* and *f*, as in the figure, which two points ſerve to receive the tooth and gum between them. When the inſtrument is uſed, hook two chains, *g* and *h*, on the lower end of the wires, holding the tooth and gum between

the other end of the wires, *c* and *f*: put the end of the chain *g* round the bottom of an electric jar, and let a perſon hold the chain *h*, hanging down from his hand, and both chains being clear of the table, and not touching each other. Then, having charged the jar, let the perſon who holds the chain *h*, ſtrike the end of it againſt the prime conductor; this will diſcharge the jar, and give the patient a ſhock only in his tooth and gum, and which ſeldom fails to cure the tooth-ach, if the cauſe be a cold.

The *Electrometer* is an inſtrument to ſhew the kind and quantity of electricity, of which there are ſeveral ſorts; the moſt ſimple is that which conſiſts of a linen thread, having a ſmall cork or pith ball at each end, (fig. 13.) This electrometer is ſuſpended by the middle of the thread on any conductor proper for the purpoſe. And if the conductor be charged poſitively, by applying a ſtick of ſealing-wax, excited, the balls collapſe together: and by applying an excited, ſmooth glaſs tube, they will recede further aſunder; and if the conductor be charged negatively, the reverſe will take place.

But the moſt perfect of theſe inſtruments is that called the Quadrant Electrometer, which ſhews the exact degree to which any body is electrified, and is as follows:—A is a fine rod that turns on B, the centre of a ſemi-circle, (fig. 27,) ſo as always to keep near its graduated limb, which is divided into one hundred and eighty degrees. At the end of the rod is a cork ball C; the pillar D may be either fixed to the prime conductor, or to the braſs knob of a jar or battery, or be ſet on a ſtand by itſelf. The inſtrument ſhould be made of box wood, and the ſemi-circle of ivory.

When this inſtrument begins to be electrified, the rod A is repelled by the pillar D, and conſequently, begins to move over the edge of the ſemi-circle, and ſhews very exactly the degree to which the conductor is electrified, or how high any jar or battery is charged. This inſtrument

ſhould

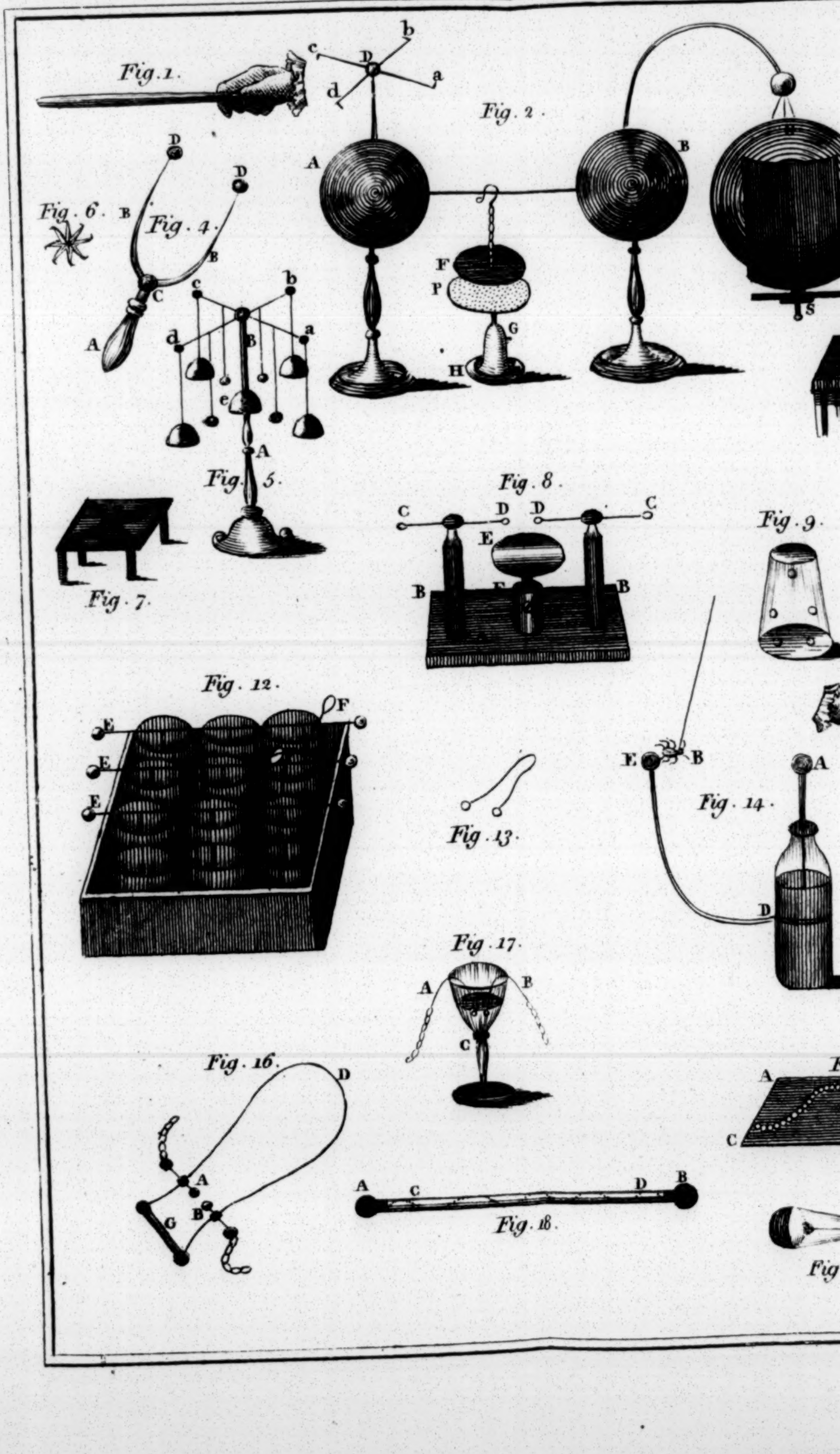
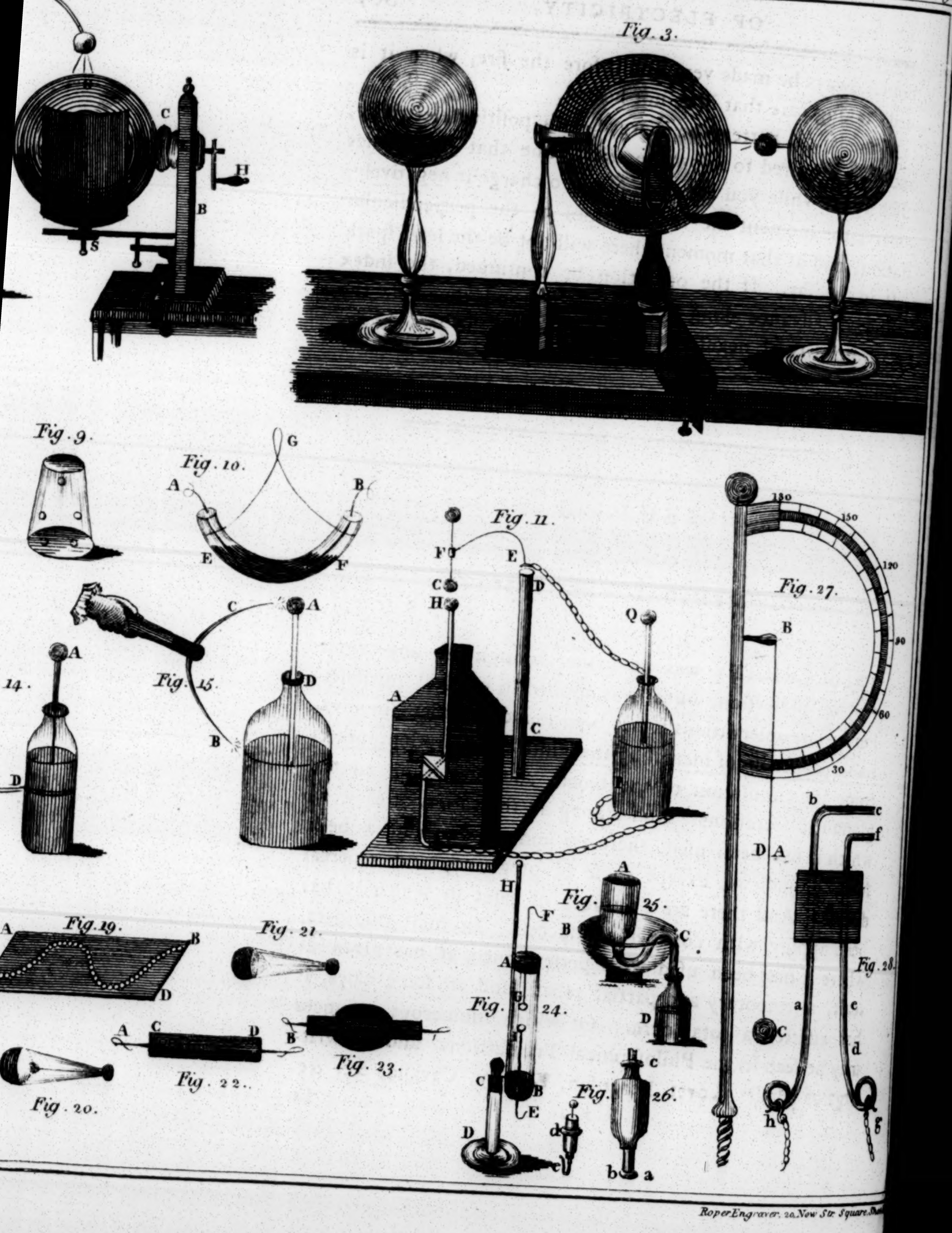


Fig. 1.
Fig. 2.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 16.
Fig. 17.
Fig. 18.

ELECTRICITY.

Plate 19.



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ſhould always be made very dry before the fire, when it is uſed, taking care that it be not heated.

If the jar or battery be charged with poſitive electricity, and it be required to know the exact time that it becomes diſcharged, while you are attempting to charge it negatively, obſerve the moment the index comes to the perpendicular ſtation, and at that moment there will not be the leaſt ſpark left in the jar. If the operation be continued, the index will again advance along the ſemi-circle: and thus ſhew the exact quantity of negative electricity, which the jar has acquired.

SECT. III.

OF MEDICAL ELECTRICITY.

ELECTRICITY was no ſooner brought to any degree of perfection, but it was applied to medical purpoſes. For by late obſervations it has been found to poſſeſs the invariable properties of increaſing the ſenſible perſpiration, quickening the circulation of the blood, and promoting all the glandular ſecretions. And among all the variety of caſes, in which it has been uſed, there are none in which it has been found prejudicial, except thoſe of pregnancy and venereal diſeaſe. And there are a number of caſes in which it has been applied with conſiderable ſucceſs. In moſt diſorders, where it has been uſed with perſeverance, it has given at leaſt, a temporary and partial relief; and in ſome caſes, it has effected a total cure. Of which, numerous inſtances may be ſeen in the Philoſophical Tranſactions, and the writings of Meſſrs. Lovet, Ferguſon, Weſtley, Cavallo, &c. &c.

To know what cases are proper to be electrified, experience shews in general, that all kinds of obstructions, whether of motion, of circulation, or secretion, are very often removed, and in general, alleviated by electricity. Likewise, nervous disorders have very often been cured; and rheumatic disorders, even of a long standing, are always relieved, and very often quite cured, by only drawing the electric fluid with a point from the afflicted part, or by drawing sparks from the conductor. It has also been found very beneficial in diseases of a long standing; and has not unfrequently been found a powerful remedy in muscular contractions.

There are three instruments generally used for administering medical electricity, besides the electrical machine, viz.— An electric jar, with Mr. Lane's electrometer; an insulated chair or stool, upon which, a common chair may be occasionally set; and the directors.

The jar used on this occasion, should be coated with tinfoil, and should be about 4 inches in diameter, and 6 in height, containing about 72 square inches of coated surface. Through the covering of the jar, passes a brass wire B, (fig. 1, plate 20,) touching the inside coating of the jar, and having a brass ball F, to which the electrometer F D C is fastened, and terminating at the top in a brass ball B, which is to touch the prime conductor and which is supposed to stand before the electrical machine. The electrometer C E D F, consists of a piece of glass F D cemented to the two brass caps D and F; from the former of which, proceeds a strong perpendicular brass wire, having at the top an horizontal spring socket, through which slides the wire C E, having the brass ball C at one end, and an open ring E at the other end: and so fixed, that the ball C is exactly the same height as the ball B, and may be set at any required distance from the ball B. This distance seldom exceeds half an inch, therefore, the electrometer may be made very small. Sometimes

times there is a ſcale on the wire C E, which ſerves to ſet the balls B C to any given diſtance from each other, with more certainty. When this inſtrument is uſed, the jar is ſo placed, that the ball B, may touch the conductor. Then ſuppoſe the ball C to be ſet at one tenth of an inch diſtant from the ball B, and the electrometer E, be connected to the outſide coating of the jar at I by a chain *x*. In this caſe, if the electrical machine be put in motion, the jar will be charged; and when the charge is ſo high, that the electric fluid accumulated within the jar, can paſs from the ball B to C, which is here ſuppoſed to be one tenth of an inch; the diſcharge will take place, the ſpark will appear between the balls, and the ſhock will paſs through the chain *x*, from E to I; for the part F D being of glaſs, and generally covered with ſealing-wax, is impervious to the electric fluid; therefore, the electric fluid has no way to paſs from the inſide to the outſide of the jar, but from the ball B to the ball C, and along the wire C E, and from thence along the chain *x*.

When the electrical ſhock is to be adminiſtered to any part of the human body, (as for example to the arm,) inſtead of the chain which muſt now be taken away, two ſmall pliable wires E L, I L, are to be faſtened, one to the ring E, and the other to a hook I, of the ſtand H I, which communicates with the outſide coating of the jar, (if the jar have not the ſtand H I, the extremity of the wire I, may be put in contact with the outſide coating of the jar in any other convenient manner;) the other end of the ſaid wires are faſtened to the braſs wires L L, of the directors K L. Each director conſiſts of a knobbed braſs wire L, connected to a glaſs handle K, by means of a braſs cap. Then the operator holding the directors by the extremities of the glaſs handles, brings their balls into contact, with the extremities of that part of the patient's body, through which the ſhock is to be ſent. Then it is evident from a view of the figure, that the diſcharge of the jar, muſt be made through that part

part of the patient's arm, which lies between the two knobs of the directors, as in the former case, the discharge was made through the chain *x*. Thus, the operator has nothing more to do, but to hold the knobs of the directors to the extremities of that part of the body, through which the shock is to pass, while an assistant keeps the machine in motion. Care must be taken that the two wires E L and I L do not touch each other; otherwise, the shock will not pass through the patient's body. By these means, any number of shocks, precisely of the same strength, may be given without altering any part of the apparatus. And when it is required to increase or diminish the force of the shock, it is only necessary to increase or diminish the distance between the balls B and C, which is done by moving the wire C E through the socket.

It is of little consequence, whether the patient stands upon the ground, upon the insulated stool, or in any other situation. Neither is it necessary to remove the clothes from the part that is to be electrified; for the shocks will readily go through them, except there be too many coverings.

In the application of electricity, the chief difficulty consists in distinguishing the proper strength of the electric force, that is requisite for a given disorder. For this purpose, it is impossible to give any general rules, the circumstances being of so complex a nature, that nothing but long experience, and strict attention to every particular phenomenon, can direct the operator. It need hardly be said, that regard must be had to the sex, and condition of the patient: and the surest rule that can be given, is to begin with more gentle treatment, at least, such that, considering the circumstances, may be thought rather weak, than strong. If, after a few days trial, this gentle treatment be found ineffectual, then the operator may gradually increase the force of the electricity, until he finds the proper degree. But when any limb of the body is deprived of motion, it must be observed, that the

the cauſe is ſometimes a contraction of the muſcles, in which caſe, electricity has often proved an effectual remedy: but the loſs of motion is ſometimes occaſioned by a relaxation, as well as contraction; as when the hand is bent inwardly, and the patient has no power to ſtraighten it. In theſe caſes, it is often difficult to diſcover the real cauſe, but the ſureſt method is to electrify both thoſe muſcles, which are contracted, and alſo their antagoniſts; for no injury can attend electrifying a ſound muſcle. In rheumatic diſorders the electric fluid ſhould be drawn from the parts afflicted; or the ſparks may be drawn from the conductor.

The operation ſhould be continued for four or five minutes, and may be repeated once or twice a day. When the ſhocks are ſtrong, their greateſt number, at one operation, ſeldom exceed a dozen or fourteen, except they be given to different parts of the body.

The *Electrophorus*, (fig. 2,) conſiſts of two plates A and B, from 6 to 18 inches diameter in general, and ſometimes much larger. The upper plate is generally made of braſs; but a tin plate will ſerve the purpoſe, having a wire turned in upon its edge, in the common manner; on the centre of this plate, is fixed the ſocket O, in which the glaſs handle I is fixed, which is 9 or 10 inches long. When the electrophorus is to be of a large diameter, a thin board covered with tin-foil, and ſuſpended by ſilken ſtrings, will anſwer exceeding well.

The lower plate A, may be made either of glaſs, ſealing-wax, or the following compoſition, viz.—reſin four parts, pitch three parts, ſhell-lac three parts, Venice turpentine two parts, melted together over a gentle fire; then poured and ſpread upon a thin linen cloth, about a quarter of an inch thick. The linen cloth is then ſtretched upon a hoop, and made as tight as poſſible.

To charge a jar with this machine, rub the coated ſide of the under plate A, with a piece of fine new flannel, or a

hare

hare or cat-ſkin; and when it is excited as much as poſſible, ſet it upon a table, and place the upper plate B upon it, and put your finger upon the upper plate; then taking your finger off, take hold of the glaſs handle I, and apply it to the knob of a coated jar. When this operation is repeated thirty or forty times, the jar will become charged.

It was with a machine of this kind, that *Mr. Cavallo*, charged a coated phial ſeveral times by only once exciting, and ſo ſtrong as to pierce a hole through a card, at every diſcharge. If a plate of glaſs be coated with ſealing-wax and excited, and then laid with the wax ſide downwards; then on making the above experiment by putting the plate upon it, and taking the ſpark with the finger, and applying it to the glaſs handle, &c. it will have the contrary electricity to what it had before.

When it is required to diſcover whether a ſmall degree of electricity be poſitive or negative, or to know how the charge advances in uſing large batteries, and of what ſtrength it is, the moſt uſeful electrometer is *Mr. Canton's* balls, which are made of pith of elder, turned perfectly globular, and ſuſpended by fine threads, from the conductor, (fig. 3.)

To know whether the inſide of the jar or battery, be charged poſitively or negatively; the balls are to be preſented to the jar or battery, which ſtands upon the table; and they are immediately attracted by the wire, and diverge from each other. This is always the caſe in both poſitive and negative electricity; and the greater the diſtance to which the balls ſeparate, and the more they repel one another, the higher is the charge. To determine whether the electricity is poſitive or negative, rub a ſmall piece of glaſs againſt the hand or coat, which will excite it poſitively, and then preſent it to the balls in their diverging ſtate, and if it makes the balls converge, it ſhews they are electrified poſitively: but if it increaſe their divergency, it ſhews their electricity to be negative. And it muſt be obſerved, that the electricity of the balls

balls are always contrary to that with which they are charged (for they do not receive any electricity from the wires of the jar or battery;) for all bodies placed within the influence of electrified bodies, are affected with the contrary electricity.

But, to discover the kind of electricity. when the charge is very small, instead of the pith balls, a piece of downy feather should be used, suspended by a single silken thread, as it comes from the worm; or at least by a very few of those threads, to render it as light as possible. If any electrified body be presented to this, the feather will be repelled by it, if it be of the same kind with its own, and attracted by it, if the electricity be contrary to it; for this light body when once electrified, either positively or negatively, will retain its virtue a long time, with very little loss.

Notwithstanding electricity might be rendered so generally useful in the application of it to medical purposes, yet it is frequently found to be ineffectual, where it might be expected to prove the most salutary, which is very often the effect of ignorance in the operator; for many persons, particularly in the metropolis, undertake to administer electricity, who are entirely ignorant of any medical knowledge, and consequently, of the cause and situation of disorders: hence, the failure of it is owing generally to an error in the application.

The Technical Terms, used by Writers on Electricity.

Battery, electrical, a number of jars combined together, to be all charged and discharged at the same time, (fig. 12, plate 19.)

Charging, throwing an additional quantity of electric fluid upon one side of a plate of glass, or a jar, while the other side is exhausted in the same proportion. All electric substances may be charged as well as glass.

 Circuit,

Circuit, those conducting substances, used to connect the two coatings of a jar or battery together, and through which the electric fluid must pass.

Conductor, a piece of metal furnished with points, to receive the electric matter from the globe. It must always be insulated, or unconnected with the Earth, by means of electric substances, as glass, baked wood, &c. Whenever it is mentioned, the prime conductor is understood.

Discharging, is restoring the equilibrium of the electric fluid after it has been disturbed by charging. It is effected by forming a communication between the overloaded and exhausted sides of a jar, battery, &c. by some conducting substance.

Discharging Rod, a brass rod, or any other instrument, (figures 4 and 8, plate 19,) used to effect a discharge.

Electric Matter or Fluid, that subtle fluid, inherent in all bodies, and supposed to be the cause of all those appearances, which we term electric.

Electrics, those bodies in which the electric powers of attraction, repulsion, &c. may be excited by friction. They are called *non-conductors*, because the electric fluid cannot pass through them. And non-electrics are called *conductors*, because the electric matter may pass through them; but no electric powers can be excited in them.

Electrometers, instruments to measure the quantity of electric matter.

Excitation, calling forth the electric powers from electric substances by friction.

Insulating, placing bodies where they are not in contact with any conducting substance, as by suspending them in the air, by silken strings, placing them on glass stands.

Negative Electricity, a less quantity of the electric fluid, than is natural to any body.

Positive Electricity, a greater quantity of the electric fluid, than its natural share.

Rubber,

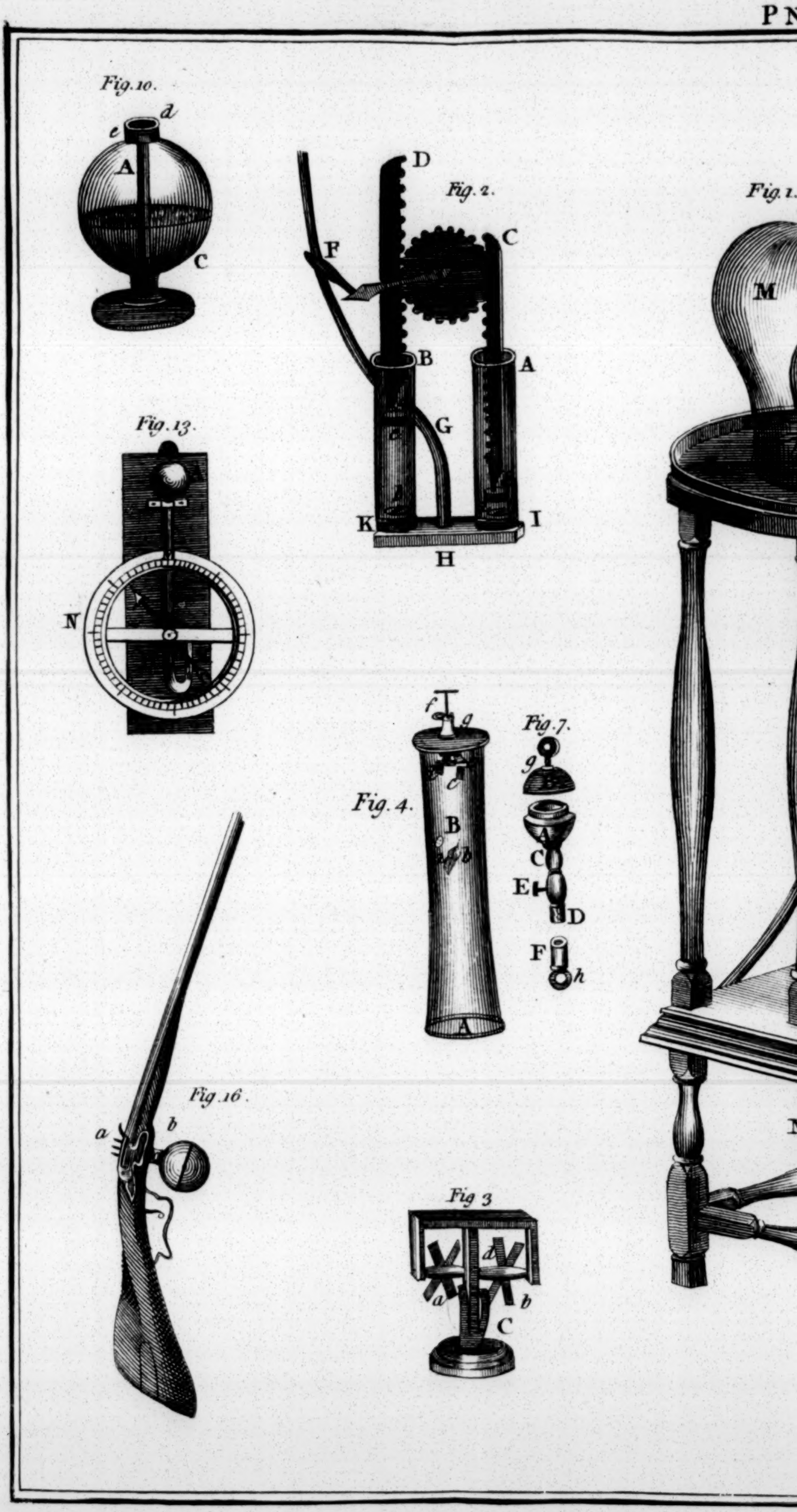


Fig. 10.
Fig. 2.
Fig. 1.
Fig. 13.
Fig. 4.
Fig. 7.
Fig. 16.
Fig 3

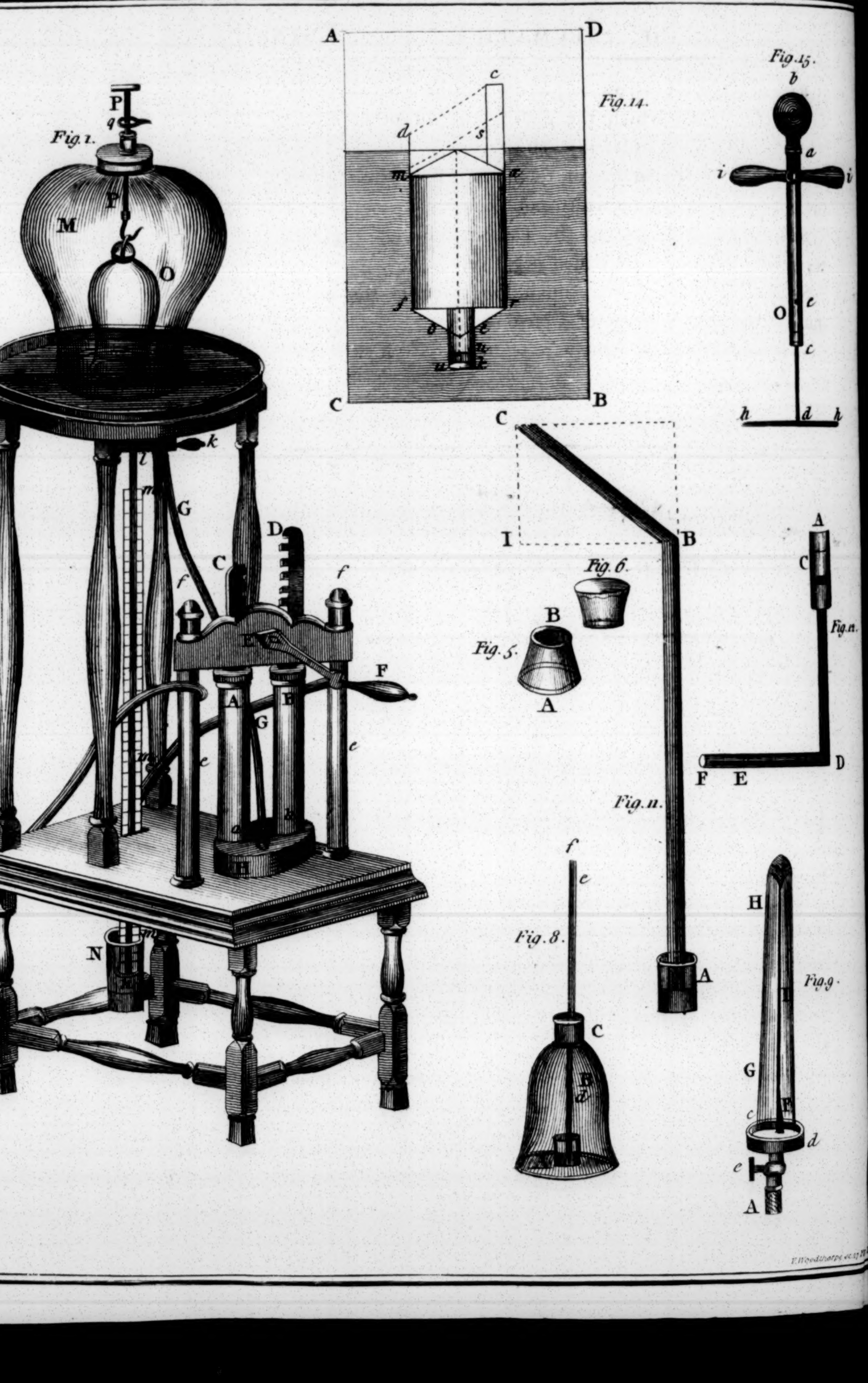


Fig. 1.
Fig. 14.
Fig. 15.
Fig. 6.
Fig. 5.
Fig. 11.
Fig. 8.
Fig. 9.

Rubber, or Cushion, a piece of leather, or any other ſubſtance, againſt which the glaſs globe, or other electric body, is rubbed, in order to excite them.

Pencil, the appearance of the electric fluid, iſſuing from the point of a body, electrified poſitively.

Star, the appearance of the electric fluid, iſſuing from the point of a body, electrified negatively.

Shock Electric, the convulſion given to the animal muſcles, by the diſcharge of a jar or battery.

Wire of a Jar, &c. the wire or metal rod which touches the inſide coating of the jar, &c.

CHAP. XV.

OF PNEUMATICS.

SECT. I.

OF THE PROPERTIES OF AIR.

PNEUMATICS, is that part of Natural Philoſophy, which treats of the weight, preſſure and elaſticity of the air, with the effects ariſing from them.

The Air, is that thin tranſparent fluid body, which ſurrounds the whole Earth to a conſiderable height; and which, together with the clouds and vapours that float in it, is called the Atmoſphere. That the air is a fluid, is evident from the following properties, which it poſſeſſes in common with all other fluids: viz.—1. It yields to the leaſt force, impreſſed on it.—2. Its parts are eaſily moved among one

another.—3. It presses according to its perpendicular height. —4. And its pressure is every way equal.

But the air differs from all other fluids, in the four following particulars:—1. It can be compressed into a much less space, than what it naturally possesseth, which no other fluid can.—2. It cannot be fixed or congealed as other fluids can.—3. It is of a different density in every part, upward from the Earth's surface; its weight decreasing the higher it rises; consequently, it must also decrease in density.—4. It is of an elastic nature, and the force of its spring, is equal to its weight.

It is evident, that air is a body, for it excludes all other bodies out of the space it possesses; thus, if a glass vessel, or jar, be inverted, and plunged into a vessel of water with a steady hand, still pressing it downwards, there will be very little water get into the jar, because the air, of which it is full, keeps the water out.—It is upon this principle, that diving bells are constructed.

Air being a body, must necessarily have gravity or weight; and its weight is determined by the following experiment. Let a bottle that holds a wine quart, be emptied of its air, by means of the air pump; then weighing the bottle, it will be found to be about sixteen grains lighter than when the air is let into it again; which shews that a quart of air weighs sixteen grains. And to find the proportion of the weight of air, to that of water, divide the weight of a certain quantity of water, by the weight of the same quantity of air; thus, a quart of water weighs 14621 grains, which divided by 16, the weight of a quart of air, quotes 914, in round numbers, which shews that water is 914 times as heavy as air, near the surface of the Earth.

The air has a different density, as we rise from the surface of the Earth, and grows continually rarer and lighter, the farther it is from the Earth; which is owing to its being of an elastic nature, and capable of being compressed into a less space, for the lower-most parts of the atmosphere being

pressed

pressed with the weight of all that is above them, must consequently be rendered more dense and compact at the Earth's surface than at any height above it; and that air toward the upper part of the atmosphere being less pressed, is consequently less dense and compact than that near the Earth; for the density of the air is always as the force that compresses it. The following table, given by Dr. COTES, shews that the rarity of the air, at the distance from the Earth's surface, encreases in a geometrical proportion, while its height from the Earth encrease in an arithmetical proportion:

At the Altitude of … Miles above the Surface of the Earth, the Air is	Times thinner and lighter than at the Earth's Surface.
7	4
14	16
21	64
28	256
35	1024
42	4096
49	16384
56	65536
63	262144
70	1048576
77	4194304
84	16777216
91	67108864
98	268435456
105	1073741824
112	4294967296
119	17179869184
126	68719476736
133	274877906944
140	1099511627776

From this table it appears, that at the height of 7 miles from the Earth, the air is four times rarer and lighter than at the Earth's surface; at the height of 14 miles it is 16 times rarer and lighter; at 21 miles, 64 times rarer, &c. From hence it may be proved, that a cubical inch of such air as we breathe, near the Earth's surface, would be so much more rarified at the height of 500 miles, that would fill a sphere equal in diameter to the orbit of Saturn.

The

The weight or preſſure of the air, is determined by what is called the Toricellian experiment, as follows:—

Fill with purified quickſilver, a glaſs tube, about three feet long, and open at one end; and putting your finger upon the open end, turn that end downwards, and immerſe it in a ſmall veſſel of quickſilver, without letting in any air; then taking away your finger, the quick-ſilver will remain ſuſpended in the tube, about twenty-nine inches and a half above the ſurface of that in the veſſel; ſometimes more or leſs, as the weight of the air is varied. In this experiment, it is evident that the quick-ſilver is raiſed in the tube by the preſſure of the atmoſphere, upon that in the baſon or veſſel; for if the baſon and tube be put under a glaſs, and the air be taken out of the glaſs, all the quick-ſilver in the tube will fall down into the baſon; and if the air be let in again, the quick-ſilver will riſe to the ſame height as before; therefore, the air's preſſure on the ſurface of the Earth, is equal to the weight of twenty-nine inches and a half depth of quick-ſilver all over the Earth's ſurface, at a mean rate. But a ſquare column of quick-ſilver twenty-nine inches and a half high, and one inch thick, weighs juſt fifteen pounds, which, therefore, is equal to the weight of the air, upon every ſquare inch on the Earth's ſurface; and the weight upon every ſquare foot or 144 inches, amounts to 2160 pounds. According to this rate, a middle ſized man, whoſe ſurface is generally about 14 ſquare feet, ſuſtains a preſſure of 30,240 pounds, when the air is of a mean gravity. This weight could not be born, if it were not that it is equal on every part of the body, and counterbalanced by the ſpring of the air within us, which is diffuſed through the whole body, and re-acts with an equal force againſt the external preſſure.

As the Earth's ſurface contains near two hundred million ſquare miles, in round numbers, and every ſquare mile 27,878,400 ſquare feet, there is 5,575,680,000,000,000 ſquare

ſquare feet on the Earth's ſurface, which multiplied by 2160 pounds, the weight on each ſquare foot, gives 12,043,468, 800,000,000,000 pounds, for the preſſure of the whole atmoſphere.

All common air, is impregnated with a certain kind of what is called *vivifying ſpirit,* which is eſſential to preſerve animal life; and in a gallon of air there is enough of it for one man, during the ſpace of a minute, but not much longer. This ſpirit is alſo in the air which is in water, as appears by the fiſh dying, when they are excluded from freſh air, as in a pond that is frozen over.

This ſpirit in air, is loſt by paſſing through the lungs of any animal; and is the reaſon why an animal dies ſo ſoon, when deprived of freſh air. The little eggs of inſects alſo, when ſtopped up in a glaſs, and excluded from the air, do not produce their young, though they be aſſiſted by warmth. The ſeeds alſo of plants, though mixed in good Earth, will not grow, if they be deprived of air.

The vivifying Quality, is alſo deſtroyed by the air's paſſing through fire, particularly charcoal fire, or the flame of ſulphur.

Air, may alſo become vitiated, by being cloſely confined in any place, for a conſiderable time; or, by being mixed with malignant ſteams; and laſtly, by the corruption of the vivifying ſpirit; as in the holds of ſhips, in oil ciſterns, wine cellars, which have been ſhut ſome time, or brewers vats. In any of them, the air may be ſo much vitiated, as to be immediate death to any animal that enters them.

When the air has loſt its vivifying ſpirit, it is called *damp*, becauſe it abounds with humid and moiſt vapours, and becauſe it deadens fire, extinguiſhes flame, and deſtroys life. The effects of theſe damps are ſufficiently known to thoſe who work in mines.

When part of the vivifying ſpirit of air in any country begins to putrify, the inhabitants of that country will be ſubject

ject to an epidemical diſeaſe, which will rage till the putrefaction is over. And as the putrifying ſpirit occaſions the diſeaſe, ſo if the diſeaſed body contributes towards the putrefaction of the air, the diſeaſe will then become peſtilential and contagious.

SECT. II.

OF THE THEORY OF THE WINDS.

THE wind is the conſequence of the rarefaction of the air; for when the air is rarified by heat, it will ſwell, and thereby affect the adjacent air; and thus, by the degrees of heat, being various in different places, there will ariſe various winds.

When the air is heated to any degree, it will aſcend upwards, and the adjacent air will ruſh in to ſupply its place; therefore, there will be a ſtream or current of air from all the adjacent parts towards the place where the heat is. This appears evident from the motion with which the air ruſhes towards any place, where there is a great fire, as into a glaſs-houſe, or through the key-hole of a door in a room, where there is a fire.

That wind, called the trade wind, which blows conſtantly from eaſt to weſt about the equator, is a neceſſary conſequence of this principle. For when the Sun ſhines perpendicularly upon any part of the globe, the air in that part will be heated, and conſequently rarefied, and will therefore, aſcend upwards; and when the Sun withdraws, the adjacent air ruſhing in to fill the place of the rarefied air, will conſequently cauſe a ſtream, or a current of air, from all parts, towards

wards that part, which is moſt heated by the Sun. But the courſe of the Sun being from eaſt to weſt, with reſpect to the Earth, the common courſe of the air, which ſupplies the place of the rarefied air, muſt be in the ſame direction, viz.—From eaſt to weſt, but on the north ſide, its courſe will be directed a little towards the north, and on the ſouth ſide, as much towards the ſouth.

This would be the general courſe of the wind about the equator, if it were not affected by other cauſes, which change its direction: as,—1. By exhalations that ariſe out of the Earth at different times, and different places, occaſioned by ſubterraneous fires, volcanos, &c.—2. By a ſudden inundation of rain, which cauſes a contraction of the air.—3. By the violent heat of ſome burning ſands, which cauſe an extraordinary rarefication of the contiguous air.—4. By high mountains, which alter the direction of the wind.—5. By the declination of the Sun, towards the north or ſouth, cauſing a greater heat in the air, on the ſame ſide of the equator.

Theſe are the principal cauſes which create ſuch a great variety and uncertainty in the winds, in moſt countries diſtant from the equator, as:—1. The variation of the winds in the different parts of Europe.—2. The Monſoons which are found in the Indian Seas.—3. Thoſe winds which always blow from weſt to eaſt, on the weſtern coaſts of America, and on the coaſts of Guinea; and the Sea breezes, which, in hot countries, blow from Sea to Land in the day-time; and the land breezes which blow towards the ſea in the night; and all thoſe other irregularities in the wind, as ſtorms, whirlwinds, hurricanes, &c.

SECT. III.

OF THE CAUSES OF THUNDER, LIGHTNING, &c.

THE effluvia and vapours, arifing from different bodies, meet and unite together in the atmofphere, which is the common receptacle of all vaporous bodies, as the fteams from moift bodies, the fmoak from bodies burnt, and the effluvia emitted from fulphurous, nitrous, acid and alkaline bodies. And every vollatile body, rifes to a certain height in the atmofphere, according to its own fpecific gravity. And when the effluvia, which arife from an acid and alkaline body, meet each other in the air, there will be a conflict between thefe two vapours, or what is vulgarly called a fermentation between them: if this fermentation be great, it will produce a fire; and if the effluvia be of a combuftible nature, the fire will run from one part of the air to another, following the inflammable matter.

Thefe things may be demonftrated by the following experiment:—Mix fome oil of cloves and glauber's fpirit of nitre together, which will immediately produce a fudden fermentation, with a fine flame; and if the ingredients be neat, there will be a fudden explofion. Thefe are the effects of the union of an acid and alkaline fluid.

From this experiment we may account for the effects of thunder and lightning, which is occafioned by the effluvia of fulphurous and nitrous bodies meeting each other in the air, where, affifted by the Sun's heat, a fermentation, fire, and explofion enfue. And when the inflammable matter is thin and light, it will afcend to the upper parts of the atmofphere,

mofphere, before the fermentation takes place; but when it is more denfe it will hover near the furface of the Earth, where, when an explofion takes place, the fire is vifible, and often dangerous; the explofion alfo has a violent force; and the heat being great, will rarefy and drive away all the adjacent air, kill men and cattle, fplit trees, rocks, &c.

Lightning differs from all other fires; for it has often been known to pafs through wood, leather, cloths, and other fubftances without hurting them; at the fame time melting iron, fteel, filver, gold, and other hard bodies. It has melted, or burnt afunder a fword, without hurting the fcabbard; and melted money in a man's pocket without hurting his clothes. So fine are the particles of this fire, that they pafs through foft, loofe bodies, without injuring them, and fpend their force upon thofe that are more denfe.

Any fteel inftruments, as knives, forks, &c. that have been ftruck with lightning, have a ftrong magnetical virtue, which they retain many years. The lightning ftriking the mariner's compafs has often turned it quite round, and made it ftand the contrary way—that is, with the north pole towards the fouth.

Thofe explofions which fometimes happen in mines, and called fire-damps, are of the fame nature with lightning, and occafioned by fulphurous and nitrous vapour, rifing from the mine, which, mixing with the air, take fire from the lights ufed in the mine. This fire, when once kindled, continues to run from one part of the mine to another, as the combuftible matter happens to lie; and as the elafticity of the air is encreafed by the heat, the air in the mine will fwell confiderably; and, for want of room, will at length explode, with a degree of force equal to the violence of the fire, the quantity of effluvia, and denfity of the vapours. This is fometimes fo ftrong as to blow up the mine; at other times, it is fo weak that when it has taken fire it may be eafily blown out.

The air that will take fire from a flame of a candle, may be produced experimentally, thus:—having pumped the air out of the receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air it will take fire.

When the combuſtible vapours are kindled in the bowels of the earth, where there is little or no vent, they produce earthquakes; and violent ſtorms or hurricanes of wind, as ſoon as they break forth in the open air.

An artificial earthquake may be produced, thus:—take ten or fifteen pounds of ſulphur, and as much of the filings of iron, and knead them with common water, into the conſiſtence of a paſte: This being buried under ground, will, in eight or ten hours time, burſt out into flames, and cauſe the earth to tremble around it to a conſiderable diſtance.

It is owing to ſubſtances of this nature that we have volcanos.

SECT. IV.

THE CONSTRUCTION AND USE OF THE AIR-PUMP, BAROMETER AND AIR-GUN.

THE air-pump is a machine to pump the air out of any veſſel, and conſtructed on the ſame principle as the water pump.

The air pump, with all its apparatus, is ſhewn (fig. 1, plate 20.) where L L is the plate, on which is placed a wet leather, and the large glaſs receiver M, placed upon the leather, ſo that the hole *i*, in the plate, may come within the

the glaſs. Then, by turning the handle F, (fig, 2.) the air will be pumped out of the receiver, which will be held down to the plate by the force of the external air or atmoſphere. For, as the handle F is turned backwards, or towards D, it raiſes the piſton *d e* in the barrel B K, by means the wheel E, and rack work D and C; and as the piſton is of leathered ſo tight as to fit the barrel exactly, no air can get between the piſton and barrel; and therefore, all the air above *d*, in the barrel, is lifted up towards B, and a vacuum is made in the barrel from *b* to *e*; upon which part of the air, in the receiver M (fig. 1,) by its ſpring ruſhes through the hole *i*, in the braſs plate L L, through thepipe G C G, which communicates with both barrels, by means of the hollow trunk, I H K (fig. 2.) and puſhing up the valve *b*, enters into the vacant place *b e* of the barrel B K. For, wherever the reſiſtance or preſſure of the air is diminiſhed, the air will run to that place, if it can find a paſſage. Then, if the handle F, be turned the contrary way, the piſton *d e*, will be lowered in the barrel: and, as the air, which came laſt into the barrel cannot be puſhed back through the valve *b*, it will aſcend through a hole in the piſton, and make its eſcape through a valve at *d*; and by that valve be prevented from returning into the barrel, below the piſton. At the next raiſing of the piſton, a vacuum is again made in the barrel, between *b* and *e*, as before, when more of the air that is left in the receiver M, (fig. 1.) eſcapes by its ſpring into the barrel B K, through the valve *b*. What is here explained concerning the barrel B K, muſt be underſtood with regard to the other barrel A I; and, as the angle F is turned backwards and forwards it raiſes and depreſſes the piſtons in each barrel alternately, raiſing one while it depreſſes the other. And, as there is a vacuum made in each barrel when its piſton is raiſed, the particles of air in the receiver M, (fig. 1.) puſh out one another by their elaſticity through the hole *i*, and pipe G G, into the

the barrels, until the receiver is ſo much exhauſted of the air, and the elaſticity of the air is ſo much weakened, that it will no longer have ſufficient force to paſs through the valves of *b d*; and then no more air can be taken out. From this it is evident, that it is impoſſible to make a perfect vacuum in the receiver, or to entirely exhauſt it of air; for the quantity of air taken out at any one ſtroke, will always be as the denſity of the air in the receiver; and therefore it is impoſſible to take it all out; for, if the receiver and barrels be both of equal capacity, there will always as much remain in the receiver as was taken out at the laſt turn of the handle.

At *k*, (fig. 1.) juſt under the pump-plate there is a cock, by the turning of which the air may, at any time, be let into the receiver again, through the hole *i*; and then the receiver becomes looſe, and may be taken off the plate. The barrels, (fig. 2.) are fixed to the frame E *e e*, (fig. 1.) by the two ſcrew-nuts, *f f*, which preſs the piece E upon the barrels; and the hollow trunk H, (fig. 2.) is covered by the box G H, (fig. 1.)

l m m m n is a glaſs tube, open at both ends, and about thirty-four inches long; the upper end communicating with the hole in the pump-plate, and the lower end immerſed in the veſſel N, which is nearly filled with quickſilver. This tube has a wooden ruler, *m m*, called the Gauge, and divided into inches, and parts of an inch, from the bottom at *n*, at the ſurface of the quickſilver, and continued upwards to *m*, about thirty-one inches.

The uſe of this rule is to diſcover the quantity of air that remains in the receiver M; for, as the air is pumped out of the receiver M, it is alſo pumped out of the tube *l m n*; becauſe the tube opens into the receiver; and as the tube is gradually emptied of its air, the quickſilver in the veſſel N is forced up the tube by the preſſure of the atmoſphere upon the quickſilver in the veſſel: And, if the receiver could be perfectly

perfectly exhausted of air, the quicksilver would stand as high in the tube, as it does at that time in the barometer; for the quicksilver, in both cases, is supported by the same power, viz. the weight of the atmosphere.

Every turn of the handle F, exhausts a portion of air from the receiver, and, consequently, raises the quicksilver in the tube; and the ascent of the quicksilver is always proportionable to the quantity of air exhausted; and the quantity of air remaining in the receiver is proportionable to the defect of the height of the quicksilver in the gauge, from its height in the barometer.

There are several experiments made with this air-pump, to shew the resistance, weight, and elasticity of the air.

The resistance of the air is measured by a small machine, having two mills, *a* and *b* (fig. 3.) which are of equal weights, and each turning freely on its own axis, independent of each other. Each mill has four thin sails fixed to its axis; those of the mill *a* have their planes at right angles to its axis; and those of *b* have their planes parallel to it. Therefore, when these mills turn round in common air, the mill *a* has but little resistance from the air, because its sails cut the air with their thin edges; but the mill *b*, is greatly resisted by the air, because it exposes the whole plane of its sails against the air. In each axle is a fine pin, near the middle of the frame, which goes quite through the axle, and stands out a little on each side of it. Upon these pins the slider *d* is made to bear, to hinder the mills from going round, when the strong pin C is set on bend, against the lower end of the pins.

Set this machine in motion, by drawing up the slider *d* to the pins on one side, and setting the spring C on bend, the opposite ends of the pins; then pushing down the slider *d*, the spring C acting with equal force upon each mill, will set them both in motion; but the mill *a* will run much longer

longer than the mill *b*, becauſe the air makes leſs reſiſtance againſt its ſails, than againſt thoſe of *b*.

Again, draw up the ſlider *d*, and ſet the ſpring C againſt the pins as before; then place the machine under the receiver M, upon the pump-plate; and having exhauſted the receiver of its air, puſh the wire P P, which runs through the collar of leathers on the neck *q*, upon the ſlider *d*, (fig. 3.) which will diſengage it from the pins, and thereby ſuffer the ſpring C to ſet the mills a-going; and as there is no air in the receiver to make any ſenſible reſiſtance, they will both move a conſiderable time longer than they did in the open air, and they will both ſtop at the ſame moment. This experiment ſhews the reſiſtance of air on bodies in motion; and that equal bodies meet with different degrees of reſiſtance, according as they expoſe a greater or leſs ſurface to the air in the planes of their motions.

Again, put a guinea and a feather on the braſs flap *c*, in the tall cylindrical receiver A B (fig. 4.) which is to be placed over the hole *i*, on the pump-plate; turn up the braſs flap *c*, ſo as to confine both the guinea and the feather; then, putting a wet leather over the top of the receiver, and covering it with the plate *g*, from which the guinea and feather tongs *e d* will hang within the receiver; pump the air out of the receiver, and by means of the wire *f*, open the tongs *e d*, and the flap *c* falling down, the guinea and feather will deſcend with equal velocities in the receiver, and both fall upon the plate at the ſame inſtant.

To ſhew the weight of the air, no more is neceſſary than a thin bottle or Florence flaſk, (whoſe contents are exactly known) having a braſs cap with a valve tied over it, fixed to the mouth. This braſs cap is to be ſcrewed into the hole *i* of the pump-plate, and the bottle exhauſted of its air. The bottle is then to be accurately weighed, when it will be found, that for every quart the bottle contains, it will weigh

16

16 grains lefs than when it was full of air, when the quickfilver ftands at $29\frac{1}{2}$ inches in the barometer.

If the fmall receiver O, be placed over the hole *i* in the pump plate, and the air be exhaufted therefrom, this fmall receiver will be preffed down to the pump plate, by the weight of the atmofphere, which will be found to be equal to as many fifteen pounds as there are fquare inches in that part of the plate, which the receiver covers; and which will hold down the receiver fo faft, that it cannot be removed until the air be let into it, by turning the cock *k*, when it will be perfectly loofe.

Place the fmall glafs A B, (fig. 5,) which is open at both ends over the hole *i*, on the pump plate L L, (fig. 1,) and having put your hand clofe upon the top of it, at B, exhauft the air out of the glafs, and your hand will be preffed down upon the glafs with a confiderable weight, and can hardly be releafed until the air be let into the glafs, by turning the cork *k*.

If a piece of wet bladder be tied over the end of the glafs, (fig. 6,) and when it is dry, the glafs be exhaufted of its air; the outer air will prefs upon the bladder, which will have a fpherical concave figure, and will grow more concave as more air is pumped out of the glafs, till, at length, it will break with a report as loud as that of a gun. If, inftead of the bladder, a flat piece of glafs, be laid on the top of this receiver, and joined to it by a ring of wet leather between them, upon exhaufting the air out of the receiver, the preffure of the outward air will foon break the flat piece of glafs to pieces.

Let the two brafs cups A and B, (fig. 7,) be joined together with a wet leather between them, having a hole in the middle of it; then fix the end of the pipe D into the hole *i* of the pump plate, and exhauft the air out of them, having turned the cock E, which permits the air to come through the pipe C D. Then turn the cock E again, to

keep out the air, and unſcrew the pipe D from the pump plate, and ſcrew on the handle F; then it will require a great force to pull theſe two cups aſunder; for if the diameter of the cups be four inches, they will be preſſed together, by the external air, with a force equal to 190 pounds. But if they be put under the large receiver M, (fig. 1,) and the air exhauſted out of the receiver, they will fall aſunder, having no external air to keep them together.

Place the veſſel A, (fig. 8,) on the pump plate, having ſome quick-ſilver in it, and cover it with the receiver B, in which is inſerted through the collar of leathers, in the braſs neck C, the tube *d e* open at the lower end; then exhauſt the air out of the receiver, and it will alſo be exhauſted out of the tube. When the receiver is ſufficiently exhauſted, puſh down the tube, ſo as to immerſe the lower end into the quick-ſilver. In this experiment, though the tube be exhauſted of air, yet none of the quick-ſilver will riſe in it, becauſe there is no air in the receiver, to preſs upon its ſurface; but if the air be let into the receiver, by the cock *k* in the pump plate, the quick ſilver will immediately riſe in the tube, and ſtand nearly as high as it does at that time in the barometer.

This experiment ſhews, that the quick-ſilver is ſupported in the tube, merely by the preſſure of the air, on its ſurface, in the open veſſel, in which the tube is immerſed; and that the more denſe and heavy the air is, the higher the quick-ſilver riſes; and, on the contrary, the thinner and lighter the air is, the leſs it will riſe. This is the reaſon why the quick-ſilver in the barometer falls before rain or ſnow, and riſes before fair weather; for, in the former caſe, the air is too thin and light to bear up the vapours; and in the latter caſe, too denſe and heavy to let them fall.

Note. In all experiments made with Mercury, by the air pump, there ſhould be a ſhort pipe ſcrewed in the hole *i* of the pump plate, ſo as to riſe about an inch above the plate, to prevent

prevent any quick-ſilver from getting into the air pipe and barrels, for ſhould any get looſe into the pipe or barrels, it ſpoils them, by looſening the ſolder, and corroding the braſs.

To ſhew the elaſticity or ſpring of the air, ſcrew the pipe A, (fig. 9,) into the pump plate, and place the receiver G H, upon the plate *c d*, which is fixed to the pipe, and exhauſt the air out of the receiver; then turn the cock *e*, to keep out the air, and unſcrew the pipe from the pump, and ſcrew it into the mouth of the copper veſſel A, (fig. 10,) which is half filled with water. Then, upon opening the cock *e*, the elaſticity of the air, which is confined in the upper part of the copper veſſel A, will force the water up through the pipe A B in a jet or fountain, into the exhauſted receiver.

There are a great number of other experiments to be made with this uſeful machine, the air pump, as:—1. To ſhew how neceſſary air is, for the ſupport of animal life, by putting any ſmall animal under a receiver, and exhauſting the air. 2. The different effects it has on different bodies, by increaſing their gravity.—3. How long it will ſupply flame or fuel by putting a lighted candle under the receiver.—4. The property of air in conveying ſound, by putting a bell under the receiver, and ſtriking it when the air is exhauſted, &c.

Of the Barometer.

This is an inſtrument uſed for meaſuring the weight of the atmoſphere, foretelling the changes of weather, and meaſuring the height of mountains, &c.

The common barometer, is formed of a glaſs tube, hermetically ſealed at one end, and filled with quick-ſilver, and defecated and purged of its air. The open end of the tube being immerſed in a veſſel of quick-ſilver, and by the preſſure of the atmoſphere on the quick-ſilver, in the open veſſel, the Mercury in the tube, will riſe to the height of

twenty-nine inches and a half, when the weight of the atmoſphere is at a mean rate. When the weight of the atmoſphere is greater, then the mercury in the tube will riſe higher; and when the weight of the atmoſphere is leſs than its mean weight, the mercury in the tube will fall lower.

To Conſtruct the Barometer. Being provided with a glaſs tube of one third or one half of an inch wide, (the wider the better,) and about thirty-four inches long, being cloſe at the top, or hermetically ſealed, pour into it well purified quick-ſilver, with a ſmall funnel, of either glaſs or paper, till it wants about half an inch of being full; then ſtopping it cloſe with the finger, invert it ſlowly, and the air in the empty part, will aſcend gradually to the other end, and collect in its way any ſmall air bubbles, which will unavoidably get in, in filling the tube: then again invert it, and thus continue to invert it ſeveral times, turning the two ends alternately upwards, till all the air bubbles are collected, and brought up to the open end of the tube; then the tube will appear like a fine poliſhed ſteel rod, without a ſpeck in it. Then pour in a little more quick-ſilver to fill the tube quite up, and ſtopping the open end of the tube with the finger, invert the tube, and immerſe the finger and end of the tube, thus ſtopped, into a baſon of purified quick-ſilver, then withdraw the finger, and the mercury will deſcend in the tube to ſome place between 28 and 31 inches above the mercury in the open veſſel, as theſe are the limits between which it always ſtands in this country, on the common ſurface of the Earth. Then meaſure from the ſurface of the quick-ſilver in the open veſſel, to the height of twenty-eight inches, and alſo to the height of thirty-one inches, dividing the three inches between theſe two numbers, into inches and tenths of an inch, which are marked on a ſcale, placed againſt the ſide of the tube; and the tenths ſub-divided into hundredth parts of an inch, by a ſliding index, carrying a *vernire* or *nonius*. Theſe three inches, between twenty-

twenty-eight and thirty-one, divided thus, will anſwer all the purpoſes of a ſtationary or chamber barometer; but for experiment on altitude and depths, it is neceſſary to have the ſcales continued higher up, and a great deal lower.

Several precautions are neceſſary in filling the tube, and fitting up the barometer, as:—1. The bore of the tube ſhould be pretty wide to allow a free motion to the quick-ſilver.—2. The baſon at bottom, ſhould alſo be pretty large, that the ſurface of the mercury in it, may not ſenſibly riſe or fall with that in the tube.—3. The bottom of the tube ſhould be cut off rather obliquely, that when it reſts on the bottom of the baſon, there may be a free paſſage for the quick-ſilver. And laſtly, It is beſt to boil the quick-ſilver in the tube, which will expel all the air from it, and render it very pure.

This inſtrument owes its invention to *Toricelli*, the diſciple of *Galileo*, who lived about the beginning of the laſt century, and who having diſcovered, that water could not be raiſed in a pump unleſs the ſucker was within 33 feet of the ſurface of the well, deſired *Toricelli* to penetrate into the cauſe of it. After ſome time, *Toricelli* diſcovered, that the preſſure of the atmoſphere was the cauſe of the aſcent of the water in the pump; and that a column of water, 33 feet high, was the juſt counterpoiſe to a column of air of the ſame baſe, and which extended up to the top of the atmoſphere: and this was the true reaſon why the water did not aſcend any higher. He alſo diſcovered, that a column of quickſilver, about $2\frac{1}{2}$ feet high, would be a counterpoiſe to a column of water of the ſame baſe, and 33 feet in height; as quickſilver is nearly 14 times heavier than water. This ſuppoſition he ſoon verified by filling a glaſs tube with quickſilver, and inverting the open end of it into a baſon of the ſame, when the mercury deſcended till its height above that in the baſon was above $2\frac{1}{2}$ feet, juſt as he expected.

It

It was not till after ſome time that it was diſcovered that the preſſure of air was various at different times. This, however, was no ſooner made known, than it was alſo obſerved, that the variations in the mercurial column were always ſucceeded by certain changes in the weather, as rain, wind, ſnow, &c. Hence, this inſtrument was ſoon uſed as the means of foretelling the change of the weather, and on this account obtained the name of *weather-glaſs*, as it did that of barometer, from its being the meaſure of the weight, or preſſure of the air.

The phenomena of the barometer are ſo various, that authors have not yet agreed upon the cauſes of them; nor is the uſe of it as a weather-glaſs, yet perfectly aſcertained; though daily obſervations lead us ſtill nearer to preciſion: And the moſt general rules for judging of the weather are thoſe delivered by *Mr. Patrick*, and are eſteemed the beſt of any, and are as follow:

1. The riſing of the mercury, preſages, in general, fair weather; and its falling, foul weather; as rain, ſnow, high winds, and ſtorms.

2. In very hot weather, the falling of the mercury indicates thunder.

3. In winter, the riſing preſages froſt; and in froſty weather, if the mercury falls three or four diviſions, there will certainly follow a thaw. But in a continued froſt, if the mercury riſes, it will certainly ſnow.

4. When foul weather happens ſoon after the falling of the mercury, expect but little of it; and, on the contrary, expect but little fair weather, when it proves fair ſhortly after the mercury has riſen.

5. In foul weather, when the mercury riſes much and high, and continues ſo for two or three days before the foul weather is over, then expect a continuance of fair weather to follow.

6. In

6. In fair weather, when the mercury falls much and low, and continues ſo for two or three days before the rain comes, then, expect a great deal of wet, and probably high winds.

7. The unſettled motion of the mercury denotes uncertain and changeable weather.

8. You are not ſtrictly to obſerve the words engraved on the plates, as the mercury riſing and falling; though, in general, it will agree with them. For, if it ſtand at the words, "*much rain*," then riſes up to *changeable*, it preſages fair weather; though not to continue ſo long as if the mercury had riſen high: and ſo, on the contrary, if the mercury ſtood at *fair*, and falls to *changeable*, it preſages foul weather; though not ſo much of it as if it had ſunk lower.

Upon theſe rules of *Mr. Patrick*, *Mr. Rowning* remarks, that it is not ſo much the abſolute height of the mercury in the tube that indicates the weather, as its motion up and down; therefore, to paſs a right judgment of what weather is to be expected, we ought to know whether the mercury is actually riſing or falling, to which end the following rules ſhould be obſerved:

1. If the ſurface of the mercury be convex, ſtanding higher in the middle of the tube than at the ſides, it is a ſign that the mercury is then riſing.

2. But, if the ſurface be concave, or hollow in the middle, it is then ſinking. And,

3. If it be plain, or rather a little convex, the mercury is ſtationary; for, mercury being put into a glaſs tube, eſpecially a ſmall one, naturally has its ſurface a little convex; becauſe the particles of mercury attract each other more forcibly than they are attracted by the glaſs,

4. Sometimes the mercury will ſtick to the ſide of the tube; therefore, when an obſervation is to be made with ſuch a tube, the tube ſhould be ſhaken a little; then, if the air be grown heavier, the mercury will riſe about a twentieth of an inch higher; but if the air be lighter it will ſink as

much:

much: and if it be the wheel barometer, tap it gently with the finger, which will give the mercury a free motion.

To the foregoing rules may be added the following, taken from latter and closer observations:

1. In winter, spring and autumn, the sudden falling of the mercury, and that for a large space, denotes high winds and storms; but in summer it denotes heavy showers, and often thunder; and it always sinks lowest of all for great winds, though not accompanied with rain; for wind and rain together it falls more than for either of them alone. Also, if, after rain, the wind change into any part of the North, with a clear and dry sky, and the mercury rise, it is a certain sign of fair weather.

2. After very great storms of wind, when the mercury has been low, it commonly rises very fast. In settled, fair, and dry weather, expect but little rain, except the barometer sink much, for a small sinking then, only denotes a little wind, or a few drops of rain; and the mercury soon rises again to its former station. In a wet season, suppose in hay time, and harvest, the smallest sinking of the mercury must be noticed, for when the constitution of the air is much inclined to showers, a little sinking then denotes more rain, as it never then stands very high; and, if in such a season it rises suddenly, very fast and high, expect not fair weather more than a day or two, but rather that the mercury will fall again very soon, and rain immediately to follow: the slow, gradual rising, and keeping on for two or three days, being most to be depended on for a week's fair weather; and the unsettled state of the quicksilver always denoting uncertain and changeable weather, especially when the mercury stands any where about the word *changeable* on the scale.

3. The greatest heights of the mercury in this country are found upon easterly and north-easterly winds; and it may often rain or snow, the wind being in those points and the barometer sink little or none, or may even be in the rising

rising state the effect of those winds counteracting. But the mercury sinks for wind as well as rain in all the other points of the compass; but rises as the wind shifts about to the north, or to the east, or between those points; but if the barometer should sink, with the wind in that quarter, expect it soon to change from thence; or else, should the fall of the mercury be much, a heavy rain is likely to ensue.

4. The barometer being lower, and continuing so longer than what can be accounted for, by immediate falls, or stormy weather, indicates the approach of very cold weather for the season; and also cold weather, though dry, is always accompanied by a low barometer, till near its termination.

5. Warm weather is always preceded, and mostly accompanied, by a high barometer; and the rising of the barometer, in the time of cold weather, is a sign of the approach of warmer weather: and also, if the wind be in any of the cold points, a sudden rise of the barometer indicates the approach of a southerly wind, which, in winter, generally brings rain.

The barometer is also found to sink in a certain ratio to its distance from the surface of the earth: it is, therefore, used for measuring any accessible heights. Various rules have been given by writers on the barometer, for applying it to this use, or computing the height ascended, from the fall of the mercury in the tube, the most accurate of which is that of *Dr. Halley*, now greatly improved by *De Luc*, by introducing into it the correction of the columns of the mercury and air, on account of heat, with other corrections and modifications. This rule is as follows, viz. $3000 \times$ logarithm of $\frac{M}{m}$ is the altitude in fathoms, in the mean temperature of 41 degrees; and for every degree of the thermometer above that, the result must be increased by so many times its 435th part, and diminished when below it. In this theorem M denotes the length of the column of Mercury in the barometer tube at the bottom of the hill or eminence; and *m* denotes

denotes the ſame at the top of the hill or eminence; and it is to be obſerved, that the reſult is always in fathoms of ſix Engliſh feet each.

As the ſcale of variation in the barometer is but ſmall, being included within three inches, viz. From 28 to 31 inches, ſeveral contrivances have been deviſed for enlarging the ſcale, to render the ſmall variations of the mercury more apparent; this has given riſe to the invention of ſo many different kinds of barometers;—a few of the moſt improved are the following:

1. *The Diagonal Barometer.*

This is the method of enlarging the natural ſcale of three inches perpendicular, by extending it to any length, B C, (fig. 11,) in an oblique direction. This barometer was invented by *Sir Samuel Moreland.* The perpendicular height of the diagonal part B C, is equal to the ſcale of variation of three inches, or C I; and conſequently, while the mercury in the common barometer riſe the whole length of the ſcale, which is three inches, and equal to I C; in this barometer, it will move from B to C: thus the ſcale is enlarged in this barometer, in the proportion of B C to I C. But it is found, that the diagonal part B C, cannot be bent from the perpendicular, more than in an angle of 45 degrees, which only encreaſes the ſcale in a proportion of 7 to 5. This form is liable to ſome inconveniences, on account of the obliquity of the part B C, which makes the mercury frequently divide into ſeveral parts, and renders it neceſſary to fill the tube again.

2. *The Horizontal Rectangular Barometer.*

This barometer, (fig. 12,) was the invention of *J. Bernoulli and Caſſini,* and conſiſts of a tube A C D F, ſealed at the upper end A, and bent to a right angle at D; the end F being

F being open. The mercury in this, ſtands in both legs from E to C. The ſcale of variation from D to F, is here made larger, and it is evident, in moving three inches from A to C, it will move as much more in the ſmall leg D F, as the area of the tube at A C, is greater than that of D F: wherefore, the motion of the mercury at E, muſt be very ſenſible. Though the end of the tube F be open, yet the mercury cannot run out, being oppoſed there by the preſſure of the atmoſphere. This inſtrument is founded on that theorem, in Hydroſtatics, that fluids of the ſame baſe, preſs according to their perpendicular altitude, and not according to the quantity of their matter; ſo that the ſame preſſure ſuſtains the quick-ſilver, that fills the tube A D F, and the ciſtern C, as would ſupport the mercury in the tube alone. This form is however liable to ſome inconveniences; for the attrition of the mercury againſt the ſide of the glaſs, and the quick motion of it in the part D F, is apt to break the mercury, and render its motions unequal: it is alſo apt to be thrown out at the open end F, by any ſudden ſhock.

3. *Dr. Hook's Wheel Barometer. (Fig. 13.)*

This barometer tube has a large ball A at the top, and is bent up at the lower end, which is open, where an iron ball floats on the top of the mercury in the tube, and connected to another ball H, hanging freely over a pulley, turning an index K about its centre. When the mercury riſes in the part K, it riſes the ball, and the other ball deſcends, and turns the pulley with the index round a graduated circle from N towards M; and the contrary way when the quickſilver and the ball ſink in the bent part of the tube. This ſcale is eaſily enlarged 10 or 12 fold, being encreaſed in proportion to the axis of the pulley to the length of the index K L. If this inſtrument could be conſtructed without any friction of the pulley and axis, it would anſwer

extremely well, but the friction often obstructs the motion of the quickfilver.

4. *Mr. Caswell's Baroscope, or Barometer.*

This instrument is the most useful of any, for enlarging the fcale of variation, and at the fame time being the most exact. A B C D (fig. 14.) is a bucket of water, in which is the barofcope, *x r e k u o f m*, which confist of a body *x r f m*; and a tube, *e k u o*, which are both concave cylinders, made of tin, or rather glafs, communicating with each other. The bottom of the tube *k u*, has a leaden weight to fink it, fo that the top of the body may just fwim even with the furface of the water by adding the weight of a few grains to the top. When the instrument is forced with its mouth downwards, the water afcends into the tube to the height of *n*. A fmall concave cyl inder or pipe, is added to the top, to keep the instrument from finking down to the bottom; *m d* is a wire; *m S d e* are two threads, oblique to the furface of the water, which anfwer as diagonals; for while the instrument finks more or lefs by an alteration in the gravity of the air, where the furface of the water cuts the thread, there is formed a fmall bubble, which afcends up the thread, while the mercury of the common barofcope afcends; and *vice-versa*.

This instrument, as the author has fhewn, marks the alteration in the air 1200 times more accurately than the common barometer. The bubble on the thread will feldom ftand ftill a minute; a fmall blaft of wind, which cannot be heard in a chamber, will make it fink fenfibly; and even a cloud paffing over it, always makes it defcend.

The common *Barometer*, or *Weather glafs*, is ufually fitted up in a neat mahogany frame, and confifts of the common tube barometer, with a thermometer by the fide of it, and a hygrometer at the top.

The

The Air Gun.

This inſtrument is an ingenious pneumatical invention, for driving a bullet, with great violence, by means of condenſed air, forced into an iron ball by a condenſor.

The condenſor (fig. 15.) has at the end *a*, a male ſcrew, on which the hollow ball *b* is ſcrewed, in order to be filled with condenſed air. In the inſide of this ball there is a valve, to prevent the air from eſcaping, after it is injected into it, until it be forced open by a pin, *a*. (fig. 16.)

When the air is to be condenſed into the ball, place your feet on the iron croſs, *h h*, in order to hold down the piſton rod *d e*; then lift up the barrel *c a* by the handles *i i*, until the piſton at *e* be brought below the hole at *o*; the barrel *a c*, and ball *b*, will then be filled with air through the hole *o*. Then thruſt down the barrel *a c*, until the piſton *e* reach the neck of the iron ball at *a*; and all the air between *o* and *a* will he forced up through the valve into the ball; and when the handles *i i* are again lifted up, the valvse in the balls will cloſe, and ſo keep in the air: thus, by rapidly continuing the ſtrokes up and down, the ball will preſently be filled; then unſcrew the ball from the condenſor, and ſcrew it upon another male ſcrew which is connected with the barrel, and goes through the ſtock of the gun, (fig. 16.) Then a ball being depoſited in the barrel of the gun, the hammer of the lock at *a* ſtrikes againſt the pin, which opens the valve in the ball, and lets out as much air as will drive a muſket ball to a conſiderable diſtance.

There are ſeveral kinds of air guns; but that here deſcribed, is the moſt improved and uſeful, as the gun need not be any larger than a ſmall fowling-piece; and ſeveral balls, filled with condenſed air, may be carried out, without filling the ball every time it is wanted. A ball of $3\frac{1}{2}$ inches diameter, may be made to contain ten or twelve penny weights

weights of air, which will diſcharge 12 or 15 bullets with conſiderable force.

There are ſome air guns that have a ſmall barrel contained within a large one; and the ſpace between the two barrels holds the condenſed air. In this inſtrument there is a valve fixed at *a*, (fig. 16,) with a condenſor fixed to the barrel at *a*, and continued through the but-end to *c*, where the piſton rod may be left in. Here the whole gun ſerves inſtead of the handles *i i*, (fig. 15.) to condenſe the air into the barrel.

The magazine air-gun differs from the others, by having a ſerpentine barrel, which contains 10 or 12 bullets; theſe are brought into the barrel of the gun ſucceſſively, by means of a lever; and theſe may be diſcharged as faſt as if they were in ſeparate guns.

CHAP. XVIII.

OF HYDROSTATICS.

Definitions.

1. HYDROSTATICS treat of the equilibrium of fluids; or the gravitation of fluid bodies remaining at reſt. When this equilibrium is removed, and the fluid body ſet in motion, the effects it then produces, are called *Hydraulics*.

2. A SYPHON is a bent tube, (fig. 5, plate 22.)

3. A Valve is a kind of flap or cover, fixed to a pipe or to the aperture of any body, and which, by opening only one way, ſuffers water or any fluid body to paſs, but not to return.

4. A Piston is a ſmall cylinder, fixed to the end of a rod, and fitted to the bow of a pipe, and frequently contains a valve.

Axioms.

1. All fluids, except air, are incompreſſible, or incapable of being compreſſed into a leſs ſpace.

2. In a veſſel of water, or any other fluid body, the preſſure of the upper parts on the lower is in proportion to the depth; and is the ſame at the ſame depth, whatever the diameter of the veſſel may be.

3. The preſſure of a fluid upwards, is equal to its preſſure downwards, at any given depth.

4. The bottom and ſides of a veſſel, are preſſed by the fluid it contains, in proportion to its height, without any regard to the quantity.

5. If fluids of different gravities be contained in the ſame veſſel, the heavieſt will be at the bottom, the lighteſt at the top, and the others will be farther diſtant from the top in proportion to their ſpecific gravities.

6. The direction of the preſſure of a fluid againſt the ſides of the veſſel, which contains it, is in lines, perpendicular to the ſides of ſuch veſſel.

7. A body that is heavier than an equal quantity of a fluid, will ſink in that fluid; but, if it be lighter, it will ſwim at the top of the ſame fluid; and if it be of the ſame gravity, it will neither ſink nor ſwim, but will remain ſuſpended in any part of the fluid.

8. A ſolid immerſed into a fluid, is preſſed by that fluid on all ſides, in proportion to the height of the fluid above the ſolid. And bodies very deeply immerſed in any fluid, may be conſidered as equally preſſed on all ſides.

9. Every

9. Every ſolid immerſed in a fluid, loſes as much of its own weight as is equal to the weight of a quantity of that fluid of the ſame dimenſion with the ſolid.

10. And the fluid, in which the ſolid is immerſed, acquires the weight, the ſolid loſes.

As the principal fluid with which we have any concern in hydroſtatics, is water, it may be neceſſary to mention a few of its diſtinguiſhing properties.

1. Water is a tranſparent, colourleſs, ſcentleſs fluid, which, with a certain degree of cold, turns to ice.

2. Water is one of the conſtituent parts of all bodies; as hath been proved by diſtillation; for the drieſt woods, earths, bones, and ſtones, pulveriſed, conſtantly yield a certain quantity of water.

3. Though fluidity is commonly regarded as an eſſential property of water, yet many philoſophers, particularly *Boyle*, *Boerhaave*, and *Dr. Black*, of Edinburgh, conſider it as an adventitious circumſtance, and produced by a certain degree of heat; and therefore aſſert its natural ſtate to be that of chryſtalline, as, when in ice.

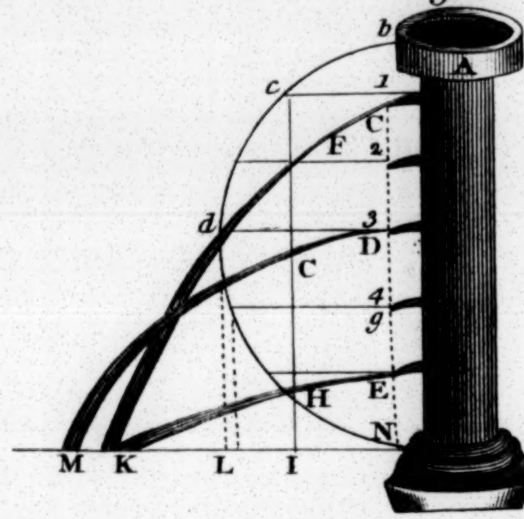
4. Water is a more penetrating body than air, though it be leſs tranſparent; for it will pervade bodies, that air will not: as is evident from its paſſing through the pores of a bladder.

5. Some bodies are diſſolved by water, as ſalts; while it conglutinates others, as bricks, ſtones, bones, &c.

6. As water owes its fluidity to heat, it is evident, from experiments made with the air-pump, that it contains no ſmall quantity of air: and the ſediment found in all water, which has not been diſtilled, always contains a quantity of earth. From which laſt element it is ſuppoſed that plants derive all their nouriſhment.

SECT.

Fig. 12.



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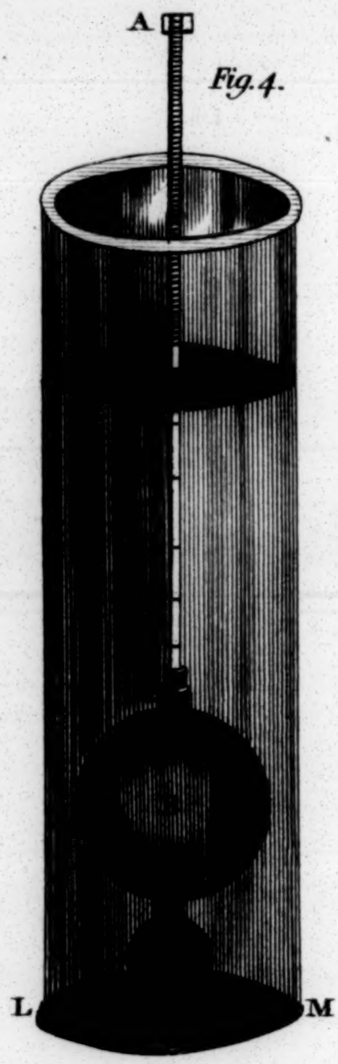

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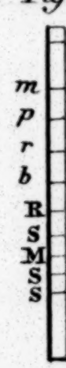

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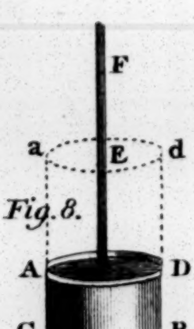

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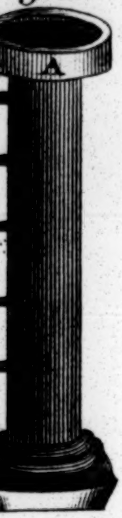
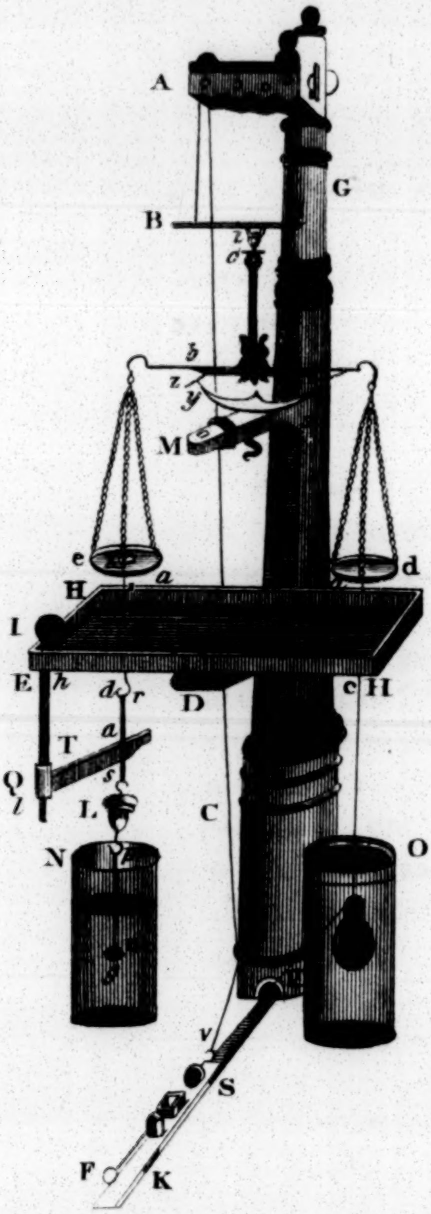


Fig. 1.



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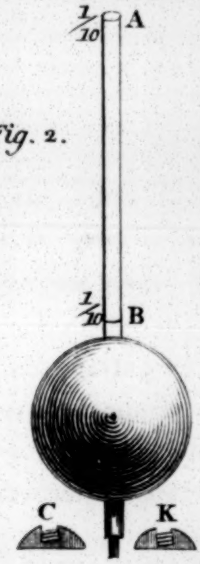


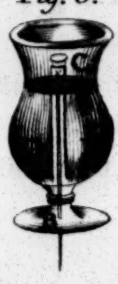
Fig. 3.

Fig. 11.

Fig. 10.

Fig. 7.

Fig. 6.



T. Woodthorpe fc. 27 Fetter Lane.

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SECT. I.

OF FLUIDITY.

A Fluid body, in Sir Iſaac Newton's definition, is, *a body yielding to any force impreſſed, and which has its parts very eaſily moved one among another*. This is the definition of a perfect fluid: if the fluid require ſome ſenſible force to move its parts, it is an imperfect fluid; and the imperfection is in proportion to that force; ſuch is, perhaps, all the fluids with which we are acquainted.

Fluids are either elaſtic, ſuch as air; or non-elaſtic, as water, mercury, &c. The latter are incompreſſible, and occupy the ſame ſpace under all preſſures or forces; but the former dilate and expand themſelves continually, by taking off the external preſſure from them. The properties of the former fluids conſtitute the doctrine of Pneumatics, before treated of: the latter contain the principles of hydroſtatics.

Fluidity differs from liquidity, or humidity, the latter implying wetting or adhering. Thus, air, ether, mercury, and other melted metals, and even ſmoke and flame are fluid bodies, though not liquid ones; while water, beer, milk, &c. are both fluids and liquids.

The modern opinion concerning the original and conſtituent parts of fluids, is, that they are ſmall, ſmooth, hard, globular particles; conſequently, each particle muſt be a ſolid globular body; and conſidered ſingly, is no fluid; but becomes a fluid, by being joined with other particles of the ſame or ſimilar kind.

That the particles of fluid bodies are very ſmall is evident, from their texture having never been diſcovered by the fineſt microſcope: that they are ſmooth, appears from that freedom wherewith they glide over one another, when ſet in motion: that they are hard and impenetrable, is plain from their being incapable of compreſſion: and that they are ſpherical is

obvious, from their being so easily put in motion; and from the interstices or vacancies, which is hereafter proved to subsist between them; which could not be the case unless they were spherical, and touched each other only in some points of their surfaces. For, upon mixing salt with water, a certain quantity of the salt will be dissolved without encreasing the dimensions of the water; which demonstrates the vacuities between the particles of the water. When a fluid becomes more buoyant, it is a proof that its specific gravity is increased, and consequently, many of its vacuities filled up; and even then it may receive a certain quantity of other dissoluble bodies, the particles whereof are adapted to the remaining vacancies, without adding any thing to its bulk, though the absolute weight of the whole fluid be thereby increased. This is demonstrated by taking the weight of a phial of rain water with a nice balance: when the water is poured out, and some salt added to it, and the phial again filled with the water, it will be found to weigh more than when before the salt was put in, from the vacuities of the fresh water being filled with saline particles.

It has also been found by experiment, that the particles whereof fluids are composed, consist of spheres of different diameters, whose interstices may be successively filled with proper ingredients; and where these interstices are smaller, the gravity of the fluid will be greater, and *vice versa.*

For example: if a barrel be filled with any large spherical bodies, as bullets, many small shot may afterwards be placed in the interstices of these bullets; the vacuities of the shot may then be filled with sea-sand; the interistices of which may again be filled with water, which will also admit of a certain quantity of salt in the vacuities; and thus the weight of the barrel may be greatly increased, without increasing the space occupied by these materials. This reasoning also holds good in fluid bodies, as well as in those which are solid; for river water will dissolve a certain quantity of

salt;

falt; after which it will diffolve a certain quantity of fugar; and after that a certain quantity of allum, and perhaps will receive other diffoluble bodies, without increafing the dimenfions of the whole.

If fluids were not compounded of fuch primary particles, but made up of one homogeneous fubftance, equally denfe without confiftence, there would be no difference in their fpecific gravities, and all fluids would be of the fame weight, which is not the cafe.

That a fluid has vacuities is evident from the following confideration, viz. if all fpace were abfolutely full of matter, the matter muft be either fluid or fixed. If it were fixed, no motion could poffibly be therein; as is evident from reafon and experience; it muft therefore be fluid. But a fluid without vacuities would be denfer, and confequently, heavier than all other fluids; and if denfer all bodies will emerge, and fwim at the top, by hydroftatical laws, and there would be no fuch thing as gravity. But as gravity exifts, all fpace therefore cannot be filled, even with a fluid.

By the experiments of *Borelli*, it has been demonftrated, that the conftituent parts of all fluids, are not fluids themfelves, but confiftent bodies; and that the elements of all bodies are perfectly firm and hard. The incompreffibility of water, proved by the florentine experiment, is a fufficient evidence that each primary particle of this fluid is a perfect impenetrable folid.

This famous experiment was firft attempted by the ingenious *Lord Verulam*, who inclofed a quantity of water in a piece of lead, and found, that the water would fooner make its way through the pores of the lead, than be reduced to lefs compafs, by any force that could be applied. This experiment was afterwards made at Florence, with a globe of filver; which being filled with water, and well clofed, was gently preffed, when a fmall quantity of water iffued through the pores of the filver in the form of dew.

The ſame experiment was afterwards made by *Sir Iſaac Newton* and others, with globes made of gold and other metals, all which experiments, was attended with the ſame phenomenon, and have tended to eſtabliſh the above theory.

As a great many of the phenomena in hydroſtatics depend upon gravity, it may be neceſſary to mention the laws of gravity, concerning bodies immerſed in fluids.

Gravity, in hydroſtatics, as well as in the other arts, is divided into abſolute and ſpecific gravity.

Specific gravity is the relative, comparative, or apparent weight of any body compared with that of another body, of the ſame bulk or magnitude; and therefore ſignifies, that gravity or weight which is peculiar to each kind of body.

A body is ſaid to be ſpecifically heavier, than another body, when it contains a greater weight than the other, under the ſame bulk or dimenſions; and thus reciprocally, that body which contains a leſs weight than another, under the ſame bulk, is ſaid to be ſpecifically lighter than the other body. Thus, if there he two equal ſpheres; ſuppoſe one foot, in diameter each; the one of lead, and the other of wood: as the leaden one is found to be heavier than the wooden one of the ſame ſize, it is ſaid to be ſpecifically heavier, that is, heavier in ſpecie, or in kind; and the wooden one is ſpecifically lighter.

Specific gravity, is by ſome called relative gravity, to diſtinguiſh it from abſolute gravity.

Abſolute gravity, is that force with which the body tends downwards, and is always in proportion to the quantity of matter in the body.

The Laws of the Specific Gravities of Bodies.

1. Two bodies of equal bulk, have their ſpecific gravities to each other, as their weights or denſities.

2. Two bodies of the ſame ſpecific gravity or denſity, have their abſolute weights in proportion to their magnitudes or bulks.

3. The

3. The ſpecific gravities, in bodies of the ſame weight, are reciprocally as their bulks.

4. The ſpecific gravities of all bodies, are in a ratio compounded of the direct ratio of their weights; and the reciprocal ratio of their magnitudes. Hence, the ſpecific gravities of bodies are as their denſities.

5. The Abſolute gravities or weights of bodies are in the compound ratio of their ſpecific gravities, and magnitudes or bulks.

6. The magnitudes of bodies are directly as their weights, and reciprocally as their ſpecific gravities.

7. When a body is immerſed in a fluid that is ſpecifically lighter than the body, the body loſes as much of its weight as is equal to the weight of a quantity of the fluid of the ſame bulk or magnitude.

Therefore, the ſpecific gravities are as the abſolute gravities, under the ſame bulk; the ſpecific gravity of the fluid, will be to that of the body immerſed in it, as the part of the weight loſt by the ſolid, is to the whole weight. Therefore, the ſpecific gravities of fluids, are as the weights loſt by the ſame ſolid immerſed in them.

8. The ſpecific gravity of any fluid or ſolid body, is found as follows:—Suſpend a globe of lead by a fine thread on one arm of a balance; and to the other arm faſten an equal weight. Immerſe the globe of the lead into the fluid, and obſerve what weight it will require then to balance it, and conſequently what weight it has loſt, which is proportional to the ſpecific gravity. Thus, the proportion of the ſpecific gravity of one fluid to another, is determined, by immerſing the globe ſucceſſively in all the fluids, and obſerving the weight the globe has loſt each time, which will be the proportions of the ſpecific gravities of the fluids.

This operation alſo gives the ſpecific gravity of the ſolid immerſed, whether it be a globe, or of any other ſhape, if the gravity of the fluid be known. For the ſpecific gravity of the fluid is to that of the ſolid, as the weight loſt is to the whole weight.

9. The

9. The ſpecific gravity of a ſolid, that is lighter than the fluid in which it is immerſed, is found by the following proceſs: to the lighter body, whoſe ſpecific gravity is required, annex another body, that is much heavier than the fluid, ſo that the compound maſs may ſink in the fluid. Weigh the heavier body and the compound maſs, ſeparately, both in water and out of it; then find how much each loſes in water, by ſubtracting its weight in water from its weight in air; and ſubtracting the leſs of theſe remainders from the greater; then ſay, as this laſt remainder is to the weight of the lighter body in air; ſo is the ſpecific gravity of the fluid, to the ſpecific gravity of that body.

10. The ſpecific gravities of bodies of equal weight, are reciprocally proportional to the quantities of weight loſt in the ſame fluid. Hence is found the ratio of the ſpecific gravities of ſolids, by weighing in the ſame fluid, maſſes of them that weigh equally in air, and noting the weight loſt by each.

11. A body deſcends in a fluid that is ſpecifically lighter, but aſcends in a fluid that is ſpecifically heavier, with a force equal to the difference between its weight and the weight of an equal bulk of the fluid.

12. A body ſinks in a fluid that is ſpecifically heavier, ſo far as that the weight of the body is equal to the weight of a quantity of the fluid of the ſame bulk as the part of the body which is immerſed in the fluid. Therefore, as the ſpecific gravity of the fluid is to that of the body, ſo is the whole magnitude of the body to the magnitude of the part immerſed.

13, In equal ſolids the ſpecific gravities are as their parts immerſed in the ſame fluid.

The foregoing theorems have been ſufficiently demonſtrated by various authors, from the principles of mechanics: they are alſo exactly conformable to experiment; as hath been ſufficiently proved from ſeveral courſes of Philoſophical Experiments.

Various

Various tables have been given, by different authors, of the ſpecific gravities of many kinds of bodies. It will be ſufficient, in this place, to give the ſpecific gravities of ſome of the moſt uſeful bodies that have been determined with greater certainty and accuracy. The numbers in this table expreſs the number of Avoirdupoiſe ounces, contained in a cubical foot of each body; that of common water being juſt 1000 ounces, or 62 $\frac{1}{2}$ pounds.

Table of Specific Gravities.

1. *Solids.*

Platina, pure	23000	Granite	3500
Fine gold	19640	White lead	3160
Standard gold	18888	Iſland chryſtal	2720
Lead	11325	Marble	2705
Fine ſilver	11091	Pebble ſtone	2700
Standard ſilver	10535	Rock Chryſtal	2650
Copper	9000	Pearl	2630
Copper halfpence	8915	Green glaſs	2600
Gun metal	8784	Flint	2570
Fine braſs	8350	Onyx ſtone	2510
Caſt braſs	8000	Common ſtone	2500
Steel	7850	Chryſtal	2210
Iron	7645	Clay	2160
Pewter	7471	Oyſter ſhells	2092
Caſt iron	7425	Brick	2000
Tin	7320	Common Earth	1984
Lapis caliminaris	5000	Nitre	1900
Loadſtone	4930	Vitriol	1880
Mean ſpecific gravity of the whole Earth	4500	Alabaſter	1874
		Horn	1840
Crude antimony	4000	Ivory	1825
Diamond	3517	Sulphur	1810

Chalk

Chalk - -	1793	Common water -	1000
Solid gun powder -	1745	Bees wax - -	955
Allum - -	1714	Butter - - -	940
Dry bone - -	1660	Oak - - -	923
Sand - - -	1520	Gunpowder, ſhaken -	922
Lignum-Vitæ - -	1327	Logwood - -	913
Coal - - -	1250	Ice - - -	908
Jet - - -	1238	Ash - - -	800
Ebony - -	1177	Maple - - -	755
Pitch - - -	1150	Beech - - -	700
Roſin - - -	1100	Elm - - -	600
Mahogany - -	1063	Fir - - -	550
Amber - -	1040	Saſſafras wood - -	482
Brazil wood - -	1031	Cork - - -	240
Box wood - -	1030	New fallen ſnow -	86

2. *Fluids.*

Quickſilver - -	13600	Ale - - -	1028
Oil of Vitriol - -	1700	Vinegar - -	1026
Oil of Tartar - -	1550	Tar - - -	1015
Honey - - -	1450	Water - - -	1000
Spirit of nitre - -	1315	Diſtilled water - -	993
Aqua-fortis - -	1300	Red wine - -	990
Treacle - -	1290	Proof ſpirits - -	931
Aqua-regia - -	1234	Olive oil - -	913
Human blood -	1054	Pure ſpirits of wine	866
Urine - -	1032	Oil of turpentine -	800
Cows milk - -	1031	Æther - - -	726
Sea water - -	1030	Common air - -	1.232
			Or, $1\frac{7}{30}$

As theſe numbers are the weights of a cubic foot, or 1728 cubic inches of each of the foregoing bodies in Avoirdupois ounces, the quantity, in any other weight, or the weight of any other quantity may be found by proportion.

For

For example, required the content of an irregular block of common ſtone, weighing one hundred weight, or 1792 ounces: here, as 2500, the ounces in a cubic foot of common ſtone, is to 1792, ſo is 1728, the inches in a cubical foot, to 1228 $\frac{4}{5}$ cubical inches, the contents.

Again, what is the weight of a block of granite, the length whereof is 63 feet, and the breadth and thickneſs each 12 feet, being the dimenſions of one of the ſtones of granite in the walls of Balbeck? Here the ſolid content of this ſtone is 9072 feet; therefore, as 1 is to 9072, ſo is 3500 ounces to 31752000 ounces, or 885 tons. 18 cwt. 3 quarters, the weight of the ſtone.

SECT. II.

THE CONSTRUCTION AND USE OF THE HYDROSTATIC BALANCE, HYDROMETER, AND HYDROSTATIC BELLOWS.

The Hydrostatic Balance.

THE hydroſtatic balance is the moſt convenient inſtrument for diſcovering the ſpecific gravity of all ſubſtances, whether fluid or ſolid. It is conſtructed in various forms; but that which is moſt generally retained, is the following:

V C G (plate 22, fig. 1.) is the ſtand or pillar of the inſtrument, which is to be fixed in a table. From the top, A, by two ſilken ſtrings, hangs the horizontal bar B B, from which is ſuſpended by a ring *i*, the fine beam of a balance *b*, which is prevented from deſcending too low on each ſide, by the gently ſpringing-piece, *l*, *x*, *y*. *z*, fixed on the ſupport M. The harneſs is annulated at *o*, to ſhew exactly the perpendicular poſition of the examen, by the ſmall pointed index fixed above it.

On each ſide of the piece A, is a pulley, over which paſſes a ſtring, which goes down to the bottom on the other ſide, and hangs over the hook at V, which hook is moveable about an inch and a quarter, backward and forward by means of the ſcrew P, ſo that the balance may be raiſed or depreſſed ſo much. But when a greater elevation or depreſſion is required, the ſliding-piece S, which carries the ſcrew-pin, is readily moved to any part of the ſquare braſs rod, V K, and fixed by means of the ſcrew.

By theſe means the motion of the balance is adjuſted; the other parts of the apparatus are as follows: D is a piece to ſupport the ſmall Board, H H, fixed under the two ſcales *d* and *e*, and is moveable up and down, by a long ſlit in the pillar above C, in which D ſlides, having a ſcrew at the back part to faſten when neceſſary. From the bottom of the middle of each ſcale, *d* and *e*, hangs a braſs wire, *a d*, and *a e*, by a fine hook: theſe wires paſs through two holes, *m m*, in the table. To the wire *a d*, is ſuſpended a curious cylindrical wire, *r s*, perforated at each end for that purpoſe. This wire *r s* is covered with paper, graduated by equal diviſions, and is about five inches long.

In one corner of the board, at E, is a fixed tube of braſs, on which a round wire, *b l*, is ſo adapted as to move ſomewhat freely, by its flat head I. Upon the lower part of this moves another tube Q, which has ſufficient friction to make it remain in any poſition required: to this is fixed an index T, which moves horizontally when the wire *b l* is turned round; and therefore may be eaſily ſet to the graduated wire, *r s*. To the lower end of the wire, *r s*, hangs a weight L, which has a wire, *p n*, with a ſmall braſs ball, *g*, at the end, about a quarter of an inch in diameter. On the other ſide of the wire, *a c*, from the other ſcale hangs, by means of a horſe hair, a large glaſs bubble, R.

To apply this inſtrument to uſe, let the weight L, be taken away, and a wire *p n*, be ſuſpended from the hook S: and

and let the bubble R, be taken away from the other ſcale, and the weight F ſuſpended in the room thereof. Suppoſe the weight F to be ſufficient to keep the parts belonging to the other ſcale in equilibrium: and that the middle point of the wire *p n*, is at the ſurface of the water in the veſſel N.—And, Note, The wire *p n*, is to be of ſuch a ſize, that the length of one inch ſhall weigh four grains.

It is evident, that as braſs is eight times heavier than water, for every inch the wire ſinks in the water, it will become an eighth part lighter, which is half a grain; and heavier in the ſame proportion, for every inch it riſes out of the water: therefore, by ſinking it two inches below the middle point, or raiſing it as much above it, the wire will become one grain lighter or heavier. Therefore, when the middle point of the wire is at the ſurface of the water, if the balance be in equilibrium, and the index T ſet to the middle point *a*, of the graduated wire *r s*, and the diſtance on each ſide *a r* and *a s*, contain an hundred equal parts; then, if in weighing bodies, the weight is required to the hundredth part of a grain, it may be eaſily found by the following method: let the body to be weighed be placed in the ſcale *d*, put the weight *x* in the ſcale *e*, and let this be ſo adjuſted as one grain more ſhall be too much, and one grain leſs too little. Then, by moving the balance gently up or down by the ſcrew P, till the equilibrium be exactly ſhewn at O; if the index T be at the middle point *a* of the wire *r s*, it ſhews that the weight put in the ſcale *e*, is juſt equal to the weight of the body.

The foregoing method, diſcovers the abſolute weight of the body: but to find the relative or ſpecific weight, it muſt be weighed hydroſtatically in water, as follows:—Inſtead of putting the body to be weighed into the ſcale *d* as before, let it hang with the weight F at the hook C, by a horſe-hair, as the weight R, ſuppoſing the veſſel of water

O were removed. Then the equilibrium being made, the index, T ſtanding between *a* and *r*, ſuppoſe at the thirty-ſixth diviſion, ſhews that the weight of the body is 109,536 grains. As it hangs thus, let it be immerſed in the water of the veſſel O, when it will become much lighter, the ſcale *e* deſcending till the beam of the balance reſt on the ſupport *x*. Then, if 100 grains, put in the ſcale *d*, will exactly reſtore the equilibrium, ſo that the index T ſtands at the thirty-ſixth diviſion above *a*; it is evident, that the weight of an equal bulk of water would be exactly 100 grains.

In the ſame manner this balance may be applied to find the ſpecific gravities of fluids.

The Hydrometer.

The hydrometer diſcovers the ſpecific gravity of fluids only; for which purpoſe it is the moſt accurate, eaſy, and expeditious inſtrument of any.

This inſtrument conſiſts of a copper ball, *B b*, (fig. 2.) to which is ſoldered a braſs wire, A B, a quarter of an inch in diameter. The upper part of this wire is filed flat, and marked *proof* at *m*, (fig. 3.) becauſe it ſinks exactly to that mark in proof ſpirits. There are two other marks at A and B, (fig. 2.) to ſhew whether the liquor be one-tenth above or below proof, according as the hydrometer ſinks to A, or riſes to B, when a braſs weight, C or K, is ſcrewed to its bottom *c*. There are alſo other weights to ſcrew on, which ſhew the ſpecific gravity of ſeveral different fluids as low as common water.

The round part of the wire, above the ball, ſhould be marked ſo as to repreſent river water, when it ſinks to R W, (fig. 3.) the weight which anſwers to that water being then ſcrewed on. When it is put into ſpring water, mineral water, ſea water, and water of ſalt ſprings, it will riſe to the marks S P, M J, S E, and S A reſpectively. On the contrary, when it is put into Briſtol water, rain water, port wine, and mountain wine, it will reſpectively ſink to the marks

marks *b r*, *r a*, *p o*, and *m o*. Inſtruments of this kind are ſometimes called Areometers. And, Note, That the globe D (fig. 2.) ſhould be made of copper: for ivory imbibes ſpirituous liquors, and conſequently alters their gravity; and glaſs globes require too much attention.

This hydrometer was the invention of *Mr. Clarke*, and anſwers very well to diſcover the ſpecific gravity of ſpirituous liquors, and to ſhew whether any ſpirit be above or below proof, and how much.

But the moſt perfect hydrometer is that repreſents (fig. 4.) which may be made to ſhew the ſpecific gravity of fluids to the greateſt degree of exactneſs. It conſiſts of a large hollow ball B, with a ſmaller bolt *b*, ſcrewed to its bottom, and partly filled with mercury or ſmall ſhot, in order to render it but little ſpecifically lighter than water. In the larger ball at C, there is a ſhort nick, into which is ſcrewed the graduating braſs wire A C, which has a ſmall weight A, at the top, to cauſe the inſtrument to deſcend in the fluid.

When this inſtrument is immerſed in any fluid contained in the jar L M, the quantity of the fluid diſplaced by it, will be equal in bulk to that part of the inſtrument which is under water, and equal in weight to the whole inſtrument. Therefore, if the weight of the whole inſtrument be four thouſand grains, we can, by theſe means, compare the different dimenſions of four thouſand grains weight of ſeveral ſorts of fluids; for, if the weight A be ſufficient to ſink the inſtrument in rain water, till the middle point of the ſtem, marked 20, come to the ſurface of the water; and after that, if it be immerſed in common ſpring water, and the ſurface of the water ſtand at one-tenth of an inch below the middle point 20, it is evident, that the ſame weight of each water differ only in bulk by the magnitude of one-tenth of an inch in the ſtem of the inſtrument.

Then, ſuppoſe the ſtem of the inſtrument to be ten inches in length, and to weigh juſt an 100 grains; every tenth

tenth of an inch will weigh juſt one grain; and as the ſtem is of braſs, which is about eight times heavier than water, the ſame bulk of water will be equal to one-eighth of a grain, and conſequently to one-eighth of one four thouſandth part, that is, to one thirty-two-thouſandth part of the whole bulk.

This inſtrument ſhould have two ſtems, either of which may be ſcrewed on, in a ſmall hole *a*. One of theſe ſtems, ſhould be a ſmooth, thin, flat, ſlip of braſs, or rather of ſteel, having on one ſide the ſeveral marks or diviſions, to which it will ſink in different ſorts of water; as, rain, river, ſpring, ſea, &c. and on the other ſide, the diviſions to which it ſinks in various lighter fluids, as hot Bath water, Briſtol water, Lincomb water, Cheltenham water, Port wine, Mountain, Madeira, &c. But in this caſe the weight A, muſt be a little leſs, than what is uſed for heavier waters. The other ſtem ſhould be cylindrical, to render it ſtronger and more ſteady, in order to try the ſtrength of ſpirituous liquors; and ſhould be ſo contrived, that when the inſtrument is immerſed, in which is called proof ſpirit, the ſurface of the ſpirit may be upon the middle point, marked 20. This is performed by adjuſting the ſmall weight A, and making the ſtem of ſuch a length, that when the inſtrument is immerſed in water, the ſurface of the water may juſt cover the ball B, and riſe to *a*; and when immerſed in pure ſpirit, it may riſe to the top, A. Then, by dividing the upper part of the ſtem, A 20, and the lower part, *a* 20, into ten equal parts each, when the inſtrument is immerſed in the ſpirituous liquor, it will immediately ſhew how much it is above or below proof, by the ſurface of the ſpirit riſing in proportion, above or below the middle point 20, which marks proof ſpirits, or that ſpirit which conſiſts of half water, and half pure ſpirit.

This is the moſt certain, and at the ſame time the moſt eaſy method of diſcovering the ſtrength of ſpirituous liquors: for it infallibly ſhews the difference of the bulks, and

and consequently the specific gravities in equal weights of spirits, and that even to the thirty, forty, or fifty-thousandth part: and is infinitely superior to the common method used by excise officers and others, of shaking the spirits in a phial, and forming a judgment of the strength by the breaking of the bubbles.

Of the Syphon.

The syphon is a bent tube, used to decant fluids from any standing vessel, and serves to perform some curious experiments. It depends upon the pressure of the air, and, may be made in various forms.

If a small syphon, whose legs are of equal length, be filled with water, and the ends turned downward, the water will remain suspended in the syphon; as long as it is held exactly level: but when it is the least inclined to either leg, whereby in effect one leg is made shorter than the other, the water will run out by the longer leg. For, the air being a fluid, whose density near the surface of the Earth is to that of water as 1 to 850; and according to the nature of all fluid bodies, pressing the surface of all things exposed to it every way equally, it must necessarily follow, that the weight of the atmosphere above, being kept off by the machine, and the air below bearing against, and repressing the water, which endeavours to fall out of either leg with equal force, keeps it in suspense, and prevents its falling. But if the syphon be the least inclined to either leg, one of the legs is in effect shortened, and the other prolonged; by which an advantage is given to the weightier fluid in the longer leg to preponderate, or overweigh the other part of the fluid in the other leg; and the water will all descend by the longer leg.

The least inclination of the syphon to either end, will be sufficient to produce this effect, which may be proved by experiment, thus:—Hang a small syphon, whose legs are of equal length, upon the edge of a jar filled with water; then from the sloping of the jar, the external leg of the

syphon

ſyphon will ſomewhat incline, and the ſyphon will ſoon begin to act, and the water will deſcend from the jar, through the ſyphon. But in practice, one leg of the ſyphon is uſually made longer than the other leg; and the ſhorter leg is put into the liquor, when the fluid will be decanted by the longer leg.

If the two legs of the ſyphon, (fig. 5,) were of equal length, terminating in the plane A B, and the ſyphon held exactly level, and then filled with liquor, no motion of the fluid would follow, till an advantage in point of gravity be given to one ſide by inclining it. But inſtead of which inclination, one of the legs is lengthened in general, perhaps a few inches, as from B to C; and which, previous to the operation, is generally filled as well as the reſt of the ſyphon with ſome fluid, many degrees heavier than air; by the gravity of which, the oppoſite ſide becomes greatly over balanced, and the liquor in this machine is decanted very rapidly.

The ſyphon, is ſometimes diſguiſed in a cup, when no liquor will flow through it, till the fluid be raiſed therein to a certain height; and when it has once begun to flow, it will continue, till the veſſel be emptied: this is called a *Syphon diſguiſed*. Thus: D D, (fig. 6,) is a cup, in the centre whereof is fixed a glaſs pipe C B, continued through the bottom at B, over this pipe is put a glaſs tube, made air tight at top, by the cork C, but left ſo open at the bottom, by holes made about D D, that the water may freely riſe between the two tubes, as the cup is filled. When any fluid is poured into this cup, no motion will take place through the ſyphon, till the fluid in the cup, ſhall have gained the top of the innermoſt pipe at C; but when the fluid is arrived to this height, it will begin to flow through the ſyphon, which runs through the bottom of the cup, and will continue to riſe up the inſide of the outer tube, and deſcend through the inner tube, till the whole fluid in the cup be run off; which is owing to the fluid at its firſt riſing through

through the tubes, expelling all the air from the tubes, while the weight of the atmoſphere preſſes on the ſurface of the fluid in the cup.

This is ſometimes called *Tantalus's* cup, and has a hollow figure, placed over the inner tube, of ſuch a length, that when the fluid is got nearly up to the mouth of the figure, the ſyphon begins to act, and empty the cup.

This is the ſame in effect, as if the two legs of the ſyphon were both in the veſſel, (fig. 7,) when the water poured into the veſſel will riſe in the ſhorter leg of the ſyphon, to its own level, and when it has gained the bend of the ſyphon, it will begin to run off by the longer leg, and continue till the veſſel be emptied as low as the extremity of the ſhorter leg of the ſyphon.

The Hydrostatical Paradox.

Any quantity of fluid, however ſmall, may be made to counterpoiſe, and ſuſtain any weight, how large ſoever. This is called the hydroſtatical paradox, and depends upon the equal preſſure of the parts of fluids every where at the ſame depth.

Let A B D G, (fig. 8,) repreſent a cylindrical veſſel, to the inſide of which, is fitted a cover, which by means of leather round the edge, will eaſily ſlide up and down in the veſſel, without permitting any water to paſs between its edges and the ſurface of the veſſel. In the cover is fixed a ſmall tube E F open at the top, and extending through the cover at the bottom. Then, the veſſel being filled with water the cover put on, and loaded with a weight, ſuppoſe of a pound, it will be depreſſed, the water will riſe in the tube to E, and the weight will be ſuſtained. If another pound be laid on the cover, the water will riſe to F, and the weight alſo be ſuſtained; and thus the water will riſe higher in the tube in proportion to the weight that is laid on the cover. And though the weight of the water in the tube, be but a few grains, yet its lateral preſſure will ſuſtain as much as the

 weight

weight of a column of water, whofe bafe is equal to that of the cylinder, and height equal to that in the tube. Thus, the column of water in the tube, produces a preffure of water, contained in the cylinder, equal to what would have been produced by the column A a, d D; and as this preffure is every way equal, the cover will be preffed upwards, equal to the weight of A a, d D; confequently, if A a, d D, would weigh a pound, the water in the tube, from the cover to E, will fuftain a pound. And the fame may be obferved of other weights. And by diminifhing the diameter of the tube, any quantity of water, however fmall, will (in theory,) fuftain any weight, however large.

The fame paradox may be fhewn by a more fimple experiment; thus, let A D G B, (fig. 9,) be a hollow cylinder of wood, into which is poured fome water, whofe furface rifes to *b g*; then, if the wooden cylinder M N, be put into the hollow one, the water will rife between the outfide furface of the inner cylinder, and the inner furface of the outer cylinder, to *a d*, and the wooden cylinder M N, will be fuftained floating. The nearer the wooden cylinder M N approaches to the fize of the hollow cylinder, the lefs quantity of water will ferve for the experiment.

The Hydrostatic Bellows.

The Hydroftatic Bellows, is the beft inftrument for demonftrating the upward preffure of fluids. It confifts of two round or oval boards, generally fixteen or eighteen inches in diameter, (fig. 10,) and joined to each other by leather nailed tight round their edges, fo that the two boards may open and fhut, like a pair of common bellows, but without any valve: only a pipe, generally three feet high is fixed into the fide of the bellows. To prove the upward preffure of fluids; let fome water be poured down the pipe of the bellows, which will run in between the two boards:

then,

then, lay ſome weights upon the upper board of the bellows, as ſuppoſe, three weights, weighing one hundred pound each, and pour more water into the pipe, which, by running into the bellows, will raiſe up the board with all the weights upon it; and if the pipe be kept full, until the weights are raiſed, as high as the board can riſe, the water will remain in the pipe, and ſupport all the weights, though the water in the pipe weigh no more than a quarter of a pound, and the weights on the bellows, three hundred pounds.

The reaſon of this experiment appears evident from what has been ſaid of the preſſure of fluids, of equal heights, without any regard to their quantities. For, if the tube be fixed in the upper board of the bellows, inſtead of the ſide, the water will riſe in it to the ſame height, as it did in the pipe, in the former caſe: and if as many tubes were fixed in the upper board, as it would contain, the water would riſe as high in each of them. The preſſure of the fluid upwards is thus computed:—If one pipe be fixed in the upper board, and, the pipe hold juſt one quarter of a pound of water; and if a perſon put his finger upon the hole of the pipe, when the forementioned weights are placed upon the bellows, he will find his finger preſſed upwards, with a force equal to a quarter of a pound: and as the ſame preſſure is equal upon equal parts of the board, each part whoſe area is equal to the area of the hole of the pipe, will be preſſed upwards with an equal force, that is, with a force equal to that of a quarter of a pound; the ſum of all which preſſures againſt the under ſide of an oval board, ſixteen inches broad, and eighteen long, will amount to three hundred pounds; and therefore, this quantity of weight, will be raiſed up, and ſuſtained by only one quarter of a pound of water in the pipe.

It is by this inſtrument, that a man may raiſe himſelf upwards, by his breath: For, if he ſtand upon the upper board, and blow through the pipe, he will raiſe the upper board

board of the bellows with himſelf upon it: and the ſmaller the bore of the pipe is, the more eaſily is the operation performed. And if he put his finger on the top of the pipe, he can ſupport himſelf up, as long as he pleaſes, provided the bellows be air tight.

This property of the upward preſſure of fluids, furniſhes us with the means of rendering any body, how heavy ſoever, buoyant in water: and upon this principle, a piece of lead may be made to ſwim in the water, by immerſing it to a proper depth, and preventing the water from getting above it. Thus, let C D, (fig. 11,) be a glaſs tube open at each end, and E F G, a flat piece of lead, placed under the lower end of the tube, having a piece of wet leather between it and the tube, to make it keep cloſe to the tube. Let the piece of lead be about half an inch thick, and in breadth equal to the diameter of the tube; let it be held cloſe to the tube by pulling the pack-thread H L upwards at L: and let the tube be immerſed in water, in the glaſs veſſel A B to the depth of ſix inches below the ſurface of the water, at K: then, the leaden bottom E F G, will be plunged to the depth of upwards eleven times its own thickneſs, at which depth, letting looſe the thread at L, the lead will not fall from the tube, but be forced upwards againſt the tube, by the upward preſſure of the water below it, occaſioned by the height of the water at K above the level of the lead; for as lead is 11.33 times as heavy as its bulk of water, and is in this experiment immerſed, to above 11.33 times its thickneſs, in a depth of water, and no water getting into the tube between it and the lead, the column of water, E a b c G, below the lead, is preſſed upwards againſt the lead, by the water K D E G I all around the tube; which water being a little more than 11.33 times as high as the lead is thick, it is ſufficient to balance and ſupport the lead, at the depth K E. If there be a little water poured into the tube, upon the lead, it will increaſe the weight

upon

upon the column of water which is under the lead, and cauſe the lead to fall from the tube to the bottom of the glaſs veſſel, as at *c d*. But if the tube be raiſed a little in the water, the lead will fall by its own weight, which will then be too great for the preſſure of the water round the tube upon the column of water below it.

Again, a piece of wood, however light, may be made to lie at the bottom of the water, by not ſuffering any water to get under it. Thus, having two pieces of wood, planed quite flat and ſmooth, ſo that no water may get between them, when they are put together; and cementing one of the pieces, as *b d*, to the bottom of the veſſel A B, place the other piece upon it, and let it be held down by a ſtick, while the water is poured into the veſſel; then, upon removing the ſtick, the upper piece of wood will not riſe from the lower one, being preſſed down both by its own weight, and the weight of all the water above it, while the contrary preſſure of the water upwards, is kept off by the wood placed beneath it: but if the top piece of wood be raiſed ever ſo little at any part of its edge, ſome water will get under it, which will be forced by all the weight of the water above, and will immediately preſs it upwards; and being lighter than its own bulk of water, it will float upon the ſurface of the water.

To prove that all fluids weigh juſt as much in their own elements, as they do in open air, put as much ſhot in a phial as when corked, will make it ſink in water; then let it be weighed, both in the air, and in the water, and the weight in each caſe wrote down; then, as the phial hangs ſuſpended in water, and counterpoiſed by another weight, pull out the cork that the water may run into it, when it will deſcend and pull down that end of the beam. Next, put as much weight into the oppoſite ſcale as will reſtore the equilibrium; which additional weight will be found to anſwer exactly to the additional weight of the phial, when it is again weighed in the air with the water in it.

The

The velocity with which water ſpouts out of a hole, or through a tube in the ſide or bottom of a veſſel, is as the ſquare root of the depth or diſtance of the hole below the ſurface of the water. Therefore, in order to make double the quantity of water run through one hole, as through another of the ſame ſize, it will require four times the preſſure of water as the other hole hath, and conſequently, it muſt be four times the depth of the other below the ſurface of the water: and for the ſame reaſon, three times the quantity of a fluid running in the ſame time through a hole of the ſame ſize, muſt run with three times the velocity; and conſequently muſt be nine times as deep below the ſurface of the fluid. Thus, let C and *g* (fig. 12) be two pipes of equal diameters, fixed in the ſide of the veſſel A B, the pipe *g* being four times as deep below the ſurface of the water at *b*, as the pipe C is: while the water runs through theſe pipes, let more water be conſtantly poured into the veſſel, to keep the ſurface ſtill at the ſame height. Then, in the ſame time that a pint of water flows from the pipe C from the pipe *g* will flow one quart.

The horizontal diſtance to which a fluid will ſpout, from an horizontal pipe fixed in any part of the ſide of an upright veſſel, below the ſurface of the fluid, is equal to twice the length of a perpendicular to the ſide of the veſſel, drawn from the mouth of the pipe to a ſemi-circle, deſcribed upon the altitude of the fluid: and therefore, the fluid will ſpout to the greateſt diſtance poſſible from a pipe at the centre of the ſemi-circle; becauſe a perpendicular to its diameter, (which is ſuppoſed parallel to the ſide of the veſſel) drawn from that point, is the longeſt that can poſſibly be drawn from any part of the diameter to the circumference of the ſemi-circle. Thus, if the veſſel A B be full of water, the horizontal pipe D, in the middle of its ſide, and the ſemi-circle N *d c b* be deſcribed upon D, as the centre with the radius D *d*. And the perpendicular D *d*, to the diameter N *b*, is

HYDRAULICS.

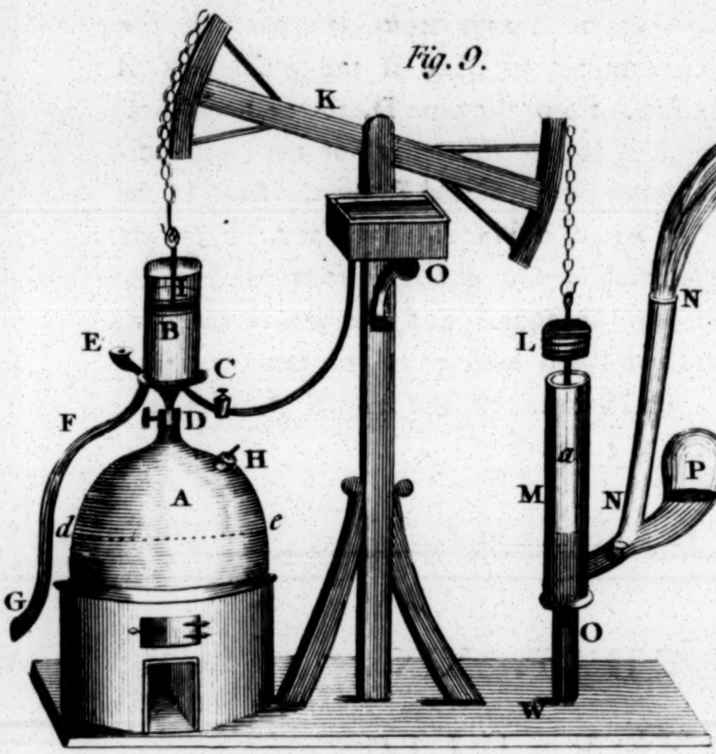


Fig. 9.

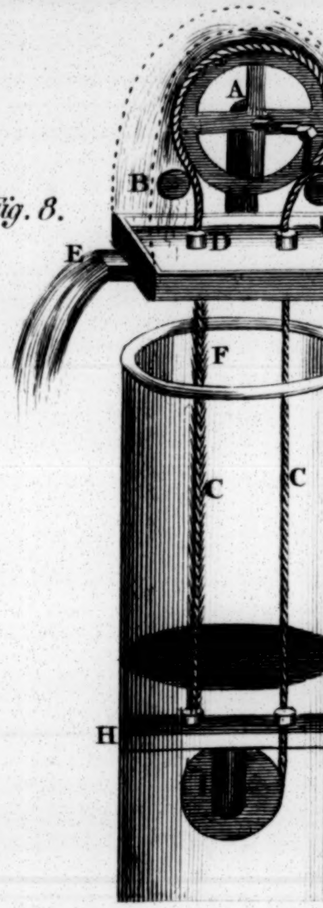


Fig. 8.

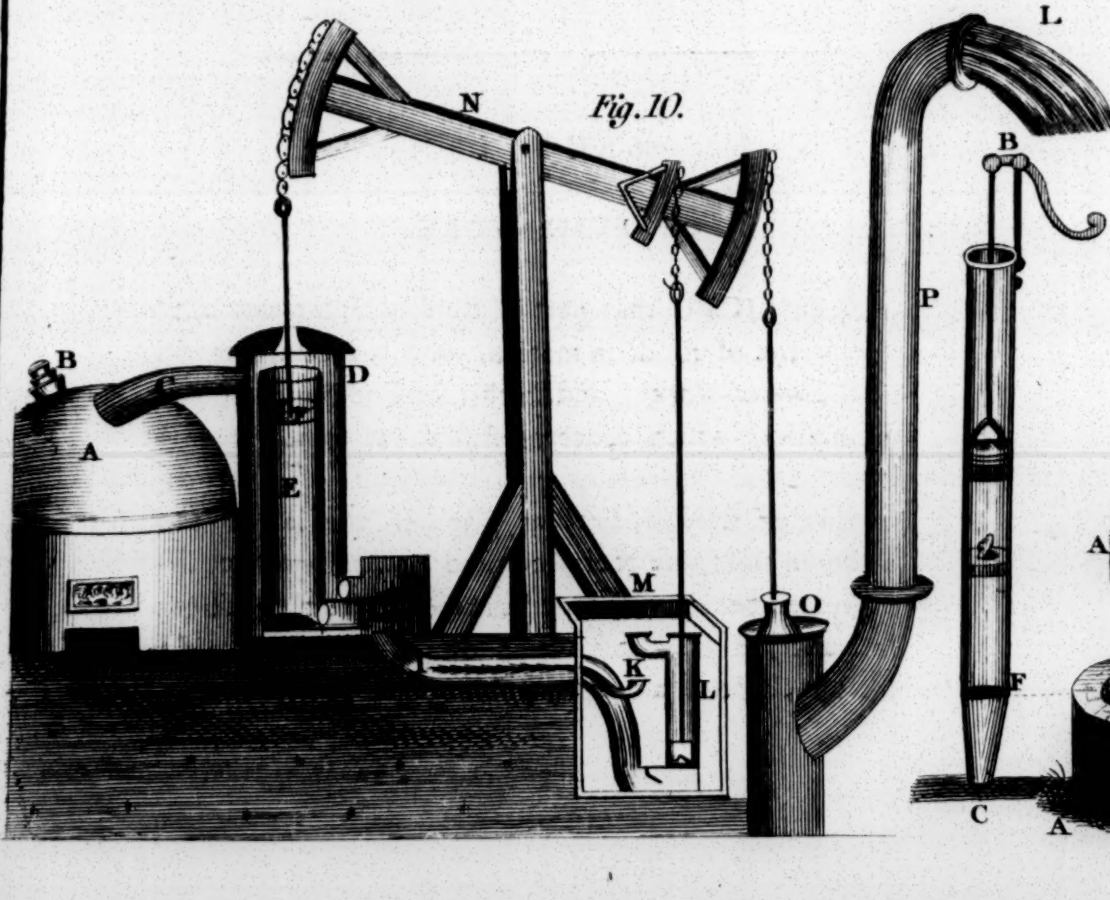


Fig. 10.

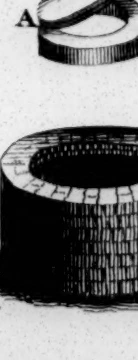
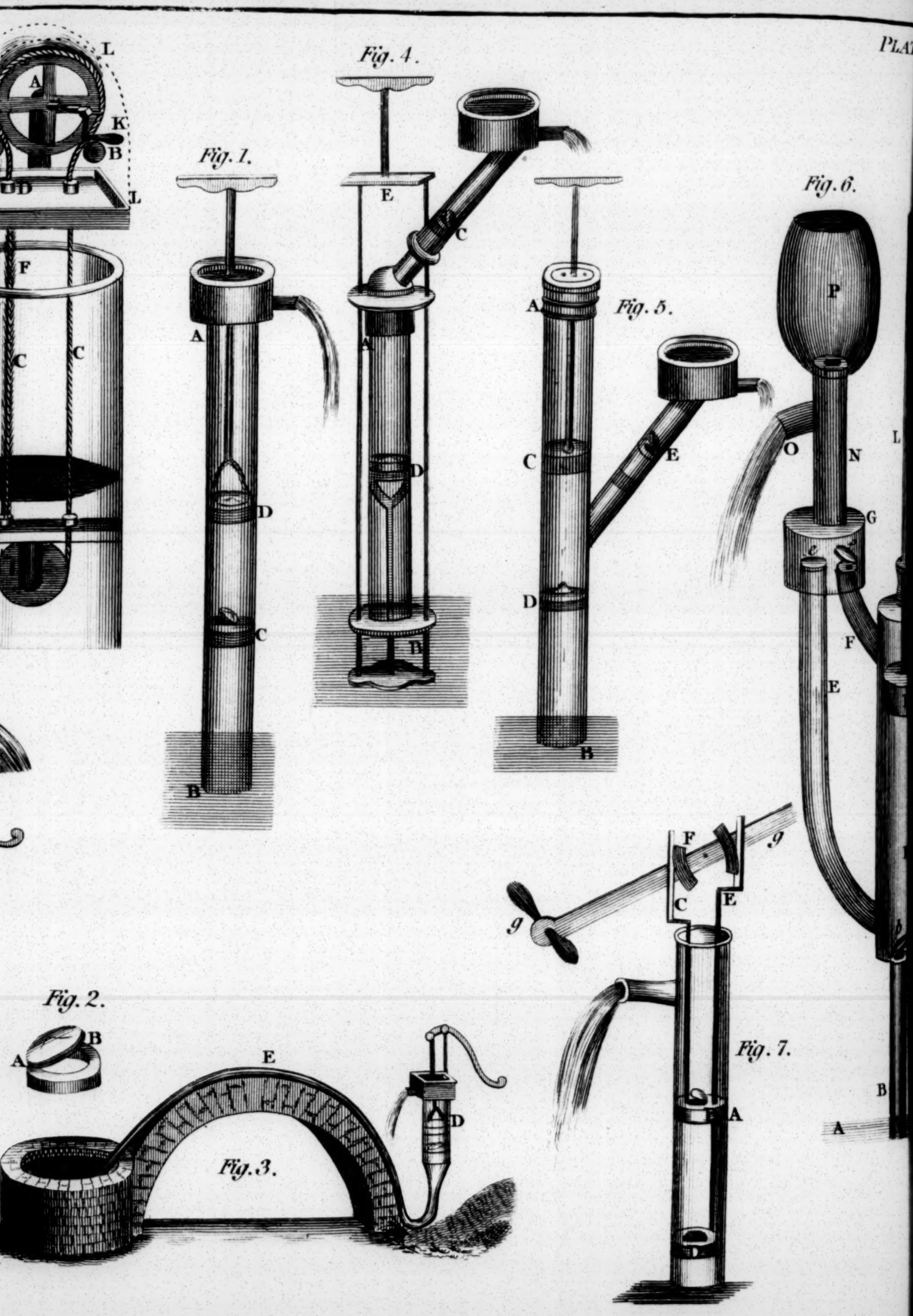


Fig. 2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is the longest that can be drawn from any part of the diameter to the circumference. And if the vessel be kept full, the jet C will spout from the pipe D, to the horizontal distance N M, which is double the length of the perpendicular D *d*. If two other pipes, C and E, be also fixed in the side of the vessel, at equal distances above and below the pipe D, the perpendicular C *c* and E *e*, from these pipes to the semi-circle will be equal, and the jets F and H, which spout from them, will each go to the same horizontal distance N K; which is double the length of either the equal perpendiculars C *c* and E *e*.

CHAP. XIX.

HYDRAULICS.

SECT. I.

OF PUMP-WORK.

HYDRAULICS is that part of fluids which treats of the properties of fluids in motion, with a special attention to artificial water-works; and in this sense it stands opposed to Hydrostatics, which concerns fluids, as they remain at rest.

The laws of fluid bodies, as given in the last chapter, obtain also in this, and therefore need not be repeated.

The greatest benefit mankind has received from the science of Hydraulics, is the construction of the water-pump, first invented by *Ctesebes*, a mathematician in Alexandria, about

120 years before Chriſt, and depends for its action upon the preſſure of the atmoſphere.

That the preſſure of the air on the ſurface of the water, is the cauſe of the water riſing in the pump, has partly been demonſtrated in pneumatics; for as the preſſure of the air cauſes the mercury to aſcend in the tube of the exhauſted barometer, ſo the ſame preſſure upon the ſurface of the water in a well, cauſes the water to aſcend in the pump, but to a far greater height; for the mercury in the barometer riſes only to twenty-nine and a half inches, at the medium; whereas the water in the tube of a pump will riſe to thirty-three feet at a medium, which is found equal in weight to a column of mercury of the ſame diameter; but of twenty-nine and a half inches in the height. The mercury being near fourteen times heavier than water.

That it is the preſſure of the atmoſphere which cauſes both the water and the mercury to aſcend, has been ſufficiently proved by numberleſs experiments: and may be ſhewn by an exhauſting ſyringe, commonly termed a ſucking ſyringe. Let this be fixed in a tranſparent tube, and the lower end thereof put in a jar of mercury or water, and the whole incloſed within a tall receiver; then, if the piſton of the ſyringe be raiſed before the air is exhauſted from the receiver, the mercury or water will immediately follow it; but after the air is exhauſted, if the piſton be raiſed, the fluid will not follow.

Therefore, what is called ſuction in hydraulic machines, is nothing more than when, by any mechanical contrivance the preſſure of the air is in any place abated, the adjacent matter being urged on by the weight of the atmoſphere, will tend to that place; and if the matter be fluid, it will riſe ſo far above its common level, till, by its abſolute weight, a juſt equality is made to preſerve that equilibrium which always obtains in nature.

Of

Of the Pump, there are ſimply three kinds, viz. The Sucking, the Forcing, and the Lifting Pump. By the former, the water is raiſed by the general preſſure of the atmoſphere on the ſurface of the water in the well, and cannot be raiſed to a greater height than thirty-three feet, as before obſerved; though in practice it is ſeldom raiſed above twenty-eight feet; becauſe the air is not always denſe enough to ſupport a column of water of thirty-three feet. By the two latter, water may be raiſed to any height, having an adequate apparatus, and ſufficient power.

Of the Sucking Pump.

The Sucking Pump is that in moſt common uſe, and conſiſts of a tube or pipe, open at each end, having within a ſliding piſton, as large as the bore of the pipe, and which, fits the pipe ſo exactly as to admit no air to paſs between it and the pipe. The pipe is called the barrel.

If the lower end of the barrel be immerſed in water, (fig. 1, plate 23) and the piſton be raiſed, a vacuum will be made in the barrel, by lifting up the column of upper air from A to D, and thereby permitting the air in the lower part of the barrel to expand itſelf; and the atmoſphere preſſing upon the ſurface of the water in the well, will force it to follow the piſton, and that even to the height of 33 feet, if the ſtroke could be of that continued length. But when the piſton is let down again in the barrel, the water will fall with it, to prevent which, there is a valve fixed (fig. 2.) in ſome convenient part of the barrel, as at C, (fig. 1.) which valve conſiſts of a wooden frame A, (fig. 2.) exactly fitted to the bore of the barrel, having a leather flap, B, lined with lead, in order to give it ſufficient weight and ſtrength. This valve opening with the upward motion of the water, and again cloſing when the piſton is let down, ſerves to retain the water above, which flows through it:

and at every riſe of the piſton, a freſh quantity of water flows through this valve.

Beſides this fixed valve, there is a moveable one placed in the piſton at D, (fig. 1,) which alſo opens the ſame way, and is called the bucket.

When the bucket deſcends, if the bore of the barrel be full of water, the reſiſtance of the water will open the bucket, and part of the water will riſe above it; and when the piſton is drawn upwards, the bucket will again cloſe under the incumbent weight of water, and the water will be raiſed by the force applied; ſo that whenever the bucket riſes, and lifts up the column of both air and water which paſſes through it, the fixed valve C, is diſcharged of its preſſure; and then a freſh quantity of water, exactly equal to that lifted up by the bucket, will, by the ordinary preſſure of the atmoſphere on the water in the well, be forced up through the valve to again ſupply the barrel. This alternate motion of the two valves may be ſeen to great advantage in the glaſs pumps.

But if there be no water in the barrel, before any water can be drawn from the well, the air in the barrel muſt be exhauſted, which may be done, if the piſton valve be tight, by the ordinary motion; but it is common to pour ſome water down the barrel, vulgarly called fetching the water, which is of no other uſe than to ſupple the leather of the valves, and render them air tight.

The firſt time the piſton is raiſed in the barrel, called the firſt ſtroke of the pump, it will make a vacuum in the barrel, and a part of the incumbent air is lifted away, upon which the air remaining in the barrel from its natural ſpring, will become conſiderably dilated; when the atmoſphere preſſing with a greater force on the ſurface of the well-water, than the dilated air does on the water in the barrel, it will cauſe the water in the barrel to riſe therein, ſo far, as, together with the included air, ſhall juſt counterpoiſe the weight of the

the atmoſphere upon the outward ſurface of the water. The ſame effect will be produced at the repetition of the ſtroke, till, by degrees, the water ſhall have reached the moving valve or bucket, and then the proceſs will go on as before deſcribed. Thus, water by this machine may be raiſed to any height whatever, provided the power be adequate to the weight, and the pipe ſtrong enough to bear the fluids natural preſſure.

The proportion of the preſſure of the water on the pipes in pump work is according to the height of the water, above the part conſidered : but the incumbent weight on the bucket of a pump in action, is nearly proportional to that of a column of water raiſed ; for though the weight of the atmoſphere on the ſurface of the water, when the bucket riſes, being really equal to the weight of thirty-three feet of water; yet this weight is exactly counterbalanced by the weight of the atmoſphere, ever incumbent on the ſurface of the water thereby raiſed. Thus, all the advantage to be obtained by the Hydraulic Machines, is ranging matters into a convenient method of being performed ; the performance itſelf depending entirely upon the moving power with all the diſadvantages of friction.

In pump-work, if both the valves be placed towards the bottom of the pipe, the pump will work as eaſy, and require no greater power than if they were fixed 30 feet, or 33 feet above the ſurface of the water.

It is generally found in practice to be more advantageous to place both the valve and bucket pretty low in the barrel ; for ſhould a leak happen beneath the bucket, which is often the caſe in a great length of pipe. the air getting through, would render the pipe uſeleſs ; whereas, ſhould the leak happen above the bucket, it will occaſion only the loſs of ſome of the water. And placing the valves under water they will always be found more ſupple and pliant, and conſequently be in a better condition for performing their office.

There is another advantage of placing the pump-work (that is the valve and piſton) near together, and at the bottom of the barrel: which is, it will in all caſes raiſe the water from the well by the ordinary pump, when another pump of an equal bore, but having its bucket higher, will not be able to raiſe any water, by reaſon of the ſhortneſs of the ſtroke, which cannot rarefy the air ſufficiently to bring the water up from the lower valve to the piſton. For example: having a ſmooth barrelled pump, 21 feet long, with its piſton placed above, fetching ſuppoſe a foot ſtroke, and the fixed valve at the other end of the barrel below. Here, by the playing of the piſton, ſuppoſe it poſſible for the water to riſe 11 feet, or which is the ſame thing, let water be poured down the barrel to the height of 11 feet above the lower valve, there will ſtill remain 9 feet of air between the bucket and the water, which cannot be ſufficiently rarefied by a ſtroke of one foot of the bucket, to open the lower valve, or fetch up more water: for in this caſe, the air will be rarefied in the proportion of 9 to 10 only; whereas it ought to be as 9 to 13 $\frac{1}{2}$, to make only a bare equilibrium with the atmoſphere: for as 22 (which is the compliment of 11 to 33 feet of water, the weight of the whole atmoſphere) is to 33 feet, or the atmoſphere, ſo is 9 to 13 $\frac{1}{2}$, to compleat which, the ſtroke ſhould be at leaſt 4 $\frac{1}{2}$ feet long. However, in a pump of this conſtruction, by filling the whole barrel with water up to the bucket, a conſtant ſupply of water may be produced.

When the bucket is placed low, as here directed, inſtead of an iron rod, to which it is fixed, a piece of oak well ſoaked, will work with leſs trouble, as it is nearly of the ſame ſpecific gravity with water.

When the pump cannot be placed perpendicularly over the well, the barrel of the pump ſhould be placed as low as the well is, and communicate with it by means of a pipe: thus, if water were to be raiſed from the well A, (fig. 3.) by

by means of the pumb B, the barrel of the pump B communicating with the well by means of the pipe C, and the bucket playing in the barrel B C, the water will rise, as if the well had been perpendicular to the pump; because the water in the well being forced by the natural pressure of the atmosphere, will replenish the barrel B, through the pipe A C.

But when it happens that the barrel of the pump cannot go down directly to the well, as in the last case, the water may be led about any other way by means of a pipe E, and thus be conveyed to the pump D. And in making this pipe of conveyance E, less in diameter than the barrel of the pump, it will sooner be exhausted of air, by moving the piston, and consequently the water will sooner follow.

But it will always be found more easy in practice to have the pipe of conveyance large, and of an equal bore throughout; because the water will have a velocity in them, and the friction will be less. This is the reason why the common pumps, made by the plumbers, do not work so easily as those which are bored out of trees; for, by making their pipe so much less than the bucket, they wire-draw the water. Therefore, in pumps that have a long pipe of conveyance, the diameter of the sucking pipe should be nearly equal to half the diameter of the barrel. For, if the barrel be four inches in diameter, and the pipe of conveyance one inch, the water will move sixteen times as fast through the pipe as it will through the barrel, which requires more labour, and is attended with a greater wear and friction of the machine.

It is also a great fault to bore a pump conically upwards, because the water cannot with freedom, be got away so fast as a vacuum may be made by the moving piston; and the reflection of the water from the sides, will always be an hindrance in the operation.

Of the Lifting Pump.

The Lifting Pump differs from the Sucking Pump in the disposition of the valves; for the Sucking Pump has the fixed valve below, and the moveable one, or bucket above; but in the Lifting Pump, just the contrary takes place, the fixed valve being above, and the moveable one below. In the Sucking Pump, the bucket or piston, is moved by a rod within the bore of the barrel; but in the Lifting Pump it is moved by means of a strong frame, fixed to a rod without it, (fig. 4.) D is the moveable valve or bucket, in the barrel of the pump, which is moved by the fixed frame A, D, B; and C is the fixed valve. This pump must always have an elbow to lead the lifted water clear of the play of the rod, which moves perpendicularly: and the friction in this pump will always be less the nearer the bend of the two barrels C and D approaches to a right line. In this pump is it also necessary to have the pump-work placed in or near the source of water, as it generally is done.

The operation of this machine, like that of the former, depends upon the pressure of the atmosphere; and if both valves be not perfectly air-tight, no water will be raised; but if they be both in good order, and no leak in the barrel, water may be raised successfully by this pump as by the Sucking Pump. And if a Sucking and a Lifting Pump be both of equal bores, and in every other circumstance alike, and wrought with equal force, they would raise an equal quantity of water in the same time.

Of the Forcing Pump.

The Forcing Pump consists of a piston or forcer, C, (fig. 5.) and two fixed valves D E, the one, D, placed in a convenient part of the sucking pipe; the other in the branching or forcing pipe at E. These valves should also be

be air-tight, and ſo diſpoſed as to let the water freely riſe, but to prevent its turning back. The forcer C is leathered upwards, that it may withſtand the preſſure of the atmoſphere from above, and that by ſucking, when raiſed, it may bring up the water to ſupply the barrel; and it is alſo leathered downwards, that when repreſſed it may reſiſt the weight of the water to be forced up.

When the forcer C is moved upwards in the barrel, it lifts up the incumbent air; and the air between that and the water having room to dilate itſelf, will be rarefied, and the water will riſe from the ſpring into the tube A B, as in the Sucking Pump: and continuing the motion of the forcer, the water will at length riſe up to the forcer, and fill the internal cavity of the pipes, between the two fixed valves D and E. And the water being prevented from deſcending again by the lower valve, will, by the forcer, be preſſed, and make its way through the upper valve E. And when the forcer riſes, this preſſure will be intermitted, and the valve at E will immediately cloſe under the weight of the upper water, and thus prevent its return, while the piſton is riſing a freſh ſupply. The ſame is repeated at every ſtroke of the forcer.

M. De La Hire's Pump, which raises water both by the ascent and descent of the piston in the Pump Barrel.

B and C are two pipes, (fig. 6.) having their ends in the well of water A A. The pipe B has a valve at top, and is ſoldered into the pump-barrel D. The pipe C alſo has a valve at top, and is ſoldered into the pipe S. The pipes E and F have each of them a valve *e f* at the ends, and communicate with the pump-barrel D, and the hollow box G.

K is a ſolid plunger, exactly fitted for the bore of the barrel. L is the rod by which it is moved up and down through the

the collar of leathers M by the pump-handle H, which turns upon its centre of motion I. This plunger never goes higher than K, nor lower than D.

When the plunger riſes from D to K, the weight of the atmoſphere acting upon the ſurface of the water in the well A A forces it up the pipe B, through the valve *b*, and thus fills the pump barrel with water up to the plunger, during which time the valves *e* and S on the tops of the pipes E and C remain ſhut. When the plunger has arrived to the height K, before it returns down again, the valve *b* ſhuts, and thereby ſtops the mouth of the pipe B, and prevents the water from returning back: and by the motion of the plunger downwards, all the water in the barrel is forced up through the crooked pipe E, and conſequently through the valve *e*, and having filled the box G, at length riſes into the pipe N, where it diſcharges itſelf by the ſpout O. During the deſcent of the plunger K, the valve *f* ſhuts, and thereby covers the mouth of the crooked pipe F; and the plunger deſcending downwards, creates a vacuum in the upper part of the pump-barrel, and conſequently in the pipes C S and F, when the preſſure of the atmoſphere on the well-water A A, forces it up the pipe C, through the valve S, and into the pump barrel, filling all the ſpace above the plunger in the barrel with water.

Again, when the plunger has deſcended to D, before it returns up again, the valve S ſhuts; and then, by raiſing the plunger, it drives all the water above it through the crooked pipe F, and through the valve *f*, into the box G, from whence it alſo aſcends in conjunction with the water that came through the pipe E, up the pipe N.

And thus, as the plunger deſcends, it forces the water below it, up the pipe E, and alſo draws the water up the pipe C, through the valve S; and as it aſcends, it forces all the water above it up the pipe F, and alſo fills the barrel with water, through the pipe B. Therefore, there is as much

much water forced up the pipe N, to the ſpout-hole, by the deſcent of the plunger, as by its aſcent ; and in each caſe as much water diſcharged at the ſpout-hole as fills that part of the pump-barrel through which the plunger moves.

P is a cloſe air veſſel, fixed on the top of the pipe O, which compreſſes the air when the water riſes up the pipe N, above the ſpout O : and this condenſed air acting on the water, cauſes it to run off by the ſpout-hole, nearly in an equal ſtream.

The pipe S, at the top of the pipe C, ſhould never be above thirty-two feet above the ſurface of the water in the well; becauſe if the pipe C be entirely exhauſted of air, the preſſure of the atmoſphere on the water in the well would not force the water up the pipe to a greater height than thirty-two feet at the moſt : but if S be within twenty-four feet of the water in the well, the pump will work ſo much the better.

The pipe N may be of any ſize required; but the pump barrel ſhould be made in proportion to the height of the ſpout-hole above the ſurface of the water in the well, as follows:

For ten feet high, the diameter of the bore of the barrel ſhould be 6.9 inches: for 15 feet high, 5.6 inches: for 20 feet, 4.9 inches : for 25 feet, 4.4 inches: for 30 feet, 4.0 inches: for 35 feet, 3.7 : for 40 feet, 3.5 inches: for 45 feet, 3.3 inches : for 50 feet, 3.1 inch: for 55 feet, 2,9 inches: for 60 feet, 2.8 inches : for 65 feet, 2.7 inches: for 70 feet, 2.6 inches : for 75 feet, 2.5 inches : for 80 feet, 2.5 inches: for 85 feet, 2.4 inches: for 90 feet, 2,3 inches: for 95 feet, 2.2 inches : and for 100 feet, 2.1 inches: or at the moſt, 2.2.

In pumps of this kind the pipes B and C ſhould be made ſufficiently large; for when they are too ſmall, the velocity of the water through them being great, the water will have too much friction to ſuffer the pump to be worked with much advantage.

Mr. Noble's Pump.

This pump is the moſt ſimple in its conſtruction of any of the ſame power, and may be conſtructed at a reaſonable charge, as it conſiſts only of one barrel and two piſtons, having each a bucket and valve; and it raiſes as much water with the ſame power, and in the ſame time, as can be raiſed by two barrels and four valves of the ſame dimenſions. The barrel conſiſts of a ſtraight tube A (fig. 7.) in which the two buckets B and D work: the bucket B being moved by the rod C, and the bucket D by the rod E, which runs through a hole in the bucket B: the two rods and buckets are moved up and down by the two circular pieces of wood F, which are fixed to the two handles *g g*, by which means, as one bucket aſcends with its load of water, the other deſcends.

A Pump, or rather Engine, for raiſing water by means of a Hair-Rope. Invented by Sieur Vera.

The wheel A, (fig. 8.) is four feet in diameter, being turned by the handle K. B B are two pullies, 14 inches in diameter, in order to keep the ropes at a proper diſtance in the well. C C is the hair-rope, near one inch in diameter, which runs under the pulley I, fixed in a frame H, below the ſurface of the water G. L L L is a box made of thin boards, in order to collect the water into the reſervoir D.

When the wheel is turned by the handle K, with a conſiderable velocity, a conſiderable quantity of water will adhere to the rope C, particularly if the well be not very deep: the rope paſſes through the tube D, which is raiſed five or ſix inches higher than the bottom of the reſervoir; and thus hinders the water from returning back into the well; and

and the water runs in a continual ſtream through the ſpout E. If this machine be conſtructed according to the above dimenſions, it will raiſe more water than any perſon unſkilled in Hydraulics would imagine.

The force required to work a pump will always be, as the height to which the water is raiſed; and as the ſquare of the diameter of the pump-bore in that part where the piſton works. Thus, if two pumps be of equal heights, and one of them be twice as wide in the bore as the other, the wider will riſe four times as much water as the narrower one; and conſequently will require four times the power to work it.

The piſton rod of a pump is always raiſed by means of a lever, whoſe longer arm exceeds the ſhorter one in length, generally five or ſix times, and the power is applied at the end of the longer arm; by which means the rod is raiſed by a fifth or ſixth part of the power which would be required to raiſe the rod without it.

The following Table ſhews the quantity of water which a common Sucking Pump will diſcharge in one minute; the pump being of any given height above the ſurface of the well, from 10 to 100 feet incluſive; and the diameter of the bore of the barrel being from 6.93 inches to 2.19 inches incluſive.

Height of the Pump above the surface of the well in feet.	The diameter of the barrel-bore in inches and decimals.	Water discharged in a minute, English Wine measure.		
		Gallons.		*Pints.*
10	6.93	81	—	6
15	5.66	54	—	4
20	4.90	40	—	7
25	4.38	32	—	6
30	4.00	27	—	2
35	3.70	23	—	3
40	3.46	20	—	3
45	3.27	18	—	1
50	3.10	16	—	3
55	2.95	14	—	7
60	2.84	13	—	5
65	2.72	12	—	4
70	2.62	11	—	5
75	2.53	10	—	7
80	2.45	10	—	2
85	2.38	9	—	5
90	2.31	9	—	1
95	2.25	8	—	5
100	2.19	8	—	1

The foregoing Table, is constructed from a pump, worked by a lever, which increases the power five times, and the power is supposed to be that of a man of ordinary strength.

Forcing pumps are the most useful machines for raising water to any given height above the surface of a river or spring; and machines may be constructed to work these pumps, either by a running stream, a fall of water, or by horses.

SECT.

SECT. II.

THE DESCRIPTION OF MR. WATT'S STEAM ENGINE.

OF all the uſes, to which ſteam has been applied, there are none where it has been uſed with greater ſucceſs than in the application of it to raiſe water from any great depth, as in that machine, called, the Steam Engine; otherwiſe, called the Fire Engine, on account of the fire employed in boiling the water to produce the ſteam.

The ſteam raiſed from hot water is an elaſticic fluid like air, and has its elaſticity proportional to its denſity, when the heat is the ſame: or proportional to the heat when the denſity is the ſame. That ſteam raiſed from boiling water of an ordinary heat is near 3000 times rarer than water, or about $3\frac{1}{2}$ times rarer than air, and its elaſticity is equal to that of common air. And it has been found by experiment that ſteam may be ſo much expanded by heat, as to occupy 14000 times the ſpace of water, and conſequently, it will become five times ſtronger than the atmoſphere. And by accidents that have happened, it has been demonſtrated that water, ſuddenly turned into ſteam, by the immediate application of great heat, is vaſtly ſtronger than the atmoſphere, or even gun-powder.

The ſteam engine is the moſt uſeful machine diſcovered in modern times; and were it not for the uſe of this moſt important invention, we ſhould never have been able to work the coal mines in England to the preſent advantage; as before the preſent century, for the want of this engine to draw the water, the attempts of our anceſtors to procure coals were always ineffectual.

Before *Mr. Watt's* improvement in this engine can be mentioned, the common ſteam engine ſhould be firſt underſtood.

The ſteam engine is commonly formed of a forcing pump, having its rod fixed to one end of a lever, which is worked by the weight or preſſure of the atmoſphere, upon a piſton, at the other end; a temporary vacuum, being made below it by ſuddenly condenſing the ſteam, which had been let into the cylinder, in which this piſton works, by a jet of cold water thrown into it. Thus, a partial vacuum being made, the weight of the atmoſphere preſſes down the piſton, and raiſes the other end of the lever, with the water from the well, &c. Then a hole is immediately opened in the bottom of the cylinder, through which a freſh quantity of hot ſteam ruſhes in from a boiler of hot water, placed below it, which proves a balance for the atmoſphere above the piſton, upon which the weight of the pump rods, fixed at the other end of the lever, cauſes that end to deſcend, and raiſes the piſton of the ſteam cylinder. The ſteam hole is then immediately ſhut, and the cock opened for injecting the cold water into the ſteam cylinder, which condenſes into water again, and thus makes another vacuum below the piſton, the atmſophere above it preſſing it down, and raiſing the pump rods with another lift of water; and this proceſs is continually repeated. Though this is the common principle of a ſteam engine, yet there are various other methods for applying the force of ſteam.

The firſt account we have of theſe engines is in a ſmall book, publiſhed in the year 1663, by the Marquis of Worcester, intitled "A Century of Inventions," being a deſcription of 100 famous diſcoveries, publiſhed that year, among which he propoſes the method of raiſing a great quantity of water by the force of ſteam; and he mentions an engine, of his own invention, which would raiſe a continual ſtream of water, forty feet high, by means of two cocks, which were alternately turned by a man, to admit the ſteam, and to re-fill the veſſel with cold water.

Captain

Captain *Thomas Savery* having read the Marquis's book, conſtructed an engine, which, after ſeveral experiments, he brought to ſome degree of perfection: upon which he bought up, and deſtroyed all the books of the Marquis he could procure, claimed the honour of the invention to himſelf, and obtained a patent for the ſame. His engine, however, would not raiſe water to any great height, or in large quantities, to anſwer the purpoſe of draining a mine. The largeſt he ever erected was for the York Buildings Company, in London, for ſupplying the inhabitants of the Strand, and that neighbourhood with water.

Several other gentlemen, both in England and France, attempted various improvements in the conſtruction and manner of working theſe engines; but with little ſucceſs, till the year 1705, when *Mr. Newcomen*, an Ironmonger, and *Mr. John Cowley*, a Glazier, both of Dartmouth, made a conſiderable improvement in theſe engines, by bringing the engine to work with a beam and piſton, (which had never been then introduced) and where the ſteam, even at the greateſt depth of mines, is not required to be greater than the preſſure of the atmoſphere: and this is the ſtructure of the engine as it has ſince been chiefly uſed. Theſe gentlemen obtained a patent for the ſole uſe of this invention, for fourteen years; and the firſt engine they erected was in the year 1712, at a Colliery, at Griff, in Warwickſhire; the cylinder of this engine being twenty-two inches in diameter. The next engine they erected was in the year 1718, at a Colliery in the county of Durham, which was alſo improved by *Mr. Henry Beighton*, F. R. S. who introduced the manner of opening and ſhutting the cocks by the hanging-beam, as at preſent uſed; and likewiſe made improvements in the pipes, valves, and ſome other parts of the machine.

When theſe engines came to be better underſtood, and their utility, particularly in draining mines, became more evident, from the great numbers of them every where

erected,

erected, they received many additional improvements, till they arrived to the preſent degree of perfection.

The following are the principles on which this engine acts, which are truly philoſophical; and when all the parts of the machine are exactly proportionate to each other, according to theſe principles, it never fails to anſwer the intention.

1. The preſſure of the atmoſphere upon every ſquare inch, at the Earth's ſurface, is about $14\frac{3}{4}$ pounds, Avoirdupois at a medium; or $11\frac{1}{2}$ pounds on a circular inch.

2. If in a cylinder, which has a moveable piſton ſuſpended from one end of a lever, equally divided, there be made a vacuum, the air will preſs down the piſton in the cylinder, with a force proportionable to the area of the ſurface, and thus raiſe an equal weight at the other end of the lever.

3. When water is rarefied into ſteam by being violently heated, the particles of it are ſo ſtrongly repellent, as to drive away air of the common denſity. This ſteam may be again condenſed into water, by a jet of cold water thrown through it; ſo that 14000 cubic inches of ſteam in a cylinder may be reduced into the ſpace of one cubic inch of water only, by which means a partial vacuum is obtained.

4. Though the preſſure of the atmoſphere be about $14\frac{3}{4}$ pounds upon every ſquare inch, or $11\frac{1}{2}$ pounds on a circular inch; yet, on account of the friction of the machine, and ſome remains of ſteam and air in the cylinder, the vacuum is very imperfect, and the piſton does not deſcend with a greater force than eight or nine pounds upon every ſquare inch of its ſurface.

5. A gallon of water of 282 cubic inches, weighs $10\frac{1}{5}$ pounds Avoirdupoiſe, or a cubic foot, $62\frac{1}{2}$ pounds; the piſton being preſſed by the weight of the atmoſphere, in the proportion of eight or nine pounds to every ſquare inch on the ſurface, depreſſes that end of the lever, and raiſes a column of water in the pump of equal weight at the other end of the lever, by means of the pump-rods ſuſpended to it. When the ſteam is again admitted, the

pump-

pump-rods ſink by their ſuperior weight, and the piſton riſes; and when that ſteam is condenſed, the piſton deſcends, and the pump-rods riſe with their quantity of water; and ſo on, alternately, as long as the piſton works.

As the piſton does not deſcend with a force exceeding eight or nine pounds upon every ſquare inch of its ſurface; and as, by reaſon of accidental frictions, and alterations in the denſity of the air, it is ſometimes leſs than this, it will be ſafeſt in practice to calculate the weight at ſomething leſs than eight pounds, viz.—At 7 pounds 10 ounces, for every ſquare inch, equal to 7.64 pounds, which is juſt 6 pounds upon every circular inch; and, as a gallon of water of 282 cubic inches, weighs $10\frac{1}{5}$ pounds, we have the dimenſions of the cylinder, pumps, &c. for any ſteam-engine, as follows:

c = the cylinder's diameter in inches.
p = the pump's diameter in inches.
f = the depth of the pit in fathoms.
g = gallons, drawn by a ſtroke of ſix feet.
h = the hogſheads drawn per hour.
s = the number of ſtrokes per minute.

Then c^2 is the area of the cylinder in circular inches; therefore $6\,c^2$ is the power of the cylinder in pounds.

And $\frac{p^2 \times .7854 \times 72}{282}$, or $\frac{1}{5}\,p^2$ is $= g$, the gallons contained in one fathom, or ſix feet of any pump; which, multiplied by f, gives $\frac{1}{5}\,p^2\,f$ for the gallons contained in f fathoms of any pump, whoſe diameter is p.

Hence, $\frac{1}{5}\,p^2 f \times 10\,\frac{1}{5}$ pound gives $2\,p^2 f$, nearly, for the weight in pounds of the column of water, which is to be equal to the power of the cylinder, which was before found equal to $6\,c^2$. Thus, we have the ſecond equation, viz.—$6\,c^2 = 2\,p^2 f$, or $3\,c^2 = p^2\,f$: the firſt equation being $\frac{1}{5}\,p^2 = g$, or $p^2 = 5\,g$.

From which two equations any particulars may be determined.

Or, if inſtead of ſix pounds for the preſſure of the air on each circular inch, that force be ſuppoſed any number as a pounds; then the power of the cylinder will be $a\,c^2$, and

 the

the ſecond equation becomes $a\ c^2 = 2\ p^2 f = 10\ f g$ by ſubſtituting $5\ g$ inſtead of p^2; and further, $63\ h = 60\ g\ s$; or $21\ h = 20\ g\ s$.

The following theorems are derived from compariſon of theſe equations, and they will determine the ſize of the cylinder and pumps of any ſteam engine capable of drawing a certain quantity of water from any given depth, with the preſſure of the atmoſphere on the piſtons.

Theſe theorems are more particularly adapted to one pump in a pit. But it often happens, that an engine has to draw ſeveral pumps of different diameters, from different depths; in this caſe the ſquare of the diameter of each pump muſt be multiplied by its depth, and double the ſum of all the products will be the weight of water drawn at each ſtroke, which is to be uſed inſtead of $2\ p^2\ f$, for the power of the cylinder.

A TABLE OF THEOREMS, FOR THE READIER *Computing the Powers of a* STEAM-ENGINE.

1	$a = \frac{2 f p^2}{c^2} = \frac{10 f g}{c^2} = \frac{21 f h}{2 c^2 s}$
2	$c = \sqrt{\frac{2 f p^2}{a}} = \sqrt{\frac{10 f g}{a}} = \sqrt{\frac{21 f h}{2 a s}}$
3	$f = \frac{a c^2}{2 p^2} = \frac{a c^2}{10 g} = \frac{2 a c^2 s}{21 h}$
4	$g = \frac{p^2}{5} = \frac{a c^2}{10 f} = \frac{21 h}{20 s}$
5	$h = \frac{4 p^2 s}{21} = \frac{20 g s}{21} = \frac{2 a c^2 s}{21 f}$
6	$p = \sqrt{5g} = \sqrt{\frac{a c^2}{2 g}} = \sqrt{\frac{21 h}{4 s}}$
7	$s = \frac{21 h}{4 p^2} = \frac{21 h}{20 g} = \frac{21 f h}{2 a c^2}$

The

The common Steam-Engine, (fig. 9.)

A is the boiler.

B the cylinder.

C the injection cock.

D the ſteam-cock, or regulator.

E the ſhifting clack.

F the eduction pipe, or ſinking pipe.

G the eduction valve.

H the ſafety valve.

I the piſton.

K the lever beam.

L weights to counterpoiſe the piſton, and preſs down the forcer in the pump-barrel M, to drive the water through the pipe N.

O a ciſtern to hold the injection water.

P an air-veſſel, which prevents the pipe N from burſting, and ſerves to keep up a regular ſtream.

The boiler A is filled with water to the height of *d e*, which being made to boil by a fire, placed beneath it, will fill the upper part A D with a very elaſtic vapour or ſteam, which, when it is of a ſufficient ſtrength, will force open the valve at H. This ſteam is let into the barrel or cylinder B, by turning the cock D; and by its elaſtic force raiſes the piſton I, which drives the air above it through a proper clack, placed at the top. Then, in order to make the piſton deſcend, a little cold water is let into the cylinder, at the bottom, from the ciſtern O, by turning the cock C, which, riſing in the form of a jet, condenſes the hot ſteam in the cylinder, into water, whereby it occupies about 13000 times leſs ſpace than it took up before; which creates a partial vacuum in the barrel, and thereby permits the piſton to deſcend by the weight of the atmoſphere, and thus raiſes the piſton *a*, in the barrel M of the forcing pump, at the other end of the lever K; which, by the pipe O, draws

the water from the depth W, and forces it to riſe and ſpout through the tube N. This is the common form of the engine uſed to draw the water from coal-pits, and deep mines. There are alſo various other engines of the ſame nature, though of a more complex ſtructure; but they all act upon the ſame principle with this.

In theſe engines there is generally a mechanical contrivance for opening and ſhutting the cocks, which could not be conveniently ſhewn in the figure. For the firſt two or three ſtrokes of the engine, when it is ſet to work, a man uſually attends to open and ſhut the cocks, after which the engine is left to turn the cocks of itſelf, which it does with greater exactneſs.

The boiler or veſſel, is uſually made of iron plates, or caſt iron, or any other materials that can ſtand the effect of the fire. The lower part, expoſed to the fire, ſhould be ſo conſtructed that the fire may act with the greateſt advantage upon it; for which purpoſe the ſides ſhould be cylindrical, and the bottom a little concave; and then the flame of the fire carried by a pipe, or iron flue, round the inſide of the boiler, and beneath the ſurface of the water, before it terminates in the chimney; for, by theſe means the water receives all the heat from the fire under the boiler, and alſo a conſiderable quantity of heat from the plate incloſed in the iron pipe, every part of which is in contact with the water.

The preſſure of the atmoſphere, or any equivalent reſiſtance is always found to prevent the production of ſteam, until the water be heated to 212 degrees of *Fahrenheit's* thermometer; but when that preſſure is removed, or the water placed in a veſſel, exhauſted of air, ſteam is produced from it, when it is colder than human blood. On the contrary, when water is preſſed by air or ſteam, which are more compreſſed than the atmoſphere, a degree of heat, of above 212 degrees, is neceſſary to produce ſteam; and the

the difference of heat at which water boils under different preſſures, increaſes in a leſs proportion than the preſſure themſelves; ſo that a double preſſure requires leſs than a double increaſe of the heat.

There are two principal defects in the common ſteam engine:—Firſt, as the vacuum in the cylinder is produced by throwing in cold water to condenſe the ſteam, the water thrown in becomes hot, and produces a ſteam from itſelf, which greatly reſiſts the motion of the piſton downwards, and thereby leſſens the power of the engine.—Secondly, upon attempting to fill a cold cylinder with hot ſteam, a great part of the ſteam will be deſtroyed; and the injection water that is let in to condenſe the ſteam not only cools the cylinder, but remains there until it be extruded at the eduction pipe by the ſteam which is afterwards let into the cylinder, which ſteam will be condenſed into water, as faſt as it enters, until all the matter it comes in contact with be nearly as hot as itſelf.

The great conſumption of fuel alſo has been a material object in theſe engines; for, it is well known, that a ſteam engine of an ordinary ſize, will conſume near three thouſand pounds worth of coals per annum, at any part near London.

Mr. Watt's Steam-Engine.

Mr. Watt has, in a great meaſure, if not wholly, remedied the foregoing inconveniencies: he preſerves an uniform heat in the cylinder of his engine, by ſuffering no cold water to touch it, and by protecting it from the air or other cold bodies, by a ſurrounding caſe filled with the ſteam, or with hot air, or water; and by coating it over with ſubſtances that tranſmit the heat very ſlowly. He makes his vacuum to approach nearly to that of the barometer, by condenſing the ſteam in a ſeparate veſſel, called the condenſer; which may be cooled at pleaſure, without cooling the cylinder, either by an injection of cold water, or by ſurrounding the

condenſer

condenſer with it, and generally by both. The injection water, and detached air, he extracts from the cylinder, or condenſer, by pumps, which are wrought by the engine itſelf; or he blows it out by the ſteam. As the entrance of air into the cylinder would ſtop the operation of the engine, and as it is hardly poſſible that ſuch enormous piſtons as thoſe of ſteam-engines can move up and down, and yet be perfectly air-tight; in the common engines, a ſtream of water is always kept running on the piſton, which prevents the entry of the air; but this mode of ſecuring the piſton would be highly prejudicial in the new engine, though it be not hurtful in the common ones. The piſton of the new engine is therefore made more accurately; and the outward cylinder having a lid, which covers it, the ſteam is produced above the piſton; and when the vacuum is introduced under it, the ſteam acts upon it by its elaſticity, as the atmoſphere does on the piſton of the common engines by its gravity. This mode of working effectually excludes the air from the inner cylinder, and gives the advantage of increaſing the power by increaſing the elaſticity of the ſteam, as ſeen in the following deſcription:

A, (fig. 10.) is the boiler.

B, the ſafety valve.

C, the pipe which conveys the ſteam to the outer cylinder.

D, the outer cylinder.

E, the inner cylinder.

F, the piſton.

G. The valve that admits the ſteam from the outer cylinder into the inner cylinder, called the ſteam valve.

H, the valve that admits the ſteam from the inner cylinder into the condenſer, called the condenſing valve.

I, the condenſer.

K, the injection valve, that admits a jet of cold water into the condenſer, to condenſe the ſteam.

L, the

L, the air-pump, that exhaufts the condenfer both of air and the injection water that is let in at every ftroke, and is fixed under water in the condenfing back M, which is full of water.

N, the lever beam.

O, the great water pump for clearing the mines, or raifing water for any other ufes through the pipes, &c.

This new engine differs from the common ones only in the following particulars: having the cylinder, the great beams, the pumps, &c. in their ufual pofitions.

The cylinder in this new engine is fmaller than ufual in proportion to the load, and is very accurately bored; and is furrounded at a fmall diftance, with another cylinder, furnifhed with a bottom and lid. The fpace between the cylinders communicates with the boiler by a large pipe, open at both ends, fo that it is always filled with fteam, and thereby preferves the inner cylinder of the fame degree of heat with the fteam, and prevents the fteam from condenfing within it, which would be more prejudicial than an equal condenfation in the outer cylinder.

The inner cylinder has a bottom and pifton as ufual; and as it does not reach up quite to the lid of the outer cylinder, the fteam in the fpace between them has always a free accefs to the upper fide of the pifton. The lid of the outer cylindor has a hole in the middle, through which the pifton rod moves up and down; and the hole is kept tight by a collar of oakum fcrewed upon it.

There are two regulating valves at the bottom of the inner cylinder; one of which admits the fteam to pafs from the fpace between the two cylinders into the inner cylinder, below the pifton, and fhuts it out at pleafure: the other opens or fhuts the end of a pipe that leads to the condenfer. The condenfer confifts of one or more pumps furnifhed with clacks and buckets, (nearly the fame as in common pumps) which are wrought by chains faftened to the great working beam of the engine. To the bottom of thefe

pumps

pumps is joined the pipe, which comes from the cylinder; and the whole condenſer ſtands immerſed in a ciſtern of cold water, ſupplied by the engine. This ciſtern is ſometimes placed in the houſe, under the floor, between the cylinder and the lever wall; and ſometimes without the houſe, between that wall and the engine-ſhaft, as may be moſt convenient.

When this engine is put in motion, the condenſer is exhauſted of air by blowing, and both the cylinders being filled with ſteam, the regulating valve, which admits the ſteam into the inner cylinder, is ſhut; and the other regulator, which communicates with the condenſer, is opened, and the ſteam ruſhes with great violence into the vacuum of the condenſer; but there it comes into contact with the ſides of the cold pipe and pumps, and meets a jet of cold water, which was opened at the ſame time with the exhauſting regulator: theſe inſtantly deprive it of its heat, and reduce it to water; and the vacuum remaining perfect, more ſteam continues to ruſh in and be condenſed, until the inner cylinder be exhauſted; then the ſteam, which is above the piſton, not being any longer counteracted by that which was below it, acts upon the piſton with its whole elaſticity, and forces it to deſcend to the bottom of the cylinder, and ſo raiſes the buckets of the pumps which are hung to the end of the beam. The exhauſtion regulator is now ſhut, and the ſteam regulator opened again, which, by letting in the ſteam, permits the piſton to be pulled up by the ſuperior weight of the pump-rods; and thus the engine is ready for another ſtroke.

Theſe improved engines have two principal advantages above the common ones:—Firſt, the cylinder being ſurrounded with the hot ſteam from the boiler, is always kept uniformly of the ſame heat with the ſteam itſelf, and therefore will not deſtroy any part of the ſteam which ſhould fill it, as the common engines do.—Secondly, the condenſer being as cold as cold water, the ſteam is perfectly condenſed, and does not, in the leaſt, oppoſe the deſcent of

the

the piston, which is therefore forced down by the full power of the steam from the boiler, which is somewhat greater than the pressure of the atmosphere.

In the common engines, when they are loaded to about seven pounds upon each square inch of the piston, and are of a middle size, the quantity of steam which is condensed, in restoring to the cylinder the heat which it had been deprived of, by the former injection of cold water, is about one full of the cylinder, besides what is really required to fill that vessel: so that twice the full of the cylinder is employed to make it raise a column of water, equal to seven pounds for each square inch; or more simply, a cubic foot of steam raises a cubic foot of water about eight feet higher, besides overcoming the friction of the engine, and the resistance of the water to motion.

But in the improved engine of *Mr. Watt*, about one full and a fourth of the cylinder is required to fill it, because the steam is one-fourth more dense, than in the common engine. This engine, therefore, raises a load equal to twelve pounds and a half upon each square inch of the piston; and each cubic foot of steam, of the density of the atmosphere, raises one cubic foot of water 22 feet high.

These engines work more regular and steady than the common ones; and the savings amount at least to two-thirds of the fuel; which is an important object where coals are dear. The new engines also will raise from twenty-thousand to twenty-four thousand cubic feet of water, to the height of twenty-four feet, by only one hundre weight of good pit coals.

CHAP. XX.

OF DRAWING.

SECT. I.

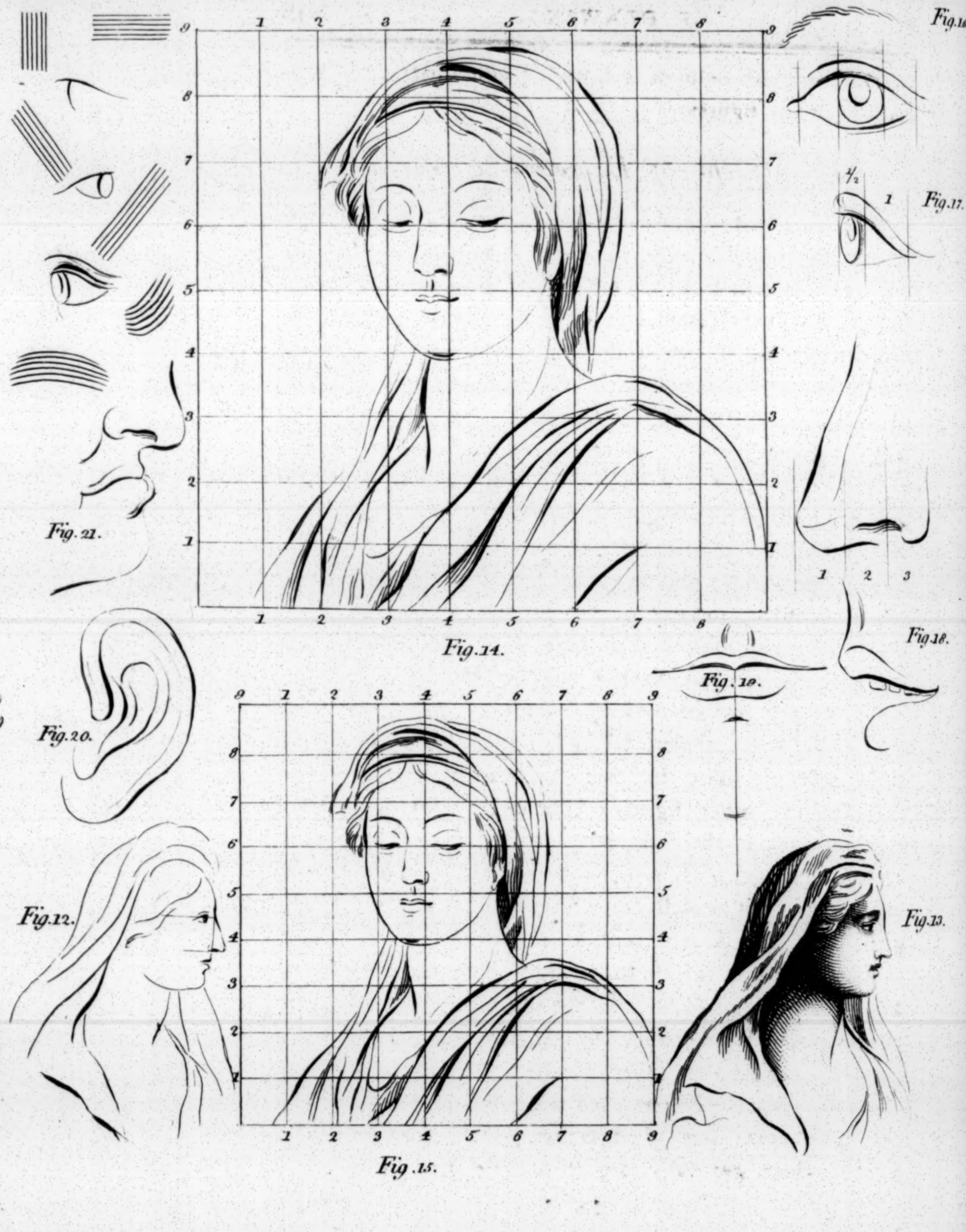
OF DRAWING.

Implements necessary for Drawing.

THE implements neceſſary for drawing are, a pair of common compaſſes, charcoal, a ruler, black and red lead pencils, India ink, crow-quill pens, camel-hair pencils, fitches, &c. and for colouring, port crayons, black, white, and red chalk, crayons, &c.

The charcoal ſhould be made of ſallow wood, ſplit into the form of pencils, and ſharpened to a point; its uſe is to draw out the deſign at firſt, that if any part be amiſs it may be rubbed out and corrected. Feathers from a duck's wing, on account of their ſtiffneſs, are alſo neceſſary to wipe out any falſe ſtrokes of the charcoal. The black and red lead pencils are uſed to draw out the draft the ſecond time, becauſe the lines drawn with theſe will not be liable to be rubbed out with the hand, when the lines are again drawn with the pen. The pens made of crow quills (though another good pen may anſwer the purpoſe) are to finiſh the work. Rulers are to draw the ſtraight lines, triangles, ſquares, &c. which are to be done at firſt, till practice render them needleſs. The compaſſes ſhould have ſteel points, which will take out, in order to uſe a black or red lead pencil, their uſe is to draw circles, ovals, arches, &c. alſo to meaſure,


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 20.



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meaſure, by the help of a ſcale, of equal parts, the proportions of the figures.

The precepts of Drawimg in general.

There is no art that depends leſs upon theory than thoſe of drawing and painting; in theſe it is principally practice and experience that can render any one a good artiſt. But in this, as in every other art, a few rules may be of ſervice: and in attending to the following rules, the young artiſt ſhould be careful in following the outlines of the figure, which is the firſt proceſs. He muſt alſo content himſelf with copying parts of objects, before he aims at any finiſhed piece; and dwell upon each part, and never begin a ſecond, till he thoroughly underſtands the proportions of all the outlines of the firſt. He muſt alſo be very ſlow in his firſt operations; and he cannot too often contemplate the length, breadth, and every other proportion of each object of his original; for this purpoſe he ſhould have it conſtantly in his eye, and cannot look too often at it.

1. The firſt part of drawing conſiſts of plain geometrical figures; as lines, angles, triangles, quadrangles, polygons, cones, and the like; for theſe are the foundation of the outlines of all other figures. The circle aſſiſts in all orbicular forms; as the ſun, moon, fruits, &c. The oval in giving a juſt proportion to the human face and mouth, the mouth of a pot or well, any cilindrical body, the foot of a glaſs, &c. The ſquare confines the picture to be copied, and ſerves to draw a ſcale to enlarge or contract the piece, &c. The triangle is uſed in the ſide face; the polygon in ground plans, fortifications, &c. angles and arches in perſpective; the cone in ſpires, ſteeples, &c. the cylinder in columns, pillars, pilaſters, and their ornaments.

2. The next part of drawing conſiſts in forming of fruits, which generally depart a little from the circular

form; as apples, pears, cherries, grapes, &c. The imitation of flowers, as roſes, tulips, &c. herbs, as thyme, mint, &c. trees; as oak, fir, &c.

3. The third part of the practice imitates beaſts, fowls and fiſhes, of which variety of prints may be purchaſed at reaſonable rates.

4. The fourth part conſiſts in imitating the human body, with all its lineaments; as the head, noſe, eyes, ears, cheeks, hands, arms, &c. which muſt be all exactly proportioned both to the whole and to one another.

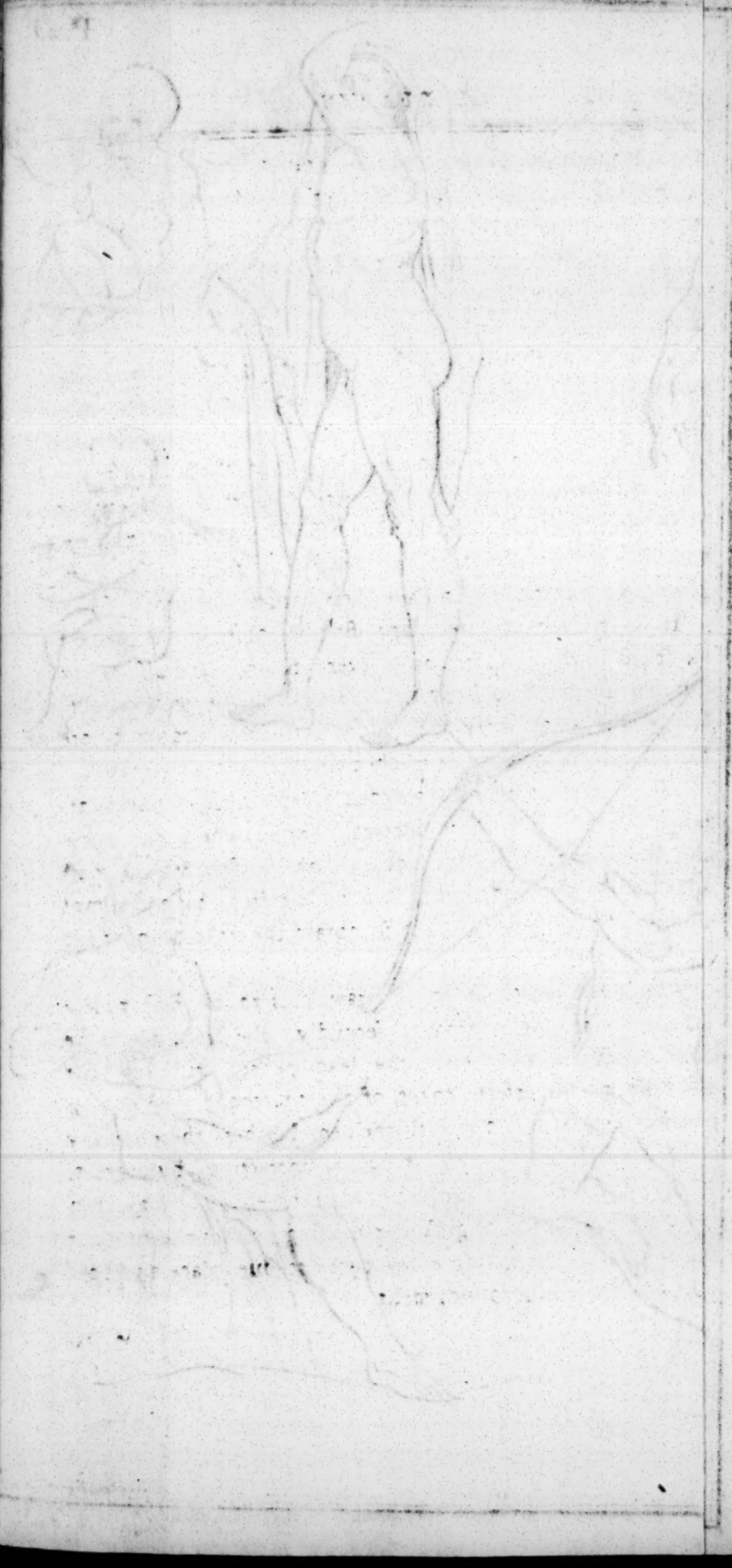
5. The fifth part of the practice conſiſts in drapery, or imitating cloathing, and ſetting off in an artificial manner, the outward coverings of the body; as cloth, ſilk, ſtuff, linen, &c. which ſhould have their natural and proper folds.

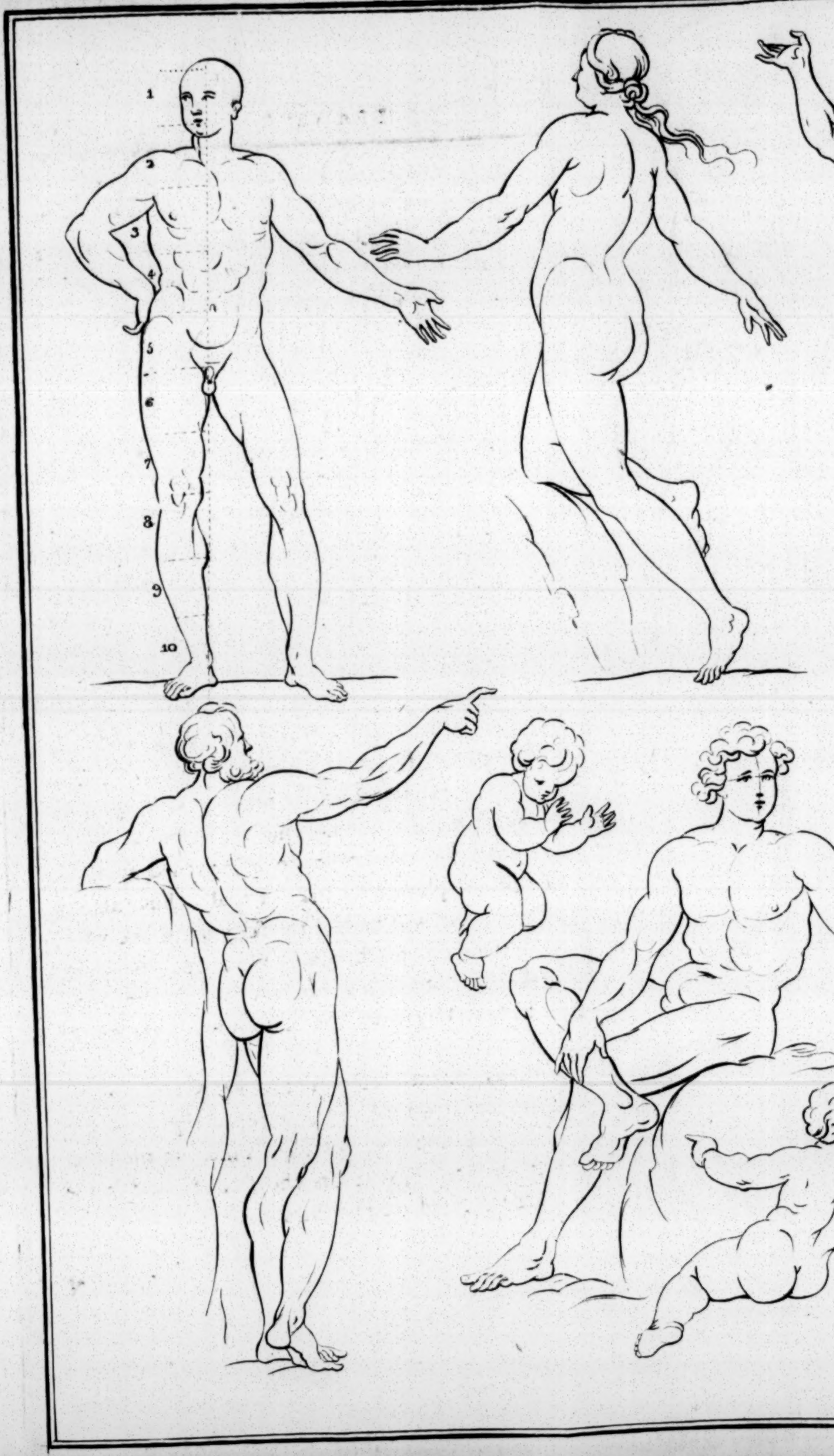
In every form it is neceſſary, firſt, to obſerve the greateſt exactneſs in the proportions of every figure; and alſo in the different parts of each figure. Secondly, the general outline of the whole work ſhould be accurately drawn before the piece be ſhadowed, or finiſhed within.

When the artiſt has proceeded ſo far in the art as to attempt figures of a more uncertain form, where the circle and ſquare cannot be uſed; as in lions, horſes, &c. he muſt work by his own judgment, and be directed, by a conſtant attention to his original, and ſo obtain the true proportions by daily practice.

The piece which is to be drawn, is to be firſt rudely ſketched with the charcoal; ſecondly, the lines are to be more exactly drawn with the lead pencil, rubbing out any falſe or imperfect ſtrokes of the charcoal; then having peruſed it well, mend it with the pencil, where there are any errors: this done peruſe it again, correcting, by degrees, all the errors of leſs magnitude, even to the leaſt jot; then compare it with the original copy, uſing neither the rule nor the compaſs, but giving every part its due place and proportion, according to judgment.

When

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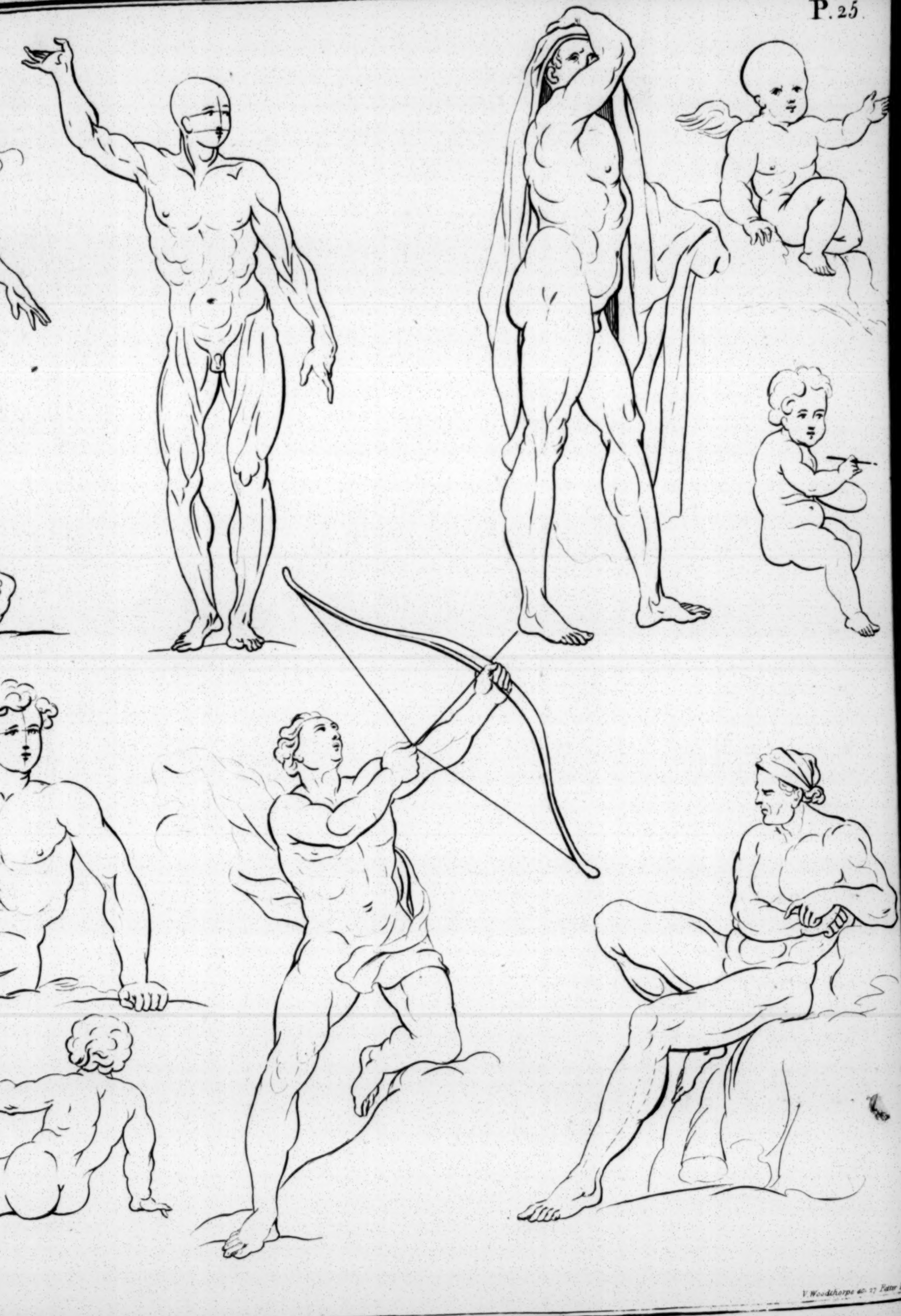

V. Woodthorpe sc. 27 Pater Los

When the artiſt has arrived to ſome perfection in the art, he may begin to copy after life, for that is the moſt correct and complete method of drawing or painting; and there only he has the largeſt liberty of imitation; but there ought to be ſome perfection in the art before he aims at this.

Particular directions for drawing.

The print or painting which is to be copied, is to be placed ſo, that the gloſs of the colours or ſhades may not fall upon the eye, and thereby prevent a perfect view of the piece; but it is to be ſo placed, that both the light and the eye may fall obliquely upon it. It muſt alſo be placed at ſuch a diſtance, that the whole may be taken into the eye at once; for which purpoſe, the larger it is the further it muſt be placed off, and ſet a little reclining.

Make a ſmall point upon the paper, to repreſent the centre, and obſerve, as a general rule, always to begin with the right ſide of the piece; for, by that means what is finiſhed will not be hidden by the hand or pencil. Obſerve alſo, the moſt perſpicuous and uppermoſt figures in the piece, (if there be more than one) which are to be touched upon the paper in their proper places, by the charcoal; thus running over the draught, there will be had the ſkeleton of the work, which is to be afterwards finiſhed and filled up. But great care is to be taken in obtaining a true draught, and the more time there is beſtowed upon it, the more it will improve the learner.

To arrive at any tolerable proficiency in this art, the ſtudent muſt particularly attend to the directions given in the following leſſons. He muſt alſo perfect himſelf in the practice of the firſt, before he attempts the ſecond; and in the ſecond, before he proceeds to the third, &c.

LESSON

LESSON I.

The first part in which the learner should perfect himself, is, in drawing the introductory lines with the several features and limbs of the human body separately; in order to which, he should be able, not only to imitate perpendicular and parallel lines without using the ruler; but also the several sorts of curved lines with their different inclinations. When he can perform this with ease, he may proceed to draw the outlines of the features of the human face; as the eyes, nose, mouth, and ears, (fig. 16, 17, 18, 19, 20, and 21, plate 24) and from thence to the limbs and other parts of the human body; (figs. 5 and 9.) These must be only faintly sketched with the charcoal, that they may be easily rubbed out in order to make the necessary alterations.

LESSON II.

When the learner has become master of the parts described in the foregoing lesson, he may then, and not before, attempt the profile, or side face, (figs. 12 and 13,) strictly observing the proportions both of the whole and the several parts to each other; he may next describe the full or oval face (fig. 2 and 10.) still observing the bearings of every feature with respect to the rest. The eye, seen in front, is divided into three equal parts, (fig. 16.) the centre part of which is the size of the sight; and the opening of the eye is generally about one-third of its length. The eye in profile, (fig. 17.) is half the size of the eye in front, having only one-third and a half of the length. The nose, seen in front, is of the same width when seen in profile, which is equal to the length of the eye, (fig. 18 and 16,) and the nostril in height is nearly one-third of the width of the nose. These proportions are to be attended to only during the learner's first practice: when he is arrived to a

little

little proficiency, he is to follow his own judgment only in the proportions, paying more regard to his original than to any verbal directions. When he is able to finifh the outline of the profile, he may proceed to the other different inclinations of the face, (as feen in the plate.)

LESSON III.

When the artift can eafily imitate the different features and limbs, he may begin to attempt whole length figures; which is to be done in the following manner: fketch the whole over lightly with the charcoal, (or if the learner be able, he may ufe the black lead pencil at firft) beginning with the head, next the fhoulders, then the body; after which the arms and hands, then the hips, legs and feet; then examine the proportion of the different parts, rubbing out any ftrokes of the charcoal where neceffary; and drawing the lines over again with the black lead pencil, to bring it as near as poffible to the original; when this is done, proceed to finifh the figure, by drawing it over again with the crow-quill pen and Indian ink, departing from the black lead lines where it may be found neceffary. Then rub out all the marks of the pencil with India rubber. The compaffes are not to be ufed till after a very minute infpection with the eye; then, if there be any fault that cannot be eafily difcovered, by applying the compaffes firft to the original, and then to the copy, the fault will be foon found. To afcertain the proportions of the feveral parts of the human body, a perpendicular line fhould be firft drawn through that part intended for the middle of the figure; which fhould be divided into feveral equal parts, and from fuch menfuration a fcale may be formed to regulate the proportions of every part of the body. Some divide the length of the human body into eight heads or parts, and others divide into ten. I fhall give the proportions of each:

When

When the human body is divided into eight heads, (fig. 1, plate 25,) the length of the head, which is the firſt diviſion of the figure, will extend from the crown or top of the head to the bottom of the chin. The ſecond diviſion will terminate in a line drawn through the paps of the breaſts. The third diviſion will fall a little below the naval. The fourth acroſs the privities, which is exactly the middle of the figure. The fifth croſſes the middle of the thigh. The ſixth is juſt below the bend of the knee. The ſeventh falls a little below the calf of the leg: and the eighth extends to the bottom of the heel. And obſerve, that the full extent from the end of the middle finger of the right hand to that of the left, when the arms are extended at full length, in a direct line, is juſt the length of the whole figure; and from the middle of the collar-bone, to the end of the middle finger, is juſt four heads, or half the length of the figure, viz. The firſt head extends to the bend of the ſhoulder; the ſecond from thence to the elbow; the third to the wriſt, and the fourth to the fingers end. Thus, from ſhoulder to ſhoulder, in a man of common ſize, meaſures exactly two heads. There is no preciſe ſtandard for the breadth of the limbs, for they vary according to the bulk of the perſon.

When the human body is divided into ten heads, the firſt diviſion extends from the crown of the head to the under lip. The ſecond a little below the collar-bone, which is juſt even with the middle of the ſhoulder. The third will fall juſt below the paps of the breaſt. The fourth will reach juſt below the navel. The fifth, being the middle of the figure, will paſs acroſs the privities. The ſixth will paſs over the middle of the thighs. The ſeventh croſſes the bend of the knee. The eighth paſſes directly through the calves of the legs. The ninth extends half from the calf to the bottom of the heel, where the laſt diviſion terminates.

The

The learner ſhould pay particular attention to theſe proportions, and retain them in his memory. It is alſo neceſſary that he pay ſome attention to anatomy, as it will enable him to judge of the proportion and diſproportion of the human figure.

LESSON IV.

The drapery, or cloathing of the figure, is next to be conſidered: having drawn the outline of the figure faintly with charcoal, correcting every part that appears faulty; proceed to draw the outline of the drapery lightly with the charcoal, with the ſeveral folds, not ſuffering them to croſs each other. The quality of the drapery ſhould alſo be conſidered; as ſtuffs and woollen cloth, are more harſh than ſilk, which is always flowing and eaſy. The drapery ſhould not ſtick too cloſe to the body, but ſhould appear to flow eaſily. If the drapery be ſuppoſed to be blown by a breeze of wind, it ſhould all flow one way, and the parts next the body ſhould be drawn before thoſe which fly off. The garments ſhould always bend with the figure; and the cloſer the drapery is to the body, the ſmaller muſt be the folds; and if it be quite cloſe to the body, there ſlould be no folds, but only a faint ſhadow, to repreſent that part of the body which it covers. But the beſt rule in this caſe is to remark the folds as they appear in the drapery of genteel perſons, if the figure be to have a modern dreſs; but a few particular rules may, however, greatly aſſiſt the learner:

1. Carefully avoid a ſuperfluity of drapery.—2. Let as much of the form of the body as poſſible appear under the drapery.—3. When the draperies are large, let them be thrown into large and graceful folds.—4. Drapery which is cloſe to the body, ſhould appear to be looſened, by ſmall folds, judiciouſly placed; for want of this caution, the figure will have a certain ſtiffneſs, and appear as if wrapped round with a bandage, inſtead of being cloathed.—5. If there be much drapery, let the greater part, if poſſible, be

thrown into ſhadow.—6. Thoſe folds which fall in the light, muſt have ſuch ſoft and tender ſhadows as to make them appear to ſit hollow from the body.—7. Let all the folds be properly contraſted; and avoid ſtraight lines as much as poſſible.—8. A judicious repetition of folds, in a circular form, greatly contributes to characterize a fore-ſhortened limb.—9. In fixed attitudes the drapery ſhould appear motionleſs, unleſs expoſed to the air. But, in figures moving with great agility, the drapery ſhould play as if agitated by the wind, and that in proportion to the velocity of the figure in motion.

It muſt, however, be obſerved, once for all, that the ſtudent, who too ſervilely copies ſtatues or paintings, will always give his figures a certain ſtiffneſs, of which it will be afterwards impoſſible to diveſt them. In this, as in every other liberal art, nature is the beſt original, and the productions of that artiſt who imitates this great miſtreſs, will ſtand the teſt of ages.

LESSON V.

When a figure has all its outlines completely finiſhed, and in their true proportions, it is laſtly to be ſhadowed, in which obſerve the following method:—

The drawing is firſt to be ſhadowed with the pencil, carefully obſerving from which ſide the light comes; which, if natural, ſhould be either from the right or left; for when the light falls in the middle, it is called an artificial light, as it proceeds from a candle or lamp, &c. The ſhades ſhould be laid on faintly at firſt, that they may be heightened where neceſſary.

The ſhades of every part of the ſame figure, and alſo of every figure in the ſame piece, muſt be all on the ſame ſide; that is, if the right ſide of the face of the figure be in ſhade, ſo muſt the right ſide of the body, arm, leg, &c. of the ſame figure; as alſo the right ſide of every other figure in the ſame drawing or piece. The ſhades ſhould be faint, as they

they approach the light the ſtrength gradually decreaſing towards the extremities.

The ſhades ſhould be rubbed with a piece of paper or glove-leather, rolled up hard, and cut almoſt to a point, like a pencil; which ſerves to blend the ſhades and ſoften them into each other; and alſo to weaken them, where they appear too ſtrrong. India ink is prepared for different ſhades, by rubbing it more or leſs in water on a marble ſtone, cut in hollows for that purpoſe, reſerving one of the hollows for the water. In ſhading a piece, as well as in drawing the outlines, the beſt rule is to follow nature, which will greatly improve the ideas in the diſpoſal of light and ſhade.

To enlarge or contract a Piece.

Divide the original into any number of equal ſquares; then divide the paper upon which the piece is to be drawn, into the ſame number of equal ſquares, either greater or leſs than thoſe in the original, and draw the ſeveral parts of the piece as they fall in the ſquares of the original, in the correſponding ſquares of the paper: then outline it with India ink, and rub out the black lead marks of the ſquares; and ſhade the piece, &c. To prevent miſtakes, the ſquares may be numbered both in the original and copy. (fig. 14 and 15, plate 24.)

Though the human body be divided into eight or ten heads, as before obſerved, and which may anſwer for ſmall figures, and where great exactneſs is not expected, yet it muſt be remembered, that theſe are not the exact proportions. Where the figures are large, the artiſt ſhould be more correct, for which purpoſe, it may be neceſſary to attend to the proportions of the ancients, as they were meaſured by *M. Audran*, from the Apollo Pythius, in the garden of the Vatican, at Rome, and the Venus Aphroditus, belonging to the family of the Medicis. And both which figures are

 ſuppoſed

ſuppoſed to ſtand upright, duly poiſed on both legs. The whole height of the former is divided into $31\frac{1}{2}$ parts, being ſeven heads, three parts, and ſix minutes: and the latter into 31 parts, being ſeven heads, and three parts; as follows:

Length of the head, and trunk of the body.

	Apollo Hhd. Pts. Min.	Venus. Hhd. Pts. Min.
From the top of the head to the bottom of the chin 4 pts. or	1 . 0 . 0	1 . 0 . 0
The bottom of the chin to the top of the ſternum, or breaſt bone - - - - - -	0 . 1 . 7	0 . 1 . 8
The top of the ſternum to the pit of the ſtomach.	0 . 3 . 10	0 . 3 . 6
The pit of the ſtomach to the navel - - - - - -	0 . 2 . 10	0 . 2 . 7
The navel to the pubis -	0 . 3 . 6	0 . 3 . 9
Length of the head and trunks of the body - - - - - -	3 . 3 . 9	3 . 3 . 6

Length of the Lower Extremities.

	Apollo	Venus
From the pubis to the ſmall of the thigh, above the patella, or knee-pan - - - - -	1 , 2 . 6	1 . 2 . 3
The ſmall of the thigh to the joint, or middle of the knee - - - - - - - - -	0 . 1 . 9	0 . 1 . 6
The joint of the knee to the ſmall of the leg, above the ancle - - - - - -	1 . 1 . 9	1 . 2 . 0
The top to the bottom of the ancle - - - - - -	0 . 1 . 0	0 . 1 . 0
The bottom of the ancle to the bottom of the heel -	0 . 0 . 9	0 . 0 . 9
Length of the lower extremities	3 . 3 . 9	3 . 3 . 6
Length of the head and trunk	3 . 3 . 9	3 . 3 . 6
Total length of the figures -	7 . 3 . 6	7 . 3 . 0

Length

Length of the fore arm, or upper extremities.

	Apollo, Hds. Pts. Min.	Venus. Hds. Pts. Min.
From the top of the ſhoulder to the elbow - - - - - -	1 . 2 . 3	1 . 2 . 3
The elbow to the hand -	1 . 1 . 2	1 . 0 . 6
The joint of the hand to the root of the middle finger - - - - - - - - -	0 . 1 . 8	0 . 1 . 6
The root to the tip of the middle finger - - - -	0 . 1 . 10	0 . 1 . 7
Length of the upper extremities	3 . 2 . 11	3 . 1 . 10
Breadth between the outward angles of the eyes -	0 . 1 . 6	0 . 1 . 7
Of the face at the temples	0 . 2 . 2	0 . 2 . 2
Of the upper part of the neck - - - - - - - - -	0 . 2 . 0	0 . 1 . 11
Over the ſhoulders - -	2 . 0 . 0	1 . 3 . 8
Of the body, below the arm-pits - - - - - -	1 . 2 . 5	1 . 1 . 8
Between the nipples - -	1 . 0 . 7	0 . 3 . 8
From the bottom of the chin to the line croſſing the nipples - - - - - - -	1 . 0 . 7	1 . 0 . 1
Of the body, at the ſmall of the waiſt - - - -	1 . 1 . 0	1 . 0 . 8
Over the loins - - - -	1 . 1 . 3	1 . 1 . 6
Over the haunches, or tops of the thigh bones - -	1 . 1 . 5	1 . 2 . 3
Of the thigh at the top -	0 . 3 . 0	0 . 3 . 1
Of the thigh below the middle - - - - -	0 . 2 . $8\frac{1}{2}$	0 . 2 . 7
Of the thigh above the knee	0 . 1 . 8	0 . 2 . 0
Of the leg below the knee	0 . 1 . 6	0 . 1 . $10\frac{1}{2}$
Of the calf of the leg -	0 . 2 . 4	0 . 2 . 3

Below

	Apollo. Hds. Pts. Min.	Venus. Hds. Pts. Min.
Below the calf - - - - -	0 . 1 . 7	0 . 1 . 11½
Above the ancle - - - -	0 . 1 . 2	0 . 1 . 2
Of the ancle - - - - -	0 . 1 . 4	0 . 1 . 3
Below the ancle - - - -	0 . 1 . 1½	0 . 1 . 1
Middle of the foot - -	0 . 1 . 4	0 . 1 . 3
At the roots of the toes -	0 . 1 . 7	0 . 1 . 7
Of arm over the biceps mufcle - - - - - - - -	0 . 1 . 8	0 . 1 . 9
Of the arm above the elbow	0 . 1 . 6	0 . 1 . 5
Of the arm below the elbow	0 . 1 . 10	0 . 1 . 7
At the wrift - - - - -	0 . 1 . 1	0 . 1 . 0
Of the hand over the firft point of the thumb - -	0 . 1 , 9	0 . 1 . 8
Of the hand over the roots of the fingers - - - - -	0 . 1 . 7	0 . 1 . 6
Breadth over the fhoulder blades - - - - - - - -	1 , 2 . 0	1 . 1 . 4
Length of both arms and hand, each of Apollo's being 3 hds. 2 pts. 11 min. and each of Venus 3 h. 1 p. 5 m.	7 . 1 . 10	6 . 2 . 10
Breadth between the tips of the middle fingers of each hand, when the arms are extended horizontally -	8 . 3 . 10	8 . 0 ; 2

Side View.

	Apollo.	Venus.
From the top of the head to the fhoulder - - - - - - -	1 . 1 . 8	1 . 1 . 6
The top of the fhoulder to to loins above the hip -	1 : 3 . 3	1 . 1 . 7
The loins to the lower part of the hip - - - - - -	1 . 0 . 2	1 . 2 . 1
The hip to the fide of the knee - - - - - - - -	1 . 2 . 0	1 . 0 . 11
The fide of the knee to the bottom of the heel - -	2 . 0 . 5	2 . 0 . 11
	7 . 3 . 6	7 . 3 . 0

From

	Apollo.			Venus.		
	Hds.	Ptss	Min.	Hds.	Pts.	Min.
From the fore to the back part of the ſkull - - - - - -	0	3	6	0	3	4
The wing of the noſe to the tip of the ear - - - - -	0	1	8½	0	1	6
The upper part of the neck	0	2	0	0	1	11
The breaſt to the back, over the nipple - - - - - -	1	0	6	1	0	6
The belly to the ſmall of the back - - - - - -	0	3	6	0	3	7
The belly above the navel to the back of the loins -	0	3	9	1	0	2
The bottom of the belly to the round of the hip - -	1	0	0	1	0	5
The fore part of the thigh to the middle of the hip -	0	3	2	0	3	7
The middle of the thigh -	0	3	3	0	3	6½
The thigh above the knee	0	2	1	0	2	3
The middle of the knee -	0	2	1	0	2	2
The leg above the knee -	0	1	9	0	1	11
The leg at the calf - -	0	1	8	0	1	9
The leg at the ancle - -	0	1	5½	0	1	4
The foot at the thickeſt part - - - - - - - -	———			0	1	3
Length of the foot - -	1	0	6	1	0	4½
The heel to the fore part of the bend of the foot - -	———			0	2	2
The arm over the biceps -	0	2	0	0	1	9
Over the elbow - - -	0	1	6	0	1	6
Below the elbow - - -	0	1	5	0	1	7
At the wriſt - - - -	0	1	1	0	0	11
Below the joint of the wriſt	0	1	0	0	0	10
The hand at the roots of the fingers - - - - - - -	0	0	5½	0	0	5
At the roots of the nails -	0	0	3½	0	0	3

These

These are the proportions of the two most admired statues extant: the proportions of some other admired statues differ somewhat from these; for example: the Laicoon measures 7h. 2p. 3m.—The Hercules 7h. 3p. 7m.—The Pyramus 7h. 2p.—The Antinons 7h. 2p.—The Grecian Shepherdess 7h. 3p. 6m. and the Mirmillo 8h. But the proportions of each part of the same figure are allowed to be perfectly harmonious, and agreeable to the characters.

It must also be observed, that the centre, or middle part, between the extremities, or head and feet of a new-born child, is in the navel; but that of an adult, or grown person, is in the os-pupis. Painters and sculptures generally divide the length of a child into four, five, or six parts, according to its age; and the head is one of those parts. Thus, a child of two years old, has about five heads in its whole length; but one of from four to five years old, has near six heads; and about the fifteenth or sixteenth year, seven heads are the proper proportion or measure, and the centre is in the upper part of the pubis. Therefore, as the body increases in growth, there is a gradual approach to the proportion of an adult, who has near eight heads in his whole length, or rather upwards of seven and three quarters.

SECT. II.

OF ETCHING COPPER PLATES.

ETCHING is a mode of engraving on copper, where the lines or strokes are eaten in with aqua-fortis, instead of being cut with the tool or graver.

Etching has several advantages above graving, as being done with more ease and expedition, requiring fewer instruments: and representing faint, distant objects, more agreeable, and according to nature: and it is usually done in all

plates

plates where there is a complexity of work, though they are often finiſhed by the graver.

The inſtruments proper for etching are needles, oil-ſtone, bruſh-pencils, burniſher, ſcraper, compaſſes, ruler, tracer: and the graver, with the hard and ſoft varniſh, prepared oil, and aqua-fortis.

The needles ſhould be of a fine grain, and ſuch as will break without bending, of which there ſhould be ſeveral ſizes. They are to be fixed in round, firm ſticks, about ſix inches in length, and the thickneſs of a large gooſe quill; and may be fixed in ſuch ſticks as have a pencil at the other end. They ſhould ſtand at leaſt a quarter of an inch out of the ſtick.

The oil-ſtone is to whet the needles upon; and note—if the points are to be round, they are to be whetted ſhort upon the ſtone, by turning them round; but if the points are to be ſloped, they are firſt to be blunted upon the ſtone, and then whetted, ſloping on one ſide only, till they come to a ſhort oval.

The bruſh-pencil is to cleanſe the work, wipe off the duſt, and ſtrike the colours even over the ground, when laid upon the plate.

The burniſher is a piece of tempered ſteel, ſomewhat round at the end, for ſmoothing and giving a luſtre to the plate.

The ſcraper is uſed for clearing the plate of any ſcratches or ſtrokes, which the burniſher will not take out.

The compaſſes ſhould have ſteel points, and are chiefly uſed in ſtriking circles, and meaſuring diſtances, &c.

The ruler is chiefly uſed to draw ſtraight hatches, or lines, upon the plate.

The tracer is uſed for drawing through all the outermoſt lines or circumference of the print or drawing, which is called etching after.

The manner in which etching is performed, is the covering of the ſurface of the plate with a proper varniſh or ground,

ground, capable of refifting the aqua-fortis: and then drawing the lines of the figure with needles quite through the varnifh upon the plate; then the plate being covered with aqua-fortis, the parts of the copper where the lines are drawn, being expofed to the action of the aqua-fortis, will be corroded, or eaten away by it; while the reft of the plate which is covered with the varnifh, will remain untouched.

There are two methods of etching, differing from each other in the varnifh ufed, and in the quality of the aqua-fortis: but the general methods of performing them are both alike. The varnifhes or grounds are diftinguifhed by the names of the hard and the foft varnifh; for their difference principally confifts in the refiftance they give to the needles in working. The hard varnifh was formerly moft in ufe, on account of its admitting the work to be made more correct, and appear more like engraving with the tool or graver; and the firmnefs of the body of the varnifh gave more opportunity of re-touching the lines, or enlarging them with the oval pointed needles; for the excellency of etching was formerly thought to confift in its near refemblance to engraving; and the principal object in thofe who practifed it, was to render it as near as poffible in appearance to engraving by the tool or graver, which was the great caufe of obftructing the improvement of the art, and cramped the talents of the ableft mafters. The foft varnifh, however, has almoft fuperfeded the ufe of the hard, except in the cafe of fome particular fubjects as it admits of a more free manner of working, and affords a power of expreffion incompatible with the inflexibility of the hard varnifh, which confine the lines and hatches to fuch a regularity and famenefs, as gives a ftiffnefs of manner and coldnefs of effect, to the work, particularly in hiftorical engraving, where there is a greater opportunity of exercifing the force of genius and fancy; and where the effect of the whole, more than the minute exactnefs of finifhing the feparate parts, conftitutes the principal value.

The

The manner of etching with the ſoft varniſh is now more frequently intermixed with the uſe of the graver: which is generally attended with great advantages, and even where the whole is intended to paſs for the work of the graver; as it gives an opportunity of ſhewing the truth and ſpirit of the outline, and gives all the variety of ſhades which the different kinds of black can produce: while the exactneſs and regularity of the lines which are required for finiſhing many kinds of deſigns, are ſupplied by the graver; and by a judicious application of both, that compleat finiſhing and effect is produced, which either of them alone would be incapable of affording.

Preparation of the soft Varnish, as directed by Mr. Lawrence, an emenent English engraver, at Paris.

"Take of virgin wax, and aſphaltum, each two ounces; of black pitch, and Burgundy pitch, each half an ounce: melt the wax and pitch in a new earthen ware glazed pot; and add to them, by degrees, the aſphaltum, finely powdered; let the whole boil, till ſuch time as that (taking a drop upon a plate) it will break when it is cold, or bending it double two or three times betwixt the fingers. The varniſh being then boiled enough, muſt be taken off the fire; and letting it cool a little, muſt be poured into warm water, that it may work the more eaſily, with the hands, ſo as to be formed into balls; which muſt be rolled up and put into a piece of taffety for uſe."

In boiling the ingredients it muſt be obſerved, firſt, that the fire be not too violent, leſt they burn; a ſlight ſimmering will be ſufficient. Secondly, while the aſphaltum is putting in, and even after it is mixed with them, the ingredients ſhould be ſtirred continually with the ſpatula: And thirdly, the water into which this compoſition is thrown, ſhould be nearly of the ſame heat to prevent a kind of cracking,

which will happen when the water is too cold: Fourthly, the varnish ought always to be harder in summer than in winter; for which it should be boiled a little longer, or else a greater proportion of asphaltum must be used. The above experiment of suffering a drop to cool, will determine the degree of hardness or softness that may be suitable to the season.

The preparation of the hard Varnish used by Callot, *commonly called the Florence Varnish.*

Take four ounces of fat oil, very clear, and made from good linseed oil: heat it in a clear pot of glazed earthen ware, and afterwards put to it four ounces of mastick, well powdered; and stir the mixture briskly, till the whole be well melted: then pass the whole mass through a piece of fine linen into a glass bottle with a long neck that can be stopped very securely, and keep it for use.

The method of using the soft Varnish.

The plate being well polished, and burnished, and also cleansed from all greasiness by chalk, or Spanish white; fix one edge of the plate where no work is to be, in a hand-vice, to serve as a handle for managing it when warm; then put it upon a chafing-dish, in which there is a moderate fire; observing to hold it so, that it may not burn: when the plate is hot enough to melt the varnish, when brought into contact with it, cover the whole plate equally with a thin coat of the varnish: and while the plate is warm, and the varnish in a fluid state upon it, beat every part of the varnish gently with a dauber or ball, made of cotton, tied up in taffaty, which smooths, and distributes, the varnish equally over the plate.

The next part of the process is to blacken the varnish or ground, by holding it over the flame of a flambeau, or of a large candle, which affords a copious smoke: some-

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times two or four candles are uſed together for diſpatch; for the varniſh muſt be blackened before it grows cold: for if it grows cold during the operation, the plate muſt be heated again, that the varniſh be in a melted ſtate when that operation is performed: but great care muſt be taken, not to ſcorch it; which may be perceived, when it happens, by the varniſh looſing its gloſs, and appearing burnt. Large plates are ſometimes ſuſpended from the ceiling, by four cords, with an iron ring about four inches diameter at the end of each cord, to hold each corner of the plate. The plate being thus ſuſpended with the varniſh ſide downwards, may be blackened very conveniently.

In blackening the varniſh the candle or flambeau ſhould be kept at a proper diſtance from the plate, that the wick may not touch the varniſh. If, after the operation, it appears, that the ſmoke has not penetrated the varniſh, the plate muſt be again heated over the chafing-diſh; and as the plate grows hot, the varniſh will gradually melt, and incorporate with the ſmoke that lay above it, in ſuch a manner, that the whole will be equally pervaded by it.

The greateſt caution is neceſſary in this operation, to keep a moderate fire all the time, to move frequently the plate, and change the place of every part of it, that the varniſh may be equally melted every where, and kept from burning, and to keep the varniſh entirely free from any filth, ſpark or duſt, till it be entirely cold.

The method of applying the hard Varnish.

This is exactly the ſame as that of applying the ſoft varniſh; being ſpread equally over the warm plate with the taffety ball, and ſmoaked in the ſame manner; but after it is ſmoaked it muſt be baked, or elſe dried over a gentle charcoal fire, till the ſmoke of the varniſh begins to decreaſe; obſerving not to heat the plate too much, which would burn, and ſoften the varniſh.

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The plate being thus prepared, the outlines of the piece are to be drawn on the varnish, as follows; having an exact drawing of the outlines of the design upon a piece of thin paper, rub the other side of the paper with chalk or Spanish white; or, which is better, with red chalk powdered, brushing all the loose chalk off with a linen rag; then the stained side of the paper is laid upon the varnish, and the corners fixed to the plate with wax, or wafers, to prevent its moving; and the drawing is slightly traced through all the lines with a blunted needle, or pointer, which communicates to the varnish the exact outlines of the design.

The design being thus chalked on the varnish, the next process is to draw the several lines with the needle through the ground upon the copper, observing, that such parts of the plate as is not to be wrought, should be covered with a sheet of white paper, upon which is a sheet of brown paper, in order to rest the hand upon, to prevent its touching the varnish.

In drawing the lines through the ground, as they consist of so great a variety, being some straight and others crooked; some small and others large, there must consequently be used several sorts of needles, bigger or less, as the work requires. The large lines are made by leaning hard on the needle, the point of which should be short and thick; or by making several lines or hatches close to one another, and passing them over again with a thicker needle: or by making them with a large needle, and permitting the aquafortis to lie longer thereon.

When the lines or hatches are to be of an equal thickness from end to end, the needle should have the same force impressed upon it; but where the lines are required to be fine, or small, the needles should be leaned upon lighter; and where they are to be deep or large, the needle should be leaned upon heavier. When the lines are too small, they should be drawn over again with a short round-pointed needle, leaning strongly where they should be more deep.

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The oval-pointed needle is moſt proper for making large and deep ſtrokes; and it ſhould be held in the ſame manner as a pen, with the flat ſide next the thumb; though it may be uſed with the face the other way; and it ſhould be held as upright and ſtraight in the hand as poſſible, ſtriking the ſtrokes freely, and firmly, which renders them neat and clear.

The fine needles, with ſlender points, are proper for fine ſtrokes, and for the faint ſtrokes of thoſe places at the greateſt diſtance in a landſcape; and alſo for thoſe places neareſt the light. And it is requiſite, when at work, to bruſh off all the looſe duſt, which is worked up by the needles.

It is hardly neceſſary to obſerve, that the ſtudent ſhould be ſo far maſter of the Art of Drawing, as to be able to copy any print exactly, before he attempts etching. It is alſo neceſſary that he be able to hatch with a pen or pencil exactly, from good copies; and then he will be able to draw from plaiſter or from the life.

In ſhading his piece, he muſt be careful to obſerve how the original is ſhadowed, how cloſe the hatches are joined, how they are laid, and how they incline, and which way the light falls, which muſt always fall one way. And if the light fall ſideways in the print, that ſide which is fartheſt from the light muſt be hatched the darkeſt.

In landſcapes the part next to the eye is to be hatched darkeſt; and the reſt to decline in its ſhadow gradually, by degrees, the farther it is off from view. The ſame is to be obſerved in etching a ſky; for that which is neareſt the eye muſt have the deepeſt ſhades; but in general, as ſoft and faint as poſſible, gradually loſing its ſhades as it comes nearer to the ground; and where they both meet, as it were, the ſky muſt be entirely loſt.

If any ſcratches, or falſe ſtrokes, happen in the working, they are to be ſtopped up with a hair pencil, dipped in the Venetian varniſh, mixed with lamp black, by which means theſe places will be defended from the aqua-fortis.

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The next operation is that of eating, or corroding the plate with aqua-fortis; in order to which, a border of ſoft wax, formed of bees-wax, melted, and mixed with a little tallow, and Venice turpentine, muſt be raiſed round the edge of the plate, about an inch high, in the form of a little wall, or rampart, to contain the aqua-fortis. A gutter is uſually formed at one of the corners of this border, to pour off the aqua-fortis when requiſite. The border being thus raiſed, take ſome refiners' aqua-fortis, and mix it with half its quantity of common water, and pour it gently upon the plate till it riſe about a finger's breadth above the ſurface of the plate, when it will be ſeen that the aqua-fortis will ſoon exert itſelf in the hatches which have been ſtrongly touched; but thoſe more weakly engraved, will appear at firſt clear, and of the ſame colour as the copper. The aqua-fortis muſt, therefore, be ſuffered to continue on the plate till its effects become viſible on the more tender parts. The aqua-fortis muſt then be poured off, the plate waſhed with clean water, and dried before the fire; and all the lighter parts of the plate covered with the pencil dipped in the Venetian varniſh. The aqua-fortis muſt then be poured on again, and ſuffered to continue a longer or ſhorter time, according to the ſtrength of the menſtrum and the nature of the engraving; when it muſt be again poured off, as before, and the plate immediately waſhed with water.

While the aqua-fortis is on the plate, the verdigreaſe that gathers in the hatches, by the action of the aqua-fortis, ſhould be cleanſed away with a feather, which gives the aqua-fortis more room to exert its action.

When the plate is ſufficiently corroded and waſhed with water, it muſt be warmed at the fire, and the wax border be moved; after which, it muſt be heated, till the varniſh melt; then it muſt be well wiped with a linen cloth, and rubbed with oil of olives, when it will be ready to be retouched and finiſhed by the graver.

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The management of the aqua-fortis is the principal matter in the whole art of etching, and on which the ſucceſs of the work chiefly depends. For the exact ſtrength of the aqua-fortis, the time it is to continue on the plate, &c. no certain rule can be given; but practice and experience alone can inform the artiſt.

Of Etching Letters.

To etch letters, the copper-plate is to have a ground laid upon it of virgin wax, which is to be ſpread very evenly with a feather, all over the plate while it is warm; then the letters being wrote on paper with a black lead pencil, the written ſide of the paper is to be laid upon the ground of the plate, and faſtened at the four corners as before directed. Then rub the back of the paper all over with a burniſher, taking care to rub every part of the paper; and taking the paper off the plate, the letters will all appear written on the wax, but reverſed; which are to be drawn through the wax on the plate with a tracer, cleaning the work from the looſe wax with a linen rag or pencil bruſh; then raiſing a border of wax, and pouring on the aqua-fortis as before, the letters will be etched. The plate being cleaned from the wax, is in the next place to be poliſhed, as follows: take a piece of good charcoal, and pulling off the rind, put fair water on the plate, and rub it with the charcoal; and by this means the plate will be cleared from all the varniſh; but the charcoal ſhould have no knots or roughneſs. After this, waſh the plate with a little aqua-fortis, added to twice its quantity of water: the plate ſhould then be wiped dry; after this, rub a little of the olive oil. Then, if any place require to be touched with the graver, it may be corrected, and the plate will be finiſhed.

SECT. III.

OF ENGRAVING.

ENGRAVING with the tool was the firſt kind of engraving practiſed, and is ſtill uſed for many purpoſes, particularly in portraits, where great regularity and exactneſs in the lines is required; as there every minute part muſt be expreſſed according to the original ſubject, without allowing the artiſt the leaſt indulgence of fancy.

The principal inſtruments uſed in engraving are gravers, ſcrapers, a burniſher, an oil-ſtone, and a cuſhion for bearing the plates.

Gravers are made in ſeveral forms, with reſpect to the points, ſome being ſquare, others lozenge; the ſquare graver is uſed to cut broad deep ſtrokes, and the lozenge for more delicate and fine ſtrokes. La Boſſe recommends a form betwixt the ſquare and lozenge; ſomewhat long and ſmall towards the point, as the moſt generally uſeful.

The burniſher is uſed to aſſiſt in the engraving, on ſome occaſions; and alſo to poliſh the plates. This inſtrument is about ſeven inches in length, and of poliſhed ſteel. The burniſher is formed at one end, and the ſcraper at the other. This alſo ſerves to take out any ſcratches that may happen on the plate; or to leſſen the effect of any parts that may be too ſtrongly marked.

The cuſhion is uſed for ſupporting the plate in ſuch a manner, that it may be turned every way with eaſe; it is formed of a bag of leather, filled with ſand, and ſhould be of the ſize that will beſt ſuit the plate it is intended to bear.

Of holding the Graver.

The graver ſhould not exceed the length of five inches and an half, including the handle, except it be uſed for ſtraight lines; and that part of the handle which is on the ſame

ſame line with the belly, or ſharp edge of the graver, ſhould be cut off flat, that it may be no obſtruction in working.

Hold the handle of the graver in the hollow of the hand, with the fore finger reſted upon the back of the graver, in order that it may be moved parallel to the plate. Great care muſt be taken that the fingers do not interpoſe between the plate and the graver, which would prevent the graver from being carried level with the plate, and render the ſtrokes not ſo clean.

Of laying the Design upon the Plate.

The plate being poliſhed ſmooth, is to be heated, ſo as to melt virgin's wax, with which it is to be rubbed thinly and equally all over, and ſuffered to cool. Then the deſign, to be laid on, muſt be drawn on paper, with a black lead pencil, and laid upon the plate, with the penciled ſide upon the wax; then it is to be preſſed cloſe to the plate, and rubbed over every part with a burniſher. Then taking the paper off the plate, every line drawn with the black lead pencil will appear upon the wax, which are to be traced through the wax upon the plate with a ſharp-pointed tool; the wax being taken off, the plate is to be engraved.

Of whetting and tempering the Graver.

Great care is required to whet the graver nicely; for which purpoſe, the two angles of the graver, which are to be held next the plate, are to be laid flat upon the ſtone, and rubbed ſteadily, till the belly riſes gradually above the plate, ſo that when the graver is laid upon it, the light may be ſeen under the point; if it be not whetted in this ſhape, it will dig into the copper, and it will be impoſſible to keep the point. In order to whet it, place the fore-finger of the left hand upon the graver, and rub it firmly upon the ſtone; then, in order to whet the face, place the flat part of the handle in the hollow of the hand, with the belly

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of the graver upwards, upon a moderate ſlope, and rub the face upon the ſtone till it has a very ſharp point, which may be tried on the thumb-nail, obſerving that the ſtone be ſupplied with oil.

When the graver is too hard, the point will frequently break; it muſt therefore be tempered by holding it upon a red hot poker, within half an inch of the point, till the ſteel change to a light ſtraw colour, then the point is put into oil to cool it: or elſe hold it cloſe to the flame of a candle, till it change colour, and cool it in the tallow; but if it be heated too much, it will turn blue, and it is then too ſoft, and muſt be tempered again. And ſometimes a little whetting will bring it to a good temper.

The dry point, or needle, is now much uſed in engraving, and is a tool like an etching point, which being drawn hard on the copper, cuts a more ſoft and delicate ſtroke than can be made any other way, and the burr which it raiſes is ſcraped off.

All the art of engraving conſiſts in the proper uſe of the graver and dry needle; for which there are no certain rules to be given, depending intirely on the genius of the artiſt. However, a few general obſervations may not be improper. It need hardly be mentioned, that the perſon who attempts engraving ſhould be a good maſter of deſign; and well acquainted with both perſpective and architecture: by which he may be enabled to throw back the figures and objects he deſigns to imitate, and to preſerve the due proportion of the ſeveral orders.

The plate being placed on the cuſhion, the graver muſt be held in the hand, in a proper manner, and moved in the proper directions for producing the lines intended; obſerving in forming ſtraight lines, to hold the plate ſteady on the cuſhion; preſſing more light where they are to be fine, and leaning with greater force where they are to be broad and deep. In making circular and curve lines, turn the plate upon the cuſhion againſt the graver. After ſome of the

the work is done, it is neceſſary to ſcrape off the roughneſs formed by the cutting of the graver, which is done with the ſcraper: or paſſing the graver over the plate in a level direction, taking care that it does not catch the copper. To render the work more viſible, it may be rubbed over with a roll of felt dipped in oil. It is neceſſary to learn to carry the graver as level as poſſible with the ſurface of the plate; for otherwiſe, if the fingers ſlip betwixt them, the line that is produced will become deeper and deeper in the progreſs of its formation, which will prevent making a ſtrokes at one cut that will be fine at the extremities, and larger in the middle, and renders it neceſſary to re-touch it. Therefore it is neceſſary to acquire the habit of making ſuch ſtrokes both ſtraight and curved, by lightning or preſſing the hand, according to the occaſion. And when the deſign is finiſhed, if any ſcratches or falſe ſtrokes appear in any part of the plate, they muſt be taken out by the burniſher.

Though perfection in this art is to be acquired chiefly by practice, yet a few rules for the uſe of the tool, and the dry needle, may not be unneceſſary.

In order to preſerve a due equality in the work, the principal objects of the deſign ſhould be ſketched out, before any of them be finiſhed. In working with the graver, the ſtrokes ſhould never be croſſed too much in a lozenge manner, particularly in repreſenting the fleſh of the human body; except in the caſe of a cloud, waves of the ſea, the ſkins of animals covered with hair, or the leaves of trees, where this method of croſſing may be admitted. In the diſpoſition of the ſtrokes, the action of the figures, and the diſpoſition of their parts ſhould be conſidered: and alſo the manner in which they advance towards, or depart from, the eye of the obſerver. The graver ſhould be ſo guided, as to mark the riſing or cavities of the muſcles, making the ſtrokes wider and fainter in the light, and cloſer and bolder in the ſhades. Thus, the hand ſhould be lightened in ſuch a manner that the outlines may be formed and terminated, without being

cut

cut too hard; and though the ſtrokes break off where the muſcle begins, yet they ought always to have a connection with each other, and in ſuch a manner, that the firſt ſtroke may, by its return, ſerve to make the ſecond, which will ſhew the freedom of the graver.

To produce the effect in engraving the fleſh of any figure, in the lighter and middle tints, round dots, with long pecks of the graver, and ſometimes a few faint lines may be judiciouſly intermixed. In engraving the hair, and beard, the chief ſhades are firſt ſketched, with a few ſtrokes, which may be afterwards finiſhed with finer ſtrokes towards the extremities.

In the repreſentation of edifices and large buildings, which are generally of ſtone, the work ſhould not be made very black, becauſe the colour does not produce very dark ſhades. In imitating ſculpture, the hair of the head or beard, muſt not be repreſented flowing as in nature, becauſe it cannot be ſo in ſculpture. Neither muſt white points be put in the pupils of the eyes, as it is in painting.

In engraving the drapery of figures, regard ſhould be had to the different kinds of cloathing. Linen ſhould be done with finer and cloſer lines than other ſorts, and be executed with ſingle ſtrokes. Woollen cloth ſhould be engraved wide, in proportion to the coarſeneſs, and with only two ſtrokes; and when the ſtrokes are croſſed, the ſecond ſhould be leſs than the firſt, and the third leſs than the ſecond. Shining ſtuffs, ſilks, and ſatin, produce flat broken folds, and ſhould be engraved more hard and ſtraight than others, with one or two ſtrokes, according as the colours are bright or brown; and the firſt ſtrokes ſhould be interlined with ſmaller ones: velvet and pluſh, are alſo expreſſed in the ſame manner.

Metals, as in armour, &c. are alſo repreſented by interlining, by clear ſingle ſtrokes. In architecture, the ſtrokes which form the rounding object, ſhould tend to the point of ſight; and when columns occur, it is proper to produce the effect by perpendicular ſtrokes. When a groſs ſtroke is

made

made, it ſhould be at right angles, and wider and thinner than the firſt ſtroke. In engraving mountains, as there are ſharp and craggy objects, the ſtrokes ſhould be frequently broken; they ſhould alſo be ſtraight, in the lozenge manner, and accompanied with long points or dots. Rocks have ſome croſs ſtrokes in them, more ſquare and even. Diſtant objects towards the horizon are very ſlightly ſhaded, as in drawing. Calm, ſtill waters, ſhould be repreſented by ſtraight ſtrokes, parallel to the horizon, and interlined with finer ſtrokes; omitting thoſe places, where the light caſt a ſhining reflection; and the form of objects reflected from the water at a ſmall diſtance upon it, or on the banks of the water, are expreſſed by the ſame ſtrokes, retouched more ſtrongly, or faintly, as it may be neceſſary. Agitated waters, as the waves of the ſea, have the firſt ſtrokes in the figure of the waves, and are interlined; and the croſs ſtrokes ſhould be very lozenge. The firſt ſtrokes in caſcades, ſhould follow the fall of the water, and be interlined. In Clouds that appear thick, and agitated, the graver muſt be turned every way, according to their form and agitation. In dark clouds, where two ſtrokes are neceſſary, they ſhould be croſſed more lozenge than the figures, and the ſecond ſtrokes ſhould be rather wider than the firſt. The flat clouds that are inſenſibly loſt in a clear ſky, ſhould be formed by ſtrokes, parallel to the horizon, but a little waving; if ſecond ſtrokes be required, they ſhould be more or leſs lozenge, and ſhould be ſo lightened at the extremities, as to have no outline. The flat and clear ſky, is repreſented by ſtrokes, parallel and perfectly ſtraight.

In all landſcapes in general, the trees, rocks, earth, and herbage, ſhould be etched as much as poſſible, leaving nothing to be done by the graver, but the perfecting, ſoftening and ſtrengthening. And obſerve, once for all, that the dry needle produces a more delicate effect, and may be uſed to much greater advantage than the graver can, particularly

in drapery, ſkies, diſtances, ice, and eſpecially in ſmall engravings: And, in general, it is moſt proper to etch the ſhadows only, leaving the lighter tints for the dry point and graver.

To reſemble the grain produced by the chalk, in chalk drawings, it is neceſſary to uſe a variety of mixed and irregular dots: for every ſtroke of the chalk on the paper may be conſidered as an infinite number of points. The drawing may alſo be imitated, and counter-proved upon the varniſh of the plate. But where this cannot be done, black or red lead pencils, or red chalk, muſt be applied to the varniſh, or oiled paper; and thus all the outlines of the original may be tranſmitted to the varniſh. The outlines muſt be formed in the hatching, by the points, whoſe magnitude and diſtance muſt be determined by the quality of the ſtrokes in the original. In forming the light and ſhade, the artiſt ſhould carefully diſtinguiſh between thoſe hatches which expreſs the perſpective, and thoſe which form the ground of it. The principal hatches ſhould be marked more ſtrongly; but the middle tints ſhould be hatched lightly, or rather finiſhed with the dry needle. There is no peculiar method of applying the aqua-fortis in this kind of engraving: but it muſt not be ſuffered to continue ſo long upon the plate as to corrode the lighter parts too much: when the lighter parts are ſufficiently corroded, they may be ſtopped up with turpentine varniſh and lamp-black mixed together; and the aqua-fortis may be applied again in the ſtronger parts: for it will not injure the work if the points which compoſe the ſhade burſt into one another, provided the extreme be avoided. When the varniſh is taken off the copper, it will be neceſſary to interſtipple with the proper points, in the fleſh, and in the ſofteſt parts of the work; for a more delicate effect will be produced by theſe means than it is poſſible to attain with the aqua-fortis only; and at the ſame time the ſtrongeſt ſhades will require to be ſomewhat deepened by the graver. In this manner drawings of chalk, of different colours, may be

be neatly imitated, if a plate be provided for every colour. And if it be well done, it will form ſuch a good deception that an able connoiſſeur cannot, from the firſt inſpection, diſtinguiſh between the original drawing and the engraved imitation: therefore, this mode of engraving is very uſeful to multiply copies of drawings left by ancient artiſts who excel in the uſe of chalks.

SECT. IV.

OF MEZZOTINTO SCRAPING.

MEZZOTINTO prints have no hatchings, or ſtrokes of the graver, but the lights and ſhades are more blended together than in etchings and engravings, and appear like a drawing of India ink.

This art is of late invention, but is greatly uſed, and is admired for the amazing eaſe with which it is executed, particularly by perſons who are deficient in drawing.

The principal tools uſed in this art, beſides thoſe uſed in etching and engraving, are the grounding tool, and the ſcraper.

General Directions for laying the Mezzotinto Ground.

Leave a ſpace upon the bottom of the plate, for the writing, coat of arms, &c. then laying the plate upon a piece of ſwan-ſkin flannel, hold the grounding-tool in your hand perpendicularly upon the plate, leaning upon it moderately hard; then rock your hand in a right line from end to end, and continue the rocking, till the plate be wholly covered with the marks of the tool in one direction: next, work the tool in the ſame manner acroſs the former ſtrokes,

and laſtly, work the tool from one corner of the plate to the oppoſite corner; uſing all poſſible care, not to let the tool cut twice in a place in the ſame direction. The plate will now be full; or in other words, all rough alike: and if an impreſſion were taken from it in this ſtate, it would appear perfectly black.

The ground being laid as above, with a piece of rag, rub the ſcrapings of black chalk all over the plate; or with the flame of a flambeau, or candles, ſmoke the plate as before directed for etching.

Then, take the print or drawing, and having rubbed the back of it with red chalk-duſt, mixed with white lake, proceed to trace it on the plate with a blunt needle or tracer, tracing only the outlines.

Directions for scraping the picture.

With a ſcraper ſcrape off the lights in every part of the plate, as clean and ſmooth as poſſible, and in proportion to the ſtrength of the lights and ſhades, in the original; ſcraping but very little and very lightly, in thoſe parts where the ſhades run deep: and ſcraping more where the ſhades run lighter. Scraping the plate quite ſmooth where there is to be no ſhade at all, uſing great caution not to hurt the outlines, in the operation. A piece of tranſparent paper may be held in the left hand ſloping, juſt over the right hand, by which the artiſt will better ſee the different tints of the work.

The extreme light parts, where there is no ſhade, as the tip of the noſe, forehead, linen, &c. where there is not to be any ſhade, is to be ſoftened or rubbed down with the burniſher; otherwiſe, theſe parts will not appear clear, when the work is proved.

There is alſo another method uſed by mezzotinto ſcrapers: which is, to etch the outlines of the original, with all the folds in the drapery, &c. marking the breadths of the ſhadows

dows by dots; then having uſed the aqua-fortis, as in etching, and the ground being taken off the plate, the mezzotinto ground is to be laid, and the work finiſhed by ſcraping as above.

When the work is to be proved, it is neceſſary to have ſome French paper, which has been wetted down four or five days; as no other paper will do for this work; and it is neceſſary for it to lie wet that length of time: then, when the proof is dry, correct it, by touching it with white chalk, where it ſhould be lighter; and with black chalk where it ſhould be darker. In re-touching the plate, proceed as before, where it ſhould be lighter, by uſing the ſcraper; and where it ſhould be darker uſe a ſmall grounding-tool, as much as is thought neceſſary to give it its proper ſhade. Then it is to be proved again, and again corrected and retouched: and thus proceed to prove, and re-touch it, till it be finiſhed.

It is to be obſerved, that the work ſhould be proved the firſt time before it is the leaſt over-ſcraped in any part; as, by this caution, it will appear more elegant: for the ſmall grounding-tool, which is uſed to deepen any ſhades that are over ſcraped, generally gives the work a coarſe appearance.

Aqua-tinta is that method lately invented of etching, by which a ſoft and beautiful ſhade is given, reſembling a drawing in water-colours, or India ink.

The principal operation, is as follows: The etching ground is to be laid on the plate as in common etching, and the outlines of the deſign etched thereon, as directed before in etching; the ground is then to be ſoftened with a little greaſe, which muſt again be wiped off with a rag, but leaving as much greaſe on the plate as to take off the glare of the copper. Then upon the plate muſt be ſifted through a fine ſieve, the powder, which is made of equal parts of aſphaltum, and fine tranſparent roſin; after which ſtrike the ſide of the plate againſt the edge of the table in order

to ſhake off any looſe powder; then hold the back of the plate over a charcoal fire, till it become ſo hot, as to give pain upon being held on the back of the hand, and the powder, which adhered to the greaſy ſurface of the plate, will now be fixed thereon. When the plate is cold, with a hair pencil dipped in turpentine varniſh, and ivory black, cover all the lights in the plate, or the places where there is no light or ſhade; then raiſe a border of bees-wax round the plate: and having reduced the aqua-fortis to the proper ſtrength by vinegar, or water, pour it on the plate, and let it ſtand five minutes for the firſt or lighteſt ſhade; then pour it off, and having waſhed the plate with water, ſet it on its edge to dry. Then again, with the varniſh, ſtop up the light ſhades, and pour on the aqua-fortis for the ſecond tint, letting it ſtand five minutes more. Proceed in the ſame manner for every tint, till the darkeſt ſhades are produced. If a bold open ground be wanted in any part, it muſt be effected by an after operation; for which purpoſe, the ground muſt be laid on as the other, by ſifting on the powder; but in this proceſs the powder is much coarſer, and the plate muſt be more heated, in order that the particles of the powder may ſpread, and form ſmall circles; even good roſin alone will do for this.

In etching landſcapes, the ſky and diſtant objects muſt be etched by a ſecond operation, and the powder ſhould be ſomewhat finer. When any part of the fore-ground requires to be higher finiſhed, or when there are trees, &c. the plate muſt be entirely cleanſed from the greaſe by bread, and a ground laid, as in common etching: then it may be finiſhed as highly as requiſite, with the needle or point, by ſtippling with dots, and biting up thoſe parts; or by a rolling wheel.

The foregoing method, will only ſerve for prints of one ſingle tint. When different colours are to be expreſſed, there muſt be as many different plates; each plate having only that part etched upon it, which is deſigned to be charged

with

with its proper colour, unleſs (as ſometimes happens) ſome of the colours are ſo diſtant from each other, as to allow the printer room to fill them in with his rubber, without blending them; in which caſe two or more different colours may be printed upon the ſame plate at once. When different plates are requiſite, there muſt be a ſeparate one, having a pin in each corner, to ſerve as a ſole or bottom to the aqua-tinta plates; and the aqua-tinta plates muſt be exactly fitted, having each a ſmall hole in their corners for paſſing over the pins of the ſole; the pins retain the plates in their due poſition, and alſo direct the printer in placing the paper exactly on each plate, ſo as not to ſhift; by which means each tint or colour will be exactly received on its proper place. This is the method practiſed by the Paris printers. Some ſubjects, however, ſuch as landſcapes, may be printed off at once in the different proper colours, by painting theſe upon the plate. In this caſe the colours muſt be pretty thick in conſiſtence, and the plate carefully wiped in the uſual way, after laying in each tint, as well as to be wiped in general, when it is charged with all the tints.

In aqua-tinta plates, it muſt be obſerved, that the aſphaltum and roſin muſt be finely powdered, and well incorporated together, before it be ſifted on the plate; for this purpoſe, it is neceſſary to ſift each of them through a fine muſlin ſieve, ſifting firſt a layer of one on the ſheet of paper, and then a layer of the other, proceeding in this manner till the whole be finely ſifted, and well incoporated together.

This art has been hitherto kept as ſecret as poſſible: but a ſtrict attention to what has been delivered, will enable the practitioner to finiſh his plate with ſucceſs.

Another method of etching in aqua-tinta is as follows:—Having prepared the copper-plate, as before directed, for etching, with the etching-ground laid upon it, etch the outlines of the deſign; then take the ground off, and clean the

plate

plate perfectly well. This being done, ſprinkle ſome gum-ſandrach, finely powdered, very thinly and evenly over the plate; then warm the plate juſt ſufficient to faſten the gum upon it; but great caution is requiſite that the plate be not ſo hot as to melt the gum, if it be, the gum muſt be taken off, and the operation repeated. When the plate is cold, with a ſmall bruſh-pencil, dipped in ſpirits of nitre, go over every part that is intended to be ſhaded, for the firſt time. This operation is to be repeated as often as is neceſſary, to bring the ſhadow to a proper depth. And, note, if the ſpirits be too ſtrong, they may be diluted with a little water.

It is by this method that Mr. P. SANDBY finiſhed ſome excellent prints, which have been greatly admired by the beſt judges of the art.

SECT. V.

OF CRAYON PAINTING.

Of the Materials necessary for Crayon Painting.

TO execute a picture with ſucceſs in Crayon Painting, it is abſolutely neceſſary that the crayons ſhould be ſoft, otherwiſe the artiſt will fail in his performance; therefore the greateſt care ſhould be obſerved in preparing them to prevent their being hard. And in all compoſitions of colours for crayons, flake-white, and white-lead, ſhould be wholly avoided, as the ſlighteſt touch with either of theſe will always turn black; for theſe whites will only ſtand in oils. Therefore, when white is required, I would recommend the ſtudent to make uſe of the common whiting, prepared in the following manner;—

Put

Put ſome whiting in a large veſſel of water, mixing them well together; when this has ſtood about half a minute, pour off the water into another veſſel, and throw the gritty ſediment away; after this water has reſted about a minute, pour it off as before, which will purify the whiting from all dirt and grittineſs. This being done, let the whiting ſettle, and pour the water from it; after which lay it on the chalk to dry, and when dry, it will be fit for uſe, either for making white crayons, or preparing tints with other colours. And, note. If the ſtudent makes the crayons of the whiting immediately after it is waſhed, it is not neceſſary to dry it on the chalk; for it may be mixed with any other colour inſtantly, whereby much trouble will be ſaved. All colours of a heavy or gritty nature, particularly blue verditer, muſt be waſhed in this manner.

The ſtudent muſt alſo be provided with a large flexible pallet-knife; alſo a large ſtone and muller, to levigate the colours, two or three large pieces of chalk, having large ſmooth ſurfaces, to abſorb the moiſture from the colours, after they are levigated; a piece of flat glaſs, to prevent the moiſture from being aborbed too much, till the colours are rolled into form. Theſe implements being provided, the ſtudent may proceed to form his crayons from the following colours:

Reds are formed either from carmine, lake, or vermilion, or a compoſition of two or more of them; though it muſt be obſerved, that it is difficult to procure either good carmine or lake; good carmine is inclined to the vermilion tint, and ſhould be an impalpable powder; a good lake ſhould incline to the carmine tint.

The *Carmine Crayons* are prepared by mixing a ſufficient quantity of good carmine with ſpirits of wine, with the levigating knife upon a grinding-ſtone, till it become ſmooth and even: and the leſs friction produced by the knife the better. Then lay it upon the chalk to abſorb the ſpirits

of

of wine; and be careful that it is laid on in a proper ſhape; for if be laid on too thin, the crayons will be too flat; and if it be laid on too thick, it will waſte the colour by its adhering to the pallet-knife; a little practice will render it familiar.

If the crayons ſhould prove too hard, they muſt be reduced to an impalpable powder in a mortar, and mixed again in the ſame manner as before, till they be ſoft enough for uſe.

The next proceſs is to form the different tints, by mixing the colour with whiting: the proportion to be obſerved, conſiſting of twenty gradations to one; which will be underſtood by the following directions;

Levigate ſome of the ſimple colour with ſpirits of wine, adding thereto one part of waſhed whiting, to three parts of carmine; which, when properly incorporated, may be formed into two parcels, for the firſt gradation. The next gradation is compoſed of equal parts of carmine and whiting, which compoſition may be formed into four crayons. Another compoſition ſhould be formed, conſiſting of one part of carmine, and three parts whiting; of this ſix crayons may be made. The laſt gradation is made of whiting, very faintly tinged with the carmine; of which compoſition eight crayons are uſually formed, making in the whole, twenty crayons. As theſe compound tints are levigated, they are to be laid immediately upon the chalk, that the moiſture may be abſorbed to a proper degree of dryneſs for forming into crayons; which may be known by looſing the greateſt part of its adheſive quality when taken into the hand. When it is brought to a proper conſiſtence it may be laid upon the glaſs, which will not abſorb any more of the moiſture: here they may remain till it be convenient to form them into crayons. And, note: that the pure carmine, when it has no whiting in it, will not bear rolling, but muſt be left on the chalk till perfectly dry.

Lake

Lake Crayons are ſomewhat difficult to form; on account of the harſhneſs of the lake; therefore the ſtudent ſhould obſerve the following particulars in forming theſe crayons: take about half the quantity of the lake intended for the crayons, and grind it very fine in ſpirits of wine; when dry pulverize it; then take the other half and grind it with ſpirits of wine; after which mix it with the pulverized lake, and lay it out directly in crayons on the chalk: thiscolour will not bear rolling. The ſimple colour being thus prepared, proceed with the compound crayons, as before directed in the carmine crayons, and in the ſame degree of gradation.

Vermillion Crayons are formed by mixing the vermillion on the ſtone with the ſpirits of wine, or even ſoft water; after which it may be rolled into crayons. The different tints are produced by mixing the ſimple colour with whiting, according to the proportions given in the carmine. And, note, that theſe crayons will ſometimes be ſo ſoft that they cannot be held in the fingers; but will break and return to powder: which may be remedied by mixing the colour with ſome thin water-gruel, well ſtrained, which will give it ſufficient coheſion.

BLUES are formed of Pruſſian blue, and blue verditer.

Pruſſian Blue Crayons are formed in the ſame manner as the lake crayons; but as the Pruſſian blue is very apt to bind, it is ſomewhat more difficult to be ſoftened than either lake or carmine. It is neceſſary to grind a large quantity of this colour, as it is chiefly uſed in draperies. The different tints may be made according to the fancy of the painter.

Blue Verditer Crayons are ſomewhat more difficult to form, on account of the coarſe gritty nature of the verditer, which requires ſome binding matter to unite it, otherwiſe it will never adhere together. Therefore, to a quantity of blue verditer, ſufficient to form two or three crayons, muſt be added a piece of ſifted plaiſter of Paris, about the ſize of a pea: theſe are to be mixed well together, and the crayons

 formed

formed upon the chalk, the blue verditer being firſt well waſhed. This is a very brilliant blue, and is uſed to a good effect in heightening draperies, &c. The different tints are formed by whitening, as before directed; and are very uſeful to produce thoſe pearly tints in painting fleſh, ſo much admired in crayon pictures. Common water will be ſufficient to mix theſe crayons.

YELLOWS are formed of yellow ochre, King's yellow, and Naples yellow.

The *King's yellow crayons* are formed by levigating with ſpirits of wine, and forming the different tints with whiting, as in the carmine crayons.

Good yellow crayons may alſo be formed with yellow ochre, and Naples yellow ground with ſpirits of wine, which may alſo have their different tints.

Orange crayons are produced by a mixture of King's yellow and vermillion, ground together with ſpirits of wine, and the tints formed as in other caſes.

GREENS are formed by a mixture of yellow and blue in different proportions; but brilliant greens are produced with great difficulty and are therefore generally procured of thoſe whoſe buſineſs it is to prepare them.

Good *green crayons* are formed ſeveral ways, 1. By grinding yellow ochre in ſpirits of wine, and mixing it with Pruſſian blue; then tempering it with the knife, and lay the crayons on the chalk without rolling them.—2. King's yellow, mixed with Pruſſian blue.—3. Brown ochre and Pruſſian blue.—4. Roman ochre and Pruſſian blue, mixed in different proportions, will alſo be found very uſeful, as they may be rolled. Various tints may be produced by the foregoing crayons, to partake, more or leſs of the blue or yellow as found neceſſary.

The brilliant green crayons is abſolutely neceſſary to complete the ſtudent's ſet of crayons; but in the crayons procured from the ſhop, there is generally a mixture of

flake

flake white, particularly in the light green crayons, which crayons will turn black on the pictures if the least damp come to them: though the dark colours will remain perfect. In order to discover whether there be any flake white in the crayons, the following experiment may be made: having bruised the crayon to a powder, mix it with an equal quantity of charcoal-dust; put the whole into a crucible, which must be placed in a fierce fire till the charcoal-dust be consumed; and if the crayon have any flake white in it, the lead will return to its original metallic state.

Browns are originally produced from Cullen's earth, or umber.

Cullen's earth crayons are of a fine dark brown, and several rich tints may be produced from a mixture of this colour with carmine in various degrees: also, this colour mixed with black and carmine, make useful tints for painting the hair. Several gradations may be made from each of these, by a mixture with whiting. Roman and brown ochre also form an excellent colour, either mixed together or compounded with carmine. Whiting, tinged in several degrees, with either of these, will prove very serviceable. Common sea-coal, ground to a fine powder, and mixed with carmine, forms a very fine brown.

Umber crayons are formed in the same manner as the above; only it is necessary to levigate the umber with spirits of wine.

Purples are formed by a mixture of blue and red. Good purple crayons may be formed with Prussian blue, ground with spirits, and mixed with pulverized lake. Also Prussian blue and carmine produce a deep purple of an excellent hue. From either of these compounds various tints may be made, by a mixture with whitening.

Black *crayons* are formed of lamp-black, as no other full black can be used with safety; all others being subject to mildew. But, as lamp-black is liable to great adulteration,

 the

the ſtudent will find it convenient to prepare it himſelf, as follows:

Fix a tin cone over the flame of a lamp, at ſuch a height that the flame may juſt reach the cone for the ſoot to gather within it. When a ſufficient quantity of ſoot is collected, take it out, and burn all the greaſe from it in a crucible. It muſt then be ground with ſpirits, and laid upon chalk to abſorb the moiſture, when it may be rolled into crayons.

Greys are formed of a mixture of lamp-black with whiting, with which a variety of tints may be made. A very good blue-black is alſo formed by black chalk finely ground.

Beſides theſe colours, there are a variety of compound colours formed, from two or more of the foregoing full colours, as, carmine and black, which form a good compound, of which five or ſix gradations ſhould be made; ſome partaking more of the black and others of the carmine; beſides ſeveral tints formed by a mixture with whiting. Cinnaber and black form another very uſeful compound, which has all its different tints. Black and Pruſſian blue alſo forms a compound very uſeful in painting the drapery.

Note, the Cullen's earth, yellew, brown, and Roman ochre, Naples yellow, umber, black chalk, and ſea-coal-duſt, ſhould all be well ground, and waſhed with boiling water, otherwiſe they will injure the picture, as they are apt to throw out a white kind of ſalt.

Of rolling the Crayons.

The colours being prepared, as above directed, the compoſition muſt be cut into a proper magnitude, in order to be rolled into paſtils. The crayon ſhould be formed in the left hand, with the ball of the right hand; firſt formed cylindrically, and then tapered at each end. If the compoſition be

be too dry, dip the finger in water; and if too wet it muſt be laid upon the chalk again to abſorb more of the moiſture. They ſhould be rolled as quick as poſſible; and when finiſhed, muſt be laid upon the chalk again to abſorb the remaining moiſture. When all the crayons of one colour are formed, the chalk, and grinding ſtone ſhould be well ſcraped, and waſhed with water, before they are uſed for another colour.

When a ſet of crayons is completed, they ſhould be ranged in ſome thin drawer, divided into a number of partitions, and diſpoſed according to the ſeveral gradations of light. The bottom of the partitions ſhould be covered with bran, to preſerve the crayons clean and prevent them from breaking.

The box in which the crayons are placed for uſe, and which ſhould be held in the lap, when the ſtudent paints, ſhould be about a foot ſquare, having nine partitions. In the upper corner, on the left hand, the black and grey crayons are uſually placed as they are the moſt ſeldom uſed; in the ſecond partition are placed the blues; in the third the greens and browns; in the firſt partition, on the left hand of the ſecond row, the carmines, lakes, vermillions, and all deep reds are depoſited; the yellows and orange are in the middle partition; and in the next are placed the pearly tints, which being of a delicate nature, muſt be kept very clean, that the different gradations of colour may be eaſily diſtinguiſhed: in the laſt row, the firſt partition contains a piece of linen rag to wipe the crayons with while they are uſing: the ſecond partition holds the pure lake and vermillion tints; and the laſt partition contains all thoſe compounded tints which cannot be claſſed with any colour.

Directions for the Artist.

To arrive at excellence in this art, the ſtudent ſhould be as particular in the outline of the work as in the diſpoſal

of

of his colours; for however juſt the colouring may be, if the work be deficient in ſymmetery and proportion, it will exhibit nothing but a glaring deformity.

The ſtudent being provided with ſome ſtrong blue paper, (the thicker the better) he is to level all the knots in it with a penknife or razor; then, the paper muſt be paſted very ſmooth on a linen cloth, which is previouſly ſtrained on a deal frame; on this the picture is to be executed; but the ſubject ſhould be firſt dead coloured, as it is called. The manner of doing this is as follows: Lay the paper with the dead colour on its face, upon a ſmooth board or table; then, with a bruſh, cover the backſide of the paper with paſte: the frame, with the ſtrained cloth, muſt then be laid on the paſte ſide of the paper, after which, turn the painted ſide uppermoſt, and lay a piece of clean paper upon it; then, by ſtroking it gently with the hand, all the air between the cloth and the paper will be expelled.

By paſting the paper in the frame, the crayons will adhere to it, much better than in any other way, and the ſtudent will be able to give a firmer body of colour and greater luſtre to his piece. The ſtudent is, however, not to proceed with the painting, till the paſte be perfectly dry.

To take a correct Copy of a Picture.

Take a piece of tiffany or black gauze, ſtrained tight on a frame, which lay upon the picture to be imitated, and with a piece of ſketching chalk trace all the outlines of the piece on the tiffany or gauze: then place the tiffany with the chalk lines upon it upon a piece of canvaſs, rubbing it over with a handkerchief, in order to make the chalk lines upon the tiffany adhere to the canvaſs, and there will be an exact outline of the picture upon the canvaſs. This is the method generally uſed by painters, and anſwers very well when the ſubject to be imitated is in oils; but if it be a crayon

crayon picture, the following method muſt be uſed on account of the glaſs.

The picture being placed upon the eſel, draw all the outlines upon the glaſs with a ſmall camel's hair pencil, dipped in lake, ground very fine in oils; then take a ſheet of paper, and place it on the glaſs, ſtroking over all the lines with the hand, by which means the colours will adhere to the paper, which is then to be pierced with pin-holes pretty cloſe to each other in all the outlines. The paper intended for the drawing is then to be laid upon the table, and the pierced paper to be laid upon it; then, with ſome fine powdered charcoal, tied up in a piece of lawn, rub over all the pierced outlines, which will give an exact outline of the piece, upon the paper under it. This is not to be bruſhed off, till the whole is drawn over with ſketching chalk, which is a compoſition made of whiting and tobacco-pipe-clay, rolled like a crayon.

But when the ſtudent paints immediately from life, it is beſt to make a correct drawing of the outlines on another paper, which he may trace by the firſt method: for, if there be any falſe ſtrokes of the ſketching chalk, they will prevent the crayons from adhering to the paper.

The ſitting poſture is the moſt proper for painting with crayons, having the box of crayons in the lap. That part of the picture which the ſtudent is at work upon, ſhould be below his face; for when it is placed too high, it will fatigue the arm. The windows of the room in which the artiſt works, ſhould be darkened to the height of ſix feet from the ground, and the ſubject to be painted ſhould be ſituated in ſuch a manner, that the light may fall on the face to the greateſt advantage, avoiding too much ſhadow, which ſeldom has a good effect in this kind of painting, particularly if the face have much delicacy.

In painting as well as drawing, the ſtudent cannot be too attentive to the ſubject; he muſt alſo learn to appropri-

ate the action or attitude to the subject; thus, if it be a child, let the action or attitude be childish; if a young female, let the figure express more vivacity than in the stately beauty of a middle-aged woman; while a person, far advanced in years, should have a greater degree of gravity. The embellishments of the piece, and the introduction of birds, flowers, animals, &c. should be regulated by the rules of consistency and propriety.

Of the Features of the Face.

The features of the face being carefully sketched with chalk, the student must first carefully draw the nostril and edge of the nose next the shadow, with a crayon of pure carmine: then with the faintest carmine first, lay in the strongest light upon the nose and forehead, which must be executed broad. He is then to proceed gradually with the second tint, and the succeeding ones, till he arrives at the shadows, which must be covered brilliant, enriched with much lake, and carmine, a little broken with brilliant green. These colours will be a good foundation toward producing a pleasing effect when the piece comes to be finished; though at first they will rather offend the eye from their crude appearance; for colours are more easily sullied when too bright, than raised into a brilliant state, when the first colouring is too dull. Those pearly tints discernable in fine complexions, must be imitated with blue verditer and white, which answers to the ultramarine tints used in oils: but when these pearly tints are used in the shade, the crayons composed of black and white must be substituted in the place of the others. And it must be observed, that though all the face, when first coloured, should be laid in as brilliant as possible, yet each part should be kept in its proper tone, which will preserve the rotundity of the piece.

Whatever

Whatever colour the iris of the eyes are, the eyes muſt be firſt drawn with a crayon, inclined to the carmine tint; the colour muſt be laid in brilliant at firſt, and executed lightly, not meddling with the pupil yet. The light of the eye ſhould incline very much to the blue caſt; for if a ſtaring white appearance is once introduced, it can ſeldom be altered: a broad ſhadow ſhould alſo be thrown on the upper part by the eye-laſh. The eye-brows ſhould be executed at firſt like a broad glowing ſhadow, on which are to be painted, in the finiſhing, the hair of the eye-brows, by which the former tints will ſhew themſelves through, and produce a pleaſing effect; but a black heavy tint is always to be avoided in firſt forming the eye-brows.

The lips ſhould be began with pure carmine and lake, ſhading them with carmine and black, and laying on the ſtrong vermillion tints afterwards. Great caution is neceſſary to avoid ſtiff, harſh lines: each colour is to be gently intermixed with the neighbouring colour; the ſhadow beneath ſhould be broad, and enriched with brilliant crayons. The corner of the mouth is formed with carmine, brown ochre, and greens, variouſly intermixed. If the hair be dark, it is neceſſary to uſe a good quantity of the lake and deep carmine tints therein, which may be eaſily overpowered by the warmer hair tints, and which, as in the eye-brows, will produce a richer effect when the piece is finiſhed, than if the lake and carmine be neglected.

When the ſtudent has dead-coloured the head, he is to ſweeten the whole together, by rubbing it over with his finger, beginning at the ſtrongeſt light upon the forehead, and paſſing his finger very lightly to the next tint to unite them together, which he muſt continue to do till the work is ſweetened together, frequently wiping his finger on a a towel to prevent ſullying the colours. In this proceſs the ſtudent muſt be careful not to ſweeten his picture too often, as that would produce a thin and ſcanty effect, and the piece

would have more of the appearance of a drawing than a ſolid painting, as nothing but a body of rich colours can produce a rich effect. Therefore, it is often neceſſary in ſweetening a picture, to repleniſh it with more crayon.

The head being brought to ſome degree of perfection, the back ground is to be laid in. This muſt be performed in a different manner; for it is to be covered as thin as poſſible, and rubbed into the paper with a leather ſtump. Near the face the paper ſhould be almoſt free from colour, which will give the head and face a good appearance. Crayons which have whiting in their compoſition, ſhould not be uſed in the back ground; but with great caution: the chief crayons for this uſe are the moſt brilliant and the leaſt adulterated. The ground ſhould alſo be very thin next the hair, whereby the edge of the hair may be painted over it in the finiſhing touches.

When the face, hair, and back ground, are entirely covered, the ſtudent muſt carefully view the piece at ſome diſtance, remarking in what reſpect it is out of keeping, that is, what parts are too light, and what too dark, marking particularly thoſe parts which have a white or chalky appearance, which muſt be ſubdued with lake and carmine.

The painting now will have the appearance of a painting principally compoſed of three colours. viz. carmine, black and white, which is the beſt preparation that can enter into a compoſition for producing a fine crayon picture.

The back ground, and the hair, is next to be completed; for, if the face be finiſhed firſt, the duſt in painting thoſe will fall on it and injure it. Next, proceed to finiſh the forehead, finiſhing each part in its order, proceeding downwards, till the whole be completed.

A great deal of ſkill is required in forming the back ground; though by young artiſts it is often neglected; for a great part of the beauty and brilliancy of the piece, particularly the face, depends upon the tints being well ſuited and adapted to each other, the darks being kept in their

proper

proper places; and the whole being perfectly ſubſervient to the beauty of the face. This is requiſite even in a ſimple back ground, where there is but one object in the piece; but more attention is required in the back ground of a picture, which has ſeveral objects.

A great variety of colours are uſed for back grounds; but they ſhould always be ſuited to the complexion of the figure. A ſtrong coloured head generally ſhould have a weak and tender-tinted ground; and, on the contrary, a delicate complexion requires ſtrong and powerful tints in the ground, by which proper contraſt between the figure and the back ground, the picture receives great force.

But when ſeveral objects are introduced in one piece, as hills, trees, buildings, &c. the general rule to be obſerved is, that each grand object be diſpoſed ſo, as to contraſt each other, not merely in their forms, but in their colour, light, ſhade, &c. For example: ſuppoſe a figure in the piece, receiving the ſtrongeſt light; and behind this figure, and near at hand, ſuppoſe there be ſtems of ſome large trees: theſe ſtems muſt have ſhade thrown over them, either from a driving cloud or ſome other interpoſing object; behind theſe ſtems or trees, and at a diſtance, ſuppoſe are ſeen trees on a riſing ground; theſe again ſhould receive the light whereby they will ſerve as a contraſt to the former; and the ſame may be obſerved in all other caſes. The ſame rule holds good in an architectural back ground; as, ſuppoſe a building at a moderate diſtance, and behind the figure which receives the light, a column or ſome other object in the ſhade intervenes to preſerve proper decorum in the piece; or what will have the ſame effect, a ſhadow may be thrown over the lower part of the building, which will have an equally good effect. In a word, it muſt always be remembered, that the light muſt be always placed againſt the dark, and the weak againſt the ſtrong; and, *vice verſa*, in order to produce force and effect.

Of finishing the Features.

When the features are to be finiſhed, the forehead is to be painted over firſt, and for the laſt time; beginning the higheſt light with the moſt faint vermillion tint, in the ſame place where the faint carmine was laid at firſt, and keeping it broad in the ſame manner. The next ſhade ſucceeding the lighteſt, muſt be worked with ſome light blue tints, compoſed of verditer and white, intermixing with them ſome of the deeper vermillion tints, ſweetening them together with great caution. (This direction, however, is only to be obſerved in the fineſt complexions, for the ſtudent muſt vary his colouring according to his ſubject.) Some brilliant yellows may alſo be uſed ſparingly; and towards the roots of the hair it is neceſſary to uſe ſtrong verditer tints intermixed with greens. Theſe muſt be followed by cooling crayons, compoſed of black and white, which ſhould gradually melt into the air. Pearly tints are to be preſerved beneath the eyes, and under the noſe, and on the temples:—Alſo beneath the lips; the tints being compoſed of verditer and white, mixing them with light greens, and vermillions.

It is neceſſary to uſe greens and blues in the face, in this kind of painting; (though it may appear ſtrange to thoſe unacquainted with the art,) as they ſerve to break and correct the other colours.

In the dead colouring the carmine ſhould predominate, as that is the beſt preparation for the ſucceeding tints; but the crudeneſs of this preparation renders it neceſſary to be intermixed with greens, blues, and yellows: the degree of carmine, and the complexion of the figure will determine which of thoſe colours are to be uſed. The blue and yellow are diametrically oppoſite to each other; therefore they ſerve to oppoſe one another; and alſo to correct the reds; the greens being compounded of both colours, are of peculiar ſervice, where the tranſition is not to be violent.

In

In finiſhing the complexion, the ſtudent ſhould be particularly attentive to Nature herſelf: for whoever carefully examines a clear and tranſparent ſkin, will diſcover a pleaſing variety of colours on the ſurface and diſcernible, through it, which will be greatly increaſed by the effect of light and ſhade: one part will appear to incline to the vermillion, another to the carmine, or lake; one to the blue, another to the green, and another to the yellow, &c. Now, in order to produce theſe effects, a good artiſt will apply thoſe colours correſponding to the tints, uſing, as often as he can, the compounded colours inſtead of the ſimple colours; as, blue and yellow inſtead of green; blue and carmine inſtead of purple; red and yellow for orange. In all other circumſtances the compounded crayons, already mixed, ſhould be uſed; but in this caſe no abſolute rule can be given; the ſucceſs of the piece depending upon the experience and diſcretion of the artiſt. And, obſerve, that it is impoſſible to give any ſet of rules for forming the complexion that will hold in every caſe, the circumſtances that require different treatments are ſo many and various; but great advantage will be derived in the commencement of this art by an able maſter, to direct the ſtudent, and point out the deformities and beauties of a piece as they occur in practice, which, to a good capacity, will ſoon become clear and intelligible.

In finiſhing the cheeks, uſe the pure lake tint, which will clear them from any duſt they may have contracted from the other crayons, mixing with the lake ſome bright vermillion; and, laſtly, (if the ſubject require it) give a few touches of the orange crayons, but with great caution. This being done, ſweeten the part with the finger, as lightly as poſſible, leſt it produce a heavy diſagreeable effect on the cheeks; for the only method of imitating a beautiful complexion, conſiſts in one colour, ſhewing itſelf through, or rather between, other colours.

The

The eye is next to be executed. This is generally found the moſt difficult feature in the face, as every part muſt be expreſſed with the greateſt nicety, and with the true proportion. The ſtudent ſhould, therefore, uſe his crayons, in ſweetening this feature, as much, and his finger as little, as poſſible. When it is neceſſary to have a point to the crayon to touch a ſmall part with, he may break off a little of his crayon againſt the box, which will give a ſharp corner to it. If the eye-laſhes be dark, he muſt uſe ſome of the carmine, and brown ochre, and the crayon of carmine and black; and with theſe he may alſo touch the iris of the eye, (if brown or hazel) making a broad ſhadow, cauſed by the eye-laſh. The corners of the eye are executed with red tints of vermilion, carmine, and lake: taking care that the eye-lids are not too red, otherwiſe they will have a diſagreeable appearance. The pupil of the eye muſt be made of pure lamp-black. Between the pupil and the lower part of the iris, the light is apt to catch very ſtrong; but it muſt not be made too ſudden, but be gently diffuſed round the pupil, till it be loſt in ſhadow. The eye-balls being ſufficiently finiſhed, the ſmall ſhining ſpeck muſt laſtly be made, with a pure white crayon, which ſhould be firſt broken to a point; the ſpot is then to be laid on firmly: and if it ſhould happen not to be perfectly round and neat, it may be corrected with a pin, taking off the redundant parts, which will render it perfectly neat.

The noſe is next to be finiſhed, in which the chief difficulty conſiſts in determining the lines ſo artfully, and blending them into the cheek, ſo as to give its true projection, and to leave no real line diſcernible, upon the cloſeſt examination. In ſome caſes, it ſhould be quite blended with the cheek, which is to appear from behind it, and determined entirely with a ſlight touch of red chalk. The ſhadow, cauſed by the noſe, is generally the darkeſt in the whole face, and has no reflection from its ſurrounding parts.

The beſt colours for this is carmine and brown ochre, carmine and black, and ſuch brilliant crayons.

The lips being firſt prepared with the ſtrongeſt lake, and carmine, &c. muſt now, with the ſame colours, be made completely correct: and in the finiſhing, have a little of the ſtrong vermilion; but, with great caution, as this colour is very predominant. This colour, if properly uſed, will give the lips an appearance, equal, if not ſuperior, to thoſe executed in oils, notwithſtanding the great advantage the latter has by glazing, of which the former is deſtitute.

In painting the neck, the ſtudent ſhould carefully avoid giving too much expreſſion to the muſcles in the ſtem; and alſo be careful that the bones appear not too eminent on the cheſt, as either of them has an unpleaſing effect, and denotes a violent agitation of the body; which is ſeldom neceſſary in portrait painting. The moſt neceſſary part to be expreſſed, is a ſtrong marking juſt above the place where the collar-bones unite. This ſhould always be expreſſed even in the moſt delicate ſubjects; and if the head be thrown much over the ſhoulders, the muſcle that riſes from behind the ear, and is inſerted into the pit, between the collar-bones, ſhould be faintly marked. But, in general, all inferior muſcles ſhould be quite avoided, and not noticed. For want of this caution, many artiſts, in the portraits of thin perſons, mark the muſcles of the neck too evidently. The neck ſhould, in general, have a ſmall addition to the length, as few necks are too long; and nothing is more ungraceful than a neck too ſhort: the ſtem of the neck ſhould have a pearly hue; and the light ſhould not appear too ſtrong as upon the cheſt. The breaſt alſo (if any part appears) ſhould be expreſſed by pearly tints, but blended with beautiful vermilion in the upper part thereof.

Of Drapery.

The drapery, by many young artiſts, is thought to require very little attention; but this is an egregious miſtake: An eminent painter being aſked, what part of a picture he thought the moſt difficult to execute? he anſwered, The Drapery:—and the beſt judges of the art, have univerſally allowed it to be a very difficult part to execute with taſte. It is not ſufficient that the ſtudent be able to give the effect of ſilk, ſattin, cloth, &c. ſo as to deceive the vulgar eye. This, the ſervile copyiſt may effect by the mere dint of labour, and to ſuch perfection as to make the imitation paſs for reality; but the eſſential attributes of good drapery are to make the folds in ſuch a judicious manner as to give grace and dignity to the figure: to clothe it uninfluenced by prejudice, faſhion, or caprice, ſo as to bear the teſt of ages: theſe it is that require the greateſt exertion of genius, and diſplay all the powers of a refined taſte.

APPENDIX.

NECESSARY RECEIPTS FOR THOSE WHO PAINT IN WATER-COLOURS.

To make Gum Water.

DISSOLVE one ounce of pure gum-arabic, and half an ounce of double-refined ſugar, in a quart of ſpring water: ſtrain it through a fine ſieve or a piece of fine muſlin, and bottle it up for uſe to keep it from the duſt.

Or, Secondly, take ſome of the whiteſt ſort of gum-arabic, bruiſe it, and tie it up in a piece of woollen cloth; and ſteep it in ſpring water, till it be diſſolved. If it be too ſtiff, add more water; and if it be too thin, more gum.

With this water, moſt of the colours are to be mixed; and in ſuch a proportion that the colour may not rub off when dry. If the colour ſhine, it is a ſign there is too much gum in the water.

To make Liquid Gold for painting on Vellum, &c.

Grind ſome of the fineſt leaf gold with ſtrong gum water, very finely, adding more gum water in the grinding as you ſee neceſſary. When it is ground very fine, waſh it in a large ſhell; then temper it with mercury-ſublimate, and bind it in the ſhell with a little diſſolved gum, ſhaking and ſpreading it equally all over the ſhell: when it is uſed, it is mixed with fair water only. The ſhells are alſo ſold, ready prepared, at the colour-ſhops.

To make Liquid Silver for the same Purpose.

The proceſs for this is the ſame with the foregoing; but in uſing this, it muſt be tempered with glare of eggs inſtead of water.

To make the Glare of Eggs.

Take the whites of eggs, and beat them with a ſpoon till they riſe in a froth: let them ſtand all night, and they will be clarified into a good glare.

To make Colours for washing Maps.

Colours proper for waſhing maps are obtained by boiling different kinds of wood, or ſtaining ſubſtances in water, as logwood for purple, cochineal for red, brazil, madder, turnſal, &c.

To keep Water-Colours from sinking.

Wet the back of the print with a ſolution of four ounces of roach allum in a pint of ſpring water, before the colours are laid on; letting the paper firſt be dried from the allum-

water.

water. This will prevent the colours from ſinking, alſo give them an additional beauty and luſtre; and likewiſe preſerve them from fading. If the paper is not good, it ſhould be waſhed three or four times, with the water drying it every time.

To make Size for Painting Scenes, or other Candle-Light Pieces.

When the colours, mixed with gum water, are laid upon any ſurface, they are apt to produce a glare by candle light; to prevent which, the colours ſhould be mixed with the following ſize, while it is warm: Steep a quarter of a pound of the cuttings of white leather for ſome time in water; or for the ſpace of two or three days: then take them out, and boil them in three quarts of water, till it be conſumed to one pint, and ſtrain it through a cloth. If it feel firm under your hand when it is cold, it is a ſign it is of a ſufficient ſtrength.

To lay Mezzotinto Prints upon Glass.

Having a clear plate of glaſs, as ſtraight as poſſible, and a little larger than the print to be laid upon it, ſoak the print in warm water for about an hour, and with a thin, flexible pallet knife, ſpread ſome Venice turpentine, or good varniſh, very thinly and evenly over one ſide of the glaſs, obſerving to keep the glaſs warm that it ſpread the better, and taking care that there be not the leaſt ſpeck in the glaſs uncovered with the turpentine; then take the print out of the water, and ſpread it between two cloths, or ſeveral folds of ſoft paper, in order to abſorb the ſuperfluous water. Next lay the print on the glaſs by degrees, beginning at one end, and ſtroking outward that part which is faſtened to the glaſs that no wind or water may lie betwixt the print

and the glaſs, as that would cauſe bliſters. The print being laid in the moſt exact manner upon the glaſs, it is next to be rubbed with the finger, or a piece of linen cloth, until all the thickneſs of the paper rolls off in ſmall rolls, and nothing is left on the glaſs but a thin film, like a ſpider's web, which will be held faſt to the glaſs by the turpentine. When the print is ſo large that ſome part of it becomes dry, before it be rubbed it ſhould be wetted with a little water on the end of the finger, which muſt be done, as often as is requiſite, to keep the paper moiſt; for when it is dry it will not rub off. Great care is neceſſary in rubbing the paper off, that no holes are made in the print, particularly in the lights, which are the moſt tender parts. When the print is rubbed till it appears tranſparent on the back, it ſhould be ſet up to dry for three or four hours, after which, varniſh it over with turpentine, or maſtic varniſh, two or three times, or till it appear tranſparent; then, it having ſtood a day or two to dry, it is to be painted with ſome of the following colours:

Colours for painting upon Glass.

The colours neceſſary for this proceſs are beſt to be procured at the colour ſhops, prepared in ſmall bladders, at a reaſonable rate; and are generally formed of flake white, lamp-black, umber, vermillion, maſticote, Pruſſian blue, verdigreaſe, &c. ultrarmine for blue, and carmine for red, are beſt kept in powder, as being leaſt liable to waſte in that ſtate, and when wanted for uſe, a ſmall quantity may be mixed up with a drop or two of nut oil, with the pallet-knife; as may alſo any of the other colours, where they cannot be had, ready prepared, from the ſhops.

To get Colours out of the Bladders.

Prick a ſmall hole near the bottom, and preſs the bladder until enough run out, for preſent uſe; for if they ſtand open they are apt to ſpoil.

With theſe colours any tints or ſhades whatever may be exactly imitated by the different ways and methods of mixing them according to judgment.

To use the Colours.

The lighter colours are to be firſt laid on the lighter parts of the print, and the darker colours are next laid over the ſhaded part, and in the regular order, in which the ſhades deepen; for when the brighter colours are once laid on, it is not material if the darker colours be laid a little over them, as the colour firſt laid on, will always hide thoſe laid on afterwards. The colours are not to be laid on too thick; and if any of them be too thick in conſiſtence, they ſhould be thinned before they are uſed, with a little oil of turpentine.

If any of the colours be too ſtrong, or dark, they may be lightened to any degree by mixing more or leſs white with them on the pallet; or if they be too light, they may be darkened to any degree, by mixing them with a deeper ſhade of the ſame colour.

Note. It is neceſſary to have a pencil for each colour; but that pencil which has been uſed for green, ſhould never be uſed for any other colour, without firſt waſhing it well with oil of turpentine, as green will always appear predominant when the colours are dry. And it is alſo neceſſary to waſh all the pencils in oil of turpentine after uſing them.

To wash any Powder very fine for Colours.

Fill a large wine glaſs with clear water, and put therein half an ounce of the colour you intend to waſh; ſtir it well with a knife; then, after it has ſtood about ten ſeconds, in order to let the gritty parts ſettle to the bottom, pour it into another glaſs, and there let it ſtand till the next day; then pour off the water, and the powder will be left very fine, which is to be dried and put up for uſe. Some powders will require a longer time to ſettle in the water, and therefore muſt be permitted to ſtand longer.

To make the best Drying Oil for Painting in Oil Colours.

Boil ſome linſeed oil in which is a little litharge of gold, over a ſlow fire, obſerving that it does not boil too much, otherwiſe it will prove too thick, and will not be fit for uſe.

To make the Turpentine Varnish.

Put one ounce of Venice turpentine into an earthen pot or pipkin, which place over a ſlow fire, and when it is diſſolved add thereto two ounces of oil of turpentine; when they boil take them off the fire; and when it is perfectly cold bottle it up for uſe: for this, as well as all other varniſhes, ſhould be cloſe ſtopped up, to ſecure it from the air. With this varniſh prints on glaſs, or other things, may be varniſhed, in order to render them tranſparent: if the varniſh ſhould prove too thick, it may be thinned with a little oil of turpentine; and if too thin, add a little Venice turpentine.

To

To make the Mastic Varnish.

Put two ounces of the cleareſt gum maſtic, finely powdered, into a bottle, with ſix ounces of oil of turpentine: ſtop the bottle cloſe, and ſhake them well together, in order to incorporate them with each other. Then hang the bottle in a veſſel of boiling water for half an hour, taking it out three or four times to ſhake it. If it be neceſſary to make the varniſh ſtronger, it may hang a quarter of an hour longer in the boiling water.

To make Camp Paper, with which a Person may write or draw, without Pen, Ink, or Pencil.

Mix ſome hard ſoap with lamp-black and water, into the conſiſtence of a jelly; with this mixture bruſh over one ſide of the paper, and let it dry. When you uſe the paper, put it between two ſheets of clean paper, with its black ſide downwards: then with a pin, a ſtick, or any other ſubſtance with a ſharp point, draw or write upon the clean paper; and where the point has touched, there will be the impreſſion upon the lowermoſt ſheet of paper, as if it had been drawn or written with a pen.

This camp paper may be made of any other colour, by mixing the ſoap with different colours.

By this paper alſo any print or drawing may be exactly copied by laying it under the ſame, and tracing the outlines, &c.

The

The Method of taking Impressions from Moulds, Medals, &c. in Plaister of Paris, and thereby counterfeiting the same.

Oil the surface of the mould or medal with a piece of cotton or camel hair pencil, dipped in oil of olives; put a hoop of paper, or pasteboard round the medal, exactly equal to the thickness you would choose your impression to be made: then take some plaister of Paris, and mix it with water, to the consistence of a thick cream, and with a brush lay it on the surface of the mould or medal, and immediately after, lay on more plaister, to make it of a sufficient thickness. By rubbing it on the surface of the medal with a brush, it will entirely prevent any air-holes from appearing on the surface of the impressions. After it has stood about half an hour, it will be so hard, that it may be safely taken off without breaking; then pare it smooth on the back and round the edges, and it will be done. If the weather be cold or damp, it should be dried before a brisk fire. Also, in the operation, when the plaister is laid on to a sufficient thickness, it should be sprinkled with some of the dry powder of plaister of Paris, which makes it harder, and dry sooner. If the face of the medal only, be covered with the fine plaister, the coarser sort will answer to fill it up, which will be a considerable saving. And, note, No more plaister should be mixed at one time than is used, otherwise that which is mixed will thicken and spoil; for, adding water to it to thin it, will totally prevent it from setting hard a second time.

Having taken an impression from a medal, &c. according to the foregoing directions, a plaister mould may be prepared from it, according to the following receipt, whereby a metallic, or wax impression, may be again taken from the plaister mould, so as to exactly resemble the original.

To

To prepare a Plaister Mould so as to take an Impression from it.

Having prepared a plaiſter mould, according to the foregoing receipt, and letting it be quite dry, dip it in the following mixture: half a pint of boiled linſeed oil, and one ounce of ſpirits of turpentine; theſe are to be mixed well together in a bottle, and when wanted, the ſurface of the mould is to be dipped into it, and then ſuffered to dry. When the mould has ſucked up the oil on its ſurface, it is to be dipped again in the oil. This operation is to be repeated till the mould will imbibe no more oil, and the oil begins to ſtagnate upon it: then, with a little cotton wool, rolled up hard, wipe all the looſe oil off the mould, and put in a dry place for a day or two to dry, and the mould will acquire a very hard ſurface from the effect of the oil. When it is to be uſed, it muſt be oiled with oil of olives, in the ſame manner as before directed. By theſe two methods any medal, ſeal, or impreſſion, may be ſo exactly imitated as hardly to be diſtinguiſhed.

The Method of casting Brimstone, and to give it a metallic Gloss.

Melt ſome ſtone brimſtone over the fire, in an iron ladle, and let it flame for about five or ſix minutes, then take it off the fire, and extinguiſh the flame by covering the mouth of the ladle with a piece of board: when it is a little cool ſo as not to feel gluey, or run ropy, it is then fit for uſe, and may be poured into the mould, in which it ſhould ſtand five or ſix minutes and then be taken off; part it as before, and rub the ſurface of the impreſſion over with ſome cotton, and the beſt black lead in powder, which will give it a very fine metallic gloſs.

To make Sulphur red or green, and cast it in Moulds like Marble.

Melt two ounces of the beſt ſtone brimſtone over a gentle fire, without permitting it to flame; when melted, mix it well with one ounce of vermillion. Pour the compoſition over the ſurface of the mould, and immediately pour it off again, and fill up the mould to a proper thickneſs, with common brimſtone; let it ſtand the ſame time as before, then pare and rub over the ſurface with ſome clean cotton, which will give it a poliſh; the more impreſſions there are made at once, melting the brimſtone, the better it will be, becauſe the brightneſs of the red fades the oftener it is melted. If it be required to be green, it is done in the ſame manner, but adding a ſmall quantity of ſmalt inſtead of vermillion; and it requires more ſtirring to make it mix. It may alſo be made to imitate a beautiful marble, by mixing ſeveral colours ſeparately, and made in ſmall ſquares of equal ſizes, which break into exact lengths, and diſpoſe them according to your fancy; after which melt them together, and the colours will unite in a pleaſing manner, and each will appear diſtinct.—And, note, When the brimſtone is melting, be careful not to ſhake it; and ſuffer it to cool by degrees.

FINIS.